A SEGMENTED LOGIT MODEL FOR PRODUCT LINE RESOURCE ALLOCATION

By

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TABLE OF CONTENTS

Page

Abstract of Dissertation Presented to the Graduate School
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Requirements for the Degree of Doctor of Philosophy

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Contemporary package goods firms offer an ever-increasing assortment of brands and brand variants within common product categories. Frequently, several brands or brand-size combinations will be marketed by the same firm. With few exceptions, normative marketing resource allocation models fail to take into consideration the competitive relationships within the set of items marketed by the firm and instead focus on brand-level objectives. Such an oversight could be seen as a limitation in the current state of development of marketing models.

This dissertation seeks to address this limitation by presenting a marketing resource allocation model which is normative with respect to a given set of (possibly) competing brands, and is based on a semi-disaggregate (segment) level brand choice process. The segments in this model are composed of consumers with similar observed purchase sets

of brands. Choice within segment is modeled as a nested process in which consumers first decide whether or not to make a purchase, and then decide which brand to purchase.

The proposed choice model is empirically estimated on Universal Product Code (UPC) scanner panel and store sales data, and the results are compared with the results of aggregate choice models and econometric models of sales. The parameters from the choice model are then used to find optimal levels of price for different objective functions.

Results of the model highlight its usefulness in addressing issues in market structuring and segmentation, brand choice modeling, and resource allocation.

CHAPTER I

INTRODUCTION

The proliferation of brands and brand variants continues to be a major trend for frequently purchased consumer goods (Wind 1982). This fact is underscored by noting the extraordinary number of items marketed within relatively narrowly defined product categories: for example, Progressive Grocer (1982) sampled 20 supermarkets and recorded the number of items stocked within each of 28 categories. In the soft drink category, the survey recorded 189 different items: overall, an average of 46.9 items per category were found. In the ready-to-eat breakfast cereal category, UPC codes exist for over 170 distinct brand names, not counting for the different sizes that are available for each brand. The sheer number of brands in most categories is not only large, but also growing. Estimates of new product introductions in consumer package goods range from 3000 to 9000 per year (New York Times 1987; Advertising Age 1987).

Interestingly, the number of firms marketing this ever-expanding set of brands has not increased. If anything, the current spate of mergers and acquisitions has probably reduced the number of firms competing for individual product markets.

From these observations one might rightly deduce that, in general, a small number of firms have increased significantly the sizes of their brand portfolios, or product lines. The rationale, from the

firm's perspective, of such a tactic is relatively straightforward: by the marketing of a portfolio of differentiated brands, the firm can better satisfy the individual tastes of consumers and thus develop a sustainable competitive advantage (Porter 1980). Such an advantage would theoretically allow the firm to charge higher prices to consumers (due to the lack of equivalent substitutes) and leave the firm less vulnerable to competitive activity than if the firm marketed a single undifferentiated "for everybody" brand. Through differentiation and product line design, the firm achieves economies of "scope" rather than economies of scale (Porter 1980).

Among the first consumer product firms to adopt such a strategy was the Procter & Gamble Company, who introduced Camay soap in 1930 to complement the established Ivory brand (New York Times 1987). To increase the competitiveness of their new brand, P&G instituted a system of brand management. In this system, each brand manager was to act as an internal advocate for his brand and compete in the market as if he were a separate firm. The brand management system quickly became, and remains, the model of organizational structure for almost all large consumer-product companies (Business Week 1987). This decentralized approach to management has several advantages over a centralized decision-making structure from the perspective of the firm. First, the brand manager has a more intimate knowledge of the intricacies of his market due to his proximity to day-to-day market activity. Second, the brand manager's independence allows him to respond quickly and effectively in response to competitive activity. Third, the brand management position is often used as a crucible in which promising middle managers can be evaluated before promotion to higher positions

within the firm. Finally, as brand managers are most often compensated on the basis of brand level performance, their incentives are consistent with the incentives of the firm--outcomes maximizing the brand manager's income are also those outcomes which maximize firm profit. Inasmuch, the brand manager can be considered a "good" agent, and brand level profit maximization is equivalent to firm level profit maximization.

While the brand management system has several appealing features, it can lead to dysfunctional outcomes for the firm (Zenor 1988). The advantages of the brand management system rest largely on the assumption that each brand competes in a separable market, or that each brand is highly differentiated. In a mathematical sense, this approach assumes that the individual demand functions for brands within the firm's product line are independent. When there is competition within the firm's product line, brand outcomes are no longer necessarily consistent with firm outcomes. For example, the manager of brand A might maximize brand A profits by increasing advertising by 10%; however, some proportion of the increased profit is obtained by "cannibalizing" sales that would have normally gone to sister brand B, thereby decreasing B's profits. As the firm accrues profits from both brands, the marketing strategy that is optimal as viewed by the brand manager is not necessarily the strategy that is optimal from the view of the firm.

Seen in this light, the difficulties and drawbacks inherent in the brand management system become manifest. The competing brand managers spend more and more of their effort on the same desired market; advertising and promotion budgets are duplicated and prices decreased in a concerted effort to draw customers away, not only from outside competitors, but from each other. In a recent conversation with the

author, a brand manager at a large dog food manufacturer illustrated such a situation. After several years of intense new product development and heavy promotion, the company's overall share of the dry dog food market had remained stable, suggesting that new brand market share was obtained at least in part at the expense of existing brands. When asked who he considered to be his strongest competition, he named several of the other brands marketed by his company. Despite such strong evidence of significant intraproduct-line competition, the firm in question maintained a decentralized decision-making structure in which brand managers were given latitude over decisions regarding price, promotion and advertising, and were compensated on the basis of brand performance.

An assumption of demand independence can be seen as weak at best for most large consumer product firms. For example, it is reasonable to expect that Diet Coca-Cola and Diet Sprite (both produced by the Coca-Cola company) compete against one another for that segment of the consumer population which prefers diet soft drinks. Many firms have product lines consisting of products which might be considered as substitutes by a significant portion of the consumer populations.

In practice, firms often attempt to mitigate the amount of competition within the product line by differentiating new brands and repositioning existing brands via advertising appeals, changes in ingredients, packaging, etc. This strategy may be successful for some period of time, but it may be limited. First, the heterogeneity of tastes among the consumer population within a category is, to some degree, finite. Attempts to differentiate brands with respect to attributes that are not salient to consumers are unlikely to be successful. Second, the evidence in the brand loyalty literature

indicates that few consumers are loyal to one particular brand (Jacoby 1971; Wind 1977) and that brand loyalty, in general, is declining (Wall Street Journal 1982; Johnson 1982; Business Week 1983). Despite the lack of brand loyalty within category, empirical work has shown that evoked set size is generally small.

Attempts to differentiate not withstanding, recent trends in consumer product marketing may have actually increased the competition within firms' brand lines. Over time, most consumer product firms have shifted the allocation of total marketing budget from advertising to short-term promotional activities such as couponing, price promotion, and trade deals. Bowman (1988) reports that while total advertising expenditures rose at an annual rate of 10% between 1980 and 1987, promotional spending rose at an annual rate of 14% during the same period. While promotional spending often exerts a tremendous impact on short-term sales (Guadagni and Little 1983; Abraham and Lodish 1987; Little 1987), evidence suggests it may tend to inhibit the formation of brand loyalty. Dodson, Tybout, and Sternthal (1978) found that the offer and subsequent retraction of deal prices resulted in less brand loyalty (more switching) relative to flat pricing. One effect of widespread promotion is to inundate consumers with price information, and since less of the budget is devoted to advertising, fewer messages are produced which reinforce real or perceived differences among brands (Johnson 1984). Through a continuous process of cuing only price or deal information, promotion may thus create a consumer choice agenda (Tversky and Sattath 1979; Hauser 1986) where price is the paramount consideration. The recall of price in turn may function to inhibit the recall (and thus the importance) of other brands and non-price

information (Alba and Chattopadhyay 1985). Hence, promotion may have a long-term effect of eroding the real or perceived distinctions between brands within a market, including those brands which are produced by a common firm.

Despite its pernicious long-run effects, promotion will probably continue to be favored by marketers for a number of reasons (Strang 1976; Quelch 1982). First, most firms have incentive structures tied to short-run performance, making promotion an attractive strategy for brand managers. Second, more and more financial power in consumer product marketing has shifted from manufacturers to retailers. Retailers often exercise this power by demanding trade discounts, promotional support, and large "slotting allowances" from manufacturers before allocating shelf space. Third, McAlister (1987) has noted that promotion resembles a prisoner's dilemma in which each player has a substantial incentive to promote, but where all firms would be better off if there were no promotion. Such situations are difficult to rectify except with (possibly illegal) cartel agreements.

Previous Models

These changing aspects of consumer product marketing clearly indicate that the trend toward category management and product line decision making will continue. Despite the changing environment, the academic marketing community has been slow in developing analytic models to assist marketers in making optimal product line decisions. Instead, models within the marketing literature by and large tend to emphasize brand level objectives and decisions. The major feature distinguishing most normative marketing models is the set of decision variables which

are included in the objective function. Hence, most existing models can be characterized as models for setting optimal levels of advertising, price, promotion, or sales force, or models for setting optimal levels of various combinations of these resources.

A neglected dimension of the marketer's decision problem is the set of items in the objective function. The bulk of all published marketing resource allocation models are focused on brand level phenomena and brand level outcomes: models of brand sales generally omit competitive effects altogether. Implicit in such models is the assumption that firms market only one brand per product category, or that within category, objective functions are separable, i.e., the brands marketed by the same firm are neither substitutes nor complements. Such an assumption has been identified as a limitation in the theoretical base of marketing (Wind 1977; Monroe and Della Bitta 1978; Wind and Robertson 1983; Weitz 1985; Wensley 1981).

Much of the concern with intrafirm competition has been expressed with regard to its consequences on strategic planning (Wind and Robertson 1983; Ansoff 1965; Hofer and Schendel 1978; Wensley 1981). Much less attention has been given to the development of product line resource allocation models, though there have been some notable exceptions.

The largest set of models that address product line resource allocation uses a game theoretic approach. The game theoretic approaches typically propose a simplified oligopolistic market and derive optimal behavior for each player under a set of limiting assumptions. For instance, Monroe and Della Bitta (1978) provide a matrix solution for product line prices assuming that both quantity and cost are linearly

related to prices. Hill (1982, 1984) obtains analytically optimal prices for a monopolist with an interrelated product line. Katz (1984) incorporates a consumer population which varies in sensitivity to product quality, and derives optimality conditions for competing multiproduct firms. Katz's results imply that firms do not have an incentive to produce a full spectrum of products even when specialization is inefficient. Moorthy (1984) presents an algorithm for finding the optimal product line when segments are not individually addressable. Moorthy's results suggest that product line variety may have to be curtailed relative to a situation when segments can be addressed separately. Saghafi (1987) examines optimal prices for a firm with two related products for different firm objectives (profit, return to cost, total revenue). While game theoretic approaches are sometimes useful for obtaining general insights into competitive strategy, the models themselves tend to have very restrictive assumptions and are not well suited to empirical estimation. Thus, game theoretic models are of limited benefit to marketing practitioners.

The few empirical models of product line resource allocation that have been presented in the marketing literature fall into two broad categories: those in which interproduct line competition is addressed through the perceptual repositioning of new or existing brands, and those in which repositioning is not an option for the decision maker. In these models, the degree of cannibalization within a product line is at least partially controllable by the firm. Zufryden (1977) first suggested using attribute importances (estimated from conjoint analysis) as criteria for the selection of the best possible new product design. Green, Carroll, and Goldberg (1981) present a model similar to

Zufryden's, except that individual utility functions are not aggregated. The basic idea presented by Zufryden (1977) and Green et al. (1981) was extended by Green and Krieger (1984a; 1984b), who suggest conjoint weights as a means for designing an optimal product line, such that the greatest proportion of the consumer population is served and cannibalization is minimized.

A parallel series of papers, culminating in the Defender model, examines the optimal positioning of new products in a multiattribute perceptual space. Shocker and Srinivasan (1979) were the first to propose the optimal positioning paradigm. Closely related models were proposed by Albers and Brockhoff (1977), Albers (1979), and Gavish, Horsky and Srikanth (1983). The basic approach common to these models is to estimate the distribution of ideal points in the consumer population, propose a choice rule, and examine the predicted choice probability for products occupying different perceptual positions. The position (or positions) resulting in the greatest overall choice probability is then chosen as the optimal position (positions). The Defender model (Hauser and Simmie 1981; Hauser and Shugan 1983; Hauser and Gaskin 1984) is perhaps the most well known of all optimal positioning models. Rather than estimate the distribution of consumer ideal points, the Defender approach involves estimating the tradeoffs consumers make between product attributes. These tradeoffs are represented as vectors in a multiattribute space. Defender links objective product attribute levels to per-dollar perceptual attribute positions. Consumers are assumed to choose the brand with the highest projection onto their tradeoff vector. Hauser and Shugan (1983) use the Defender model to derive optimal defensive pricing and repositioning strategy when a new product is

introduced and consumer tradeoff vectors follow a binomial distribution. By changing either price or product formulation, Defender allows the manager to reposition brands to optimally cope with competitive new product offerings. The logic underlying Defender could be adapted to examine interfirm positioning issues.

Common to all of these empirical models is the assumption that firms can affect the positioning of their brands (and therefore market structure, cannibalization, and cross-elasticities) through changes in product design, advertising messages, or in the case of Defender, price. Because of the changing nature of the consumer package goods environment, it was noted that repositioning is becoming a more and more difficult option for most firms. Further, because of the widespread use of slotting allowances, new product introductions have become an expensive proposition for most firms. Therefore, the recommendations obtained from these models may not be feasible for many marketers, especially in the short run.

The latter category of empirical models (those in which repositioning is not an option) are typified by Urban (1969), Monroe and Della Bitta (1978), Little and Shapiro (1980), Corstjens and Doyle (1981), Reibstein and Gatignon (1984), and Zufryden (1986). These models have a strong econometric flavor and are rather silent as to the issues of market segmentation, brand positioning, and brand differentiation. For instance, in the seminal model of Urban (1969), the sales of each brand in the category are modeled as a function of the price and advertising of all available brands. Urban's model was quite general in that it considered multiple marketing resources simultaneously. Reibstein and Gatignon (1984) empirically estimated Urban's model on

store sales of various sizes of eggs, using price as an explanatory variable. The estimated cross-elasticities from the model were then used to the obtain price levels for each size that jointly maximized the total profit for the entire category. While Reibstein and Gatignon frame the optimization problem from the point of view of the retailer, it is analogous to the problem faced by the product category manager. Zufryden (1986) and Corstjens and Doyle (1981) present shelf space allocation models which take into account the cross-elasticity of space between different brands. The estimated cross-elasticities of space are used to obtain optimal levels of price and shelf allocation for each item. Interestingly, the structure of the demand function used in both Zufryden (1986) and Corstjens and Doyle (1981) is identical to the model proposed by Urban (1969) and implemented by Reibstein and Gatignon (1984).

These empirical models can be seen as tactical in nature as compared to the class of optimal positioning and optimal product design models. In these models, it is generally assumed that the set of products and the degree of cannibalization is fixed (at least in the short run). The decision maker in these models can only address cannibalization through pricing and/or reallocation of shelf space. Thus, these models can be seen as more reflective of the situation facing most brand and product line managers. Unfortunately, these models only do not address issues regarding the heterogeneity of the consumer population. All the above models obtain estimates of aggregate market response, and make no reference to individual differences or segments.

Objectives

As noted above, the allocation models currently available to product line managers have drawbacks which limit their estimability, implementability, and diagnosticity. The objective of this dissertation is to develop a normative model which overcomes these limitations. The proposed model should have a number of characteristics. First, the model should address the allocation of both short-run (price, promotion, shelf space) and long-run (repositioning, new product introduction) marketing variables. Second, the model should incorporate individual differences in consumer tastes and sensitivities. Third, the model should have a general form, such that it is applicable to firms with an arbitrary number of brands and an arbitrary number of competitors. Finally, the model should be estimable using readily available primary or secondary marketing research data.

The goal here is to provide an implementable, normative model which will provide a means by which optimal resource allocation is accomplished at a category (product line) level rather than a brand level. An empirical model meeting these criteria is developed and implemented in this dissertation. Using identified market structure and consumer evoked set information as a framework, the model is shown to have several advantages vis-a-vis current brand level modelling approaches. The model and its properties are presented in Chapter II.

CHAPTER II

MODEL AND METHODOLOGY

In order to provide a more detailed, disaggregate view of competition and cannibalization within a defined product category, this chapter defines a model and methodology which captures segment level differences in choice behavior and relates these differences to aggregate brand performance. This model has as a framework the view that consumers can be segmented on the basis of the composition of their consideration sets, and can be estimated using widely available UPC scanner data.

Consideration Sets and Decision Making

The idea that consumers differ in terms of the set of brands they consider has powerful implications for strategic managerial decision making. Consider a two-brand market in which one segment of consumers always chooses brand A, regardless of the level of marketing activity. In other words, this segment has a consideration set consisting only of brand A. A second group of consumers always chooses brand B, and a third group chooses either brand A or brand B. With knowledge of each consumer's consideration set, the manager for brand A might price discriminate such that those in the first group (strictly brand A loyal) are charged a high price, while those in the third group (potential switchers) are charged a lower price. Further, the manager

might design marketing programs for inducing trial among those consumers who do not currently consider the brand of interest. Thus, by segmenting the market on the basis of consumer consideration sets, the decision maker can tailor marketing resource allocation to each segment to obtain greater revenue and profit.

The consideration set framework for segmentation has even greater implications for the category manager. For example, the firm might use knowledge of consideration set structure to design new brands that will compete only against the brands of other firms and not against its current offerings. The firm could also use knowledge of consideration set structure in deciding which brands to drop from its portfolio so that the effect on total product line performance is minimized. These product line composition decisions can be seen as increasingly important, as growing numbers of brands are competing for limited retailer shelf space. Finally, the firm could use knowledge of consideration set structure to allocate marketing resources more efficiently. For example, it might set prices higher for those consumers who buy only those brands marketed by the firm, and set prices low for those consumers who switch between the firm's brands and brands marketed by competing firms.

The consideration set framework also offers advantages to the retailer. Note that the retailer is, in essence, the ultimate category manager: he is interested in the performance of <u>all</u> brands in the category, regardless of manufacturer. Thus, consideration set knowledge is of value to the retailer in decisions regarding shelf space allocation, promotional support, and pricing. For example, suppose that a substantial number of consumers consider both brand A and brand B, but

no consumers are singularly loyal to either brand. From the point of view of the retailer, short-term price promotion of either brand might be seen as wasteful. Further, the retailer could eliminate shelf facings for one of the brands without affecting category sales.

The idea that individuals might consider only a subset of the available alternatives in a choice situation was originally introduced in the marketing literature by Howard and Sheth (1969), who referred to the subset as the "evoked set." In the original view, the evocation of alternatives was seen as a deliberative, contemplative process that served as a simplifying heuristic for the choice maker. Much of the relevant empirical work in the experimental psychology and consumer behavior literature has focused primarily on how the process of brand evocation is influenced by environmental factors such as situation and context. Usage context, for example, has been found to influence which brands are retrieved from memory (Barsalou 1983; Barsalou and Sewell 1985; Nedungadi 1987). Various researchers have found that the evocation of brands facilitates the evocation of similar alternatives (Tversky 1972; Tversky 1977). The results of Alba and Chattopadhyay (1985) suggest that cueing of a subset of attributes tends to systematically inhibit the recall of particular alternatives.

While many researchers have explored how such contextual factors influence evocation of alternatives, the model adopted for this paper assumes that the consideration sets of individual consumers are stable over time. The consideration set is defined here as the set of alternatives that the consumer actively evaluates in each choice situation. Although this view conflicts with the empirical results discussed above, it is defensible on a number of grounds. First, this

view is consistent with the definitions proposed by earlier researchers (Howard and Sheth 1969; Narayana and Markin 1975; Urban 1975; Silk and Urban 1978; Brisoux and Larroche 1981). Secondly, most of the empirical studies of contextual effects on evoked sets have looked at memory-based rather than stimulus-based choice situations. The ability of contextual effects to influence evocation and choice is likely greater when consumers construct their consideration sets from long term memory. However, most choice situations involving frequently purchased consumer goods more closely resemble stimulus-based choice. For example, a consumer choosing among laundry detergents will typically make the decision at (or near) the point of purchase, where all of the available alternatives are visible. Since most or all alternatives are available for inspection, the consumer is unlikely to rely on long-term memory to reconstruct the alternatives, and inhibiting or facilitating recall cues would have little impact on the choice process. The model proposed in this dissertation is designed specifically for estimation using UPC scanner data for frequently purchased products, which are theoretically less subject to contextual effects.

The most evident application of evoked set consideration in quantitative marketing models is in the market structuring literature. It has been noted that brand partitions derived from market structuring methods resemble the evoked sets or consideration sets of consumer segments (Kalwani and Morrison 1977; Hutchinson and Zenor 1987; Grover and Srinivasan 1987). Unfortunately, market structuring approaches have been designed to provide a static view of brand-to-brand competition, and have as yet to be explicitly linked to consumer choice.

Although the consumer behavior literature has documented the significant role of alternative evocation in influencing consumer choice, it is absent in the structure of most formal choice models in the marketing literature. This is somewhat surprising, as some choice models (most notably those models with a Lucian form) rest on assumptions about evoked set structure. Most choice models incorporating an evoked set or consideration set framework use evocation to describe a iterative process (Tversky 1972; Meyer 1982; Gensch 1987; Nedungadi 1987). In Tversky's well-known Elimination-By-Aspects model (1972), each alternative is represented as a bundle of discrete features, or aspects, some of which may be possessed by other alternatives. At each stage in the choice process, consumers use a random utility model to select a feature, and all alternatives not possessing the feature are eliminated from consideration. This process is continued until only one alternative remains, and this alternative is chosen. In a similar vein, Meyer (1982) presents a model where choice is a process of sequential elimination of alternatives. Nedungadi (1987) presents a model in which the brand choice process is hierarchically divided into two stages: an evocation stage (where brands are retrieved from memory), and an evaluation stage (in which the evoked brands are compared and a choice is made).

These models can be seen as somewhat inappropriate for empirical UPC data, in that a great deal of information is needed at the individual level. In most instances, UPC data contains relatively little individual (or household) choice information, as purchase frequencies tend to be low for most categories (4-20 purchases per household per year).

The view of the choice process adopted in this paper differs from a sequential, iterative framework, in that the consideration set is assumed fixed. Choice is modeled with a relatively simple random utility function over the set of brands each segment considers. There are choice models that do model choice in this fashion, but these models typically rely on self-reports to identify individual consideration sets, and as such cannot be estimated with behavioral data such as UPC scanner data. For example, the Assessor model (Silk and Urban 1978) involves estimation of attraction-type individual choice models for a sample of consumers. Each consumer in the sample provides a self report of the set of brands that they would consider. The goal of the model presented in this paper is to use observed behavioral data rather than survey information to obtain consideration sets.

By defining segments in terms of homogeneous consideration sets, model estimation can provide a detailed view of both market structure and market response. As will be demonstrated, knowledge of the composition and distribution of consumer evoked sets can have a significant impact on firm level performance when the firm markets multiple products.

Model

Consider a product market consisting of N total consumers where each consumer considers a stable subset of the total set of brands on any particular choice occasion, and where consumers with identical consideration sets are homogeneous in their reaction to marketing activity. We then can group the total population of N consumers into X homogeneous consideration set segments. Note that since the segments are

based on consideration sets, the segments simultaneously refer to both sets of consumers and sets of brands. At any particular time, we then can model the sales for brand i as

$$Q_{it} = \sum_{x \in I} N_{xt} P_{ixt} \tag{2.1}$$

where I = the set of segments which have brand i in their consideration set

N_{xt} = the total number of consumers in segment x making a choice at time t

P_{ixt} = brand i choice probability within segment x at time t

In this model, the sales of a particular brand are modeled as a function of two components: a primary demand component (N_{xt}) and a purchase probability component (P_{ixt}). We can further develop the model by specifying the forms for each of the two components: first, for primary demand,

$$N_{xt} = \frac{N_x}{(1 + \exp - \{\gamma_{ox} + \sum_{j \in A_x} \sum_r \gamma_{jrx} Z_{jrt}\})} \tag{2.2}$$

where N_x = the total number of consumers in segment x

Z_{jrt} = value of marketing variable r for brand j at time t

γ_{jrx} = primary demand parameter for marketing variable r, brand j in segment x

Except for multiplication by a constant (N_x), the form of this component is identical to the familiar binomial logit model (Berkson 1944), where the alternatives are purchase or not purchase. Thus, equation (2.2) models the observed total number of purchases made by consumers with consideration set x. In a like fashion, we can develop the purchase probability component of (2.1):

$$P_{ixt} = \frac{\exp(\ \beta_{0ix} + \sum_{r} \beta_{irx} \ Z_{irt}\ )}{\sum_{j \in A_x} \exp(\beta_{0jx} + \sum_{r} \beta_{jrx} Z_{jrt})} \tag{2.3}$$

Which is the well-known multinomial logit model (McFadden 1974; Punj and Staelin 1978; Flath and Leonard 1979; Gensch and Recker 1979; Currim 1981; Gensch 1984), with the probabilities defined within segment (i.e., the set of alternatives consist only of those brands within the segment's consideration set). Because of the particular structure of the proposed model, it will henceforth be referred to as the Segmented Logit Model (SLM).

The SLM can be seen as a natural extension to market partitioning models of brand switching (Bass 1974; Kalwani and Morrison 1977; Grover and Dillon 1985; Hutchinson 1986; Grover and Srinivasan 1987). Using observed aggregate switching behavior as input, these models attempt to identify market partitions. These partitions are usually assumed to represent the evoked sets of a heterogeneous population or the choice process of a homogeneous population.

The SLM has some distinct advantages over similar attraction-type models. First, because it incorporates heterogeneity in consideration sets, it should be less encumbered by violations of the property of independence of irrelevant alternatives (IIA), a critical assumption of logit models (Luce 1959; McFadden 1974). The IIA assumption holds that if an alternative is removed from the choice set, the choice probabilities for the remaining alternatives will be proportional. Violations of the IIA assumption are sometimes referred to

as the "red bus-blue bus" problem. Suppose over some population, the choice probabilities for 3 modes of transportation (red bus, blue bus, car) are 1/3 for each alternative. The IIA assumption states that if the set is restricted to two alternatives (red bus, car), the choice probabilities will be .5 and .5 for the remaining alternatives. However, since the two alternatives (red bus, blue bus) are quite similar, it is likely that those who would have normally chosen the blue bus will chose the red bus in its absence; thus, the true choice probabilities for the restricted set of alternative will be closer to (2/3, 1/3). If consideration set segments are identified correctly, IIA violations will be much less severe in the proposed model. The reader might note that IIA violations are similarly avoided with the hierarchical or nested logit model (McFadden 1974). However, unlike the nested logit, the segmented logit model makes few assumptions about the aggregate structure of the choice process, i.e.,it need not be hierarchical.

Except for the inclusion of the primary demand term (2.2), the proposed SLM is similar in structure to the model presented by Gensch (1984). The major difference distinguishing the SLM model from the model of Gensch is the basis for segmentation. In Gensch's model, segmentation is based on heterogeneity of preferences for the various alternatives. Here, segmentation is based on consideration set. In Gensch's model, consumers are first segmented on the basis of a priori variables such as demographics or geographic region (in his analysis, Gensch looks at industrial consumers). Separate logit models are computed for each segment, and individual-level logit parameters are examined for heterogeneity. However, the set of alternatives in each segment (and individual) level choice model is the full set. Thus, Gench's model

implies that the consumers are heterogeneous with respect to the brands considered. In the proposed model, consideration set is the basis for segmentation, and the set of defined alternatives in the segment level logit models consists of only those brands in the consideration set.

It should be noted that the ability of the SLM to avoid IIA violations is not ecumenical. In fact, there might still be significant violations of IIA at the individual level. Further, the segmentation itself does not guarantee freedom from IIA assumptions. Several choice model forms (Currim 1982) have been proposed in the modeling literature which are not subject to IIA. Unfortunately, empirical estimation of these models tends to be cumbersome, computationally expensive, or impossible for more than a small number of alternatives.

A second advantage of the proposed segmented logit model is that it can test for differences in segment-level response sensitivity.

Noting that (2.1) is defined for individual brands, we can generalize it to specify a sales model for a firm with a product line:

$$Q_t = \sum_{i \in A} Q_{it} = \sum_{i \in A} \sum_{x \in I} N_{xt} P_{ixt} \tag{2.4}$$

where A = the set of brands within the firm's portfolio

Model Properties

Comparative statics of the SLM demonstrate its ability to incorporate product line effects in a parsimonious way. First, consider the objective of a brand manager. The brand manager is concerned with the effect of resource allocation on the performance of his brand, in terms of sales, profits, or market share. Thus, (2.1) represents a

potential objective function of the brand manager. Taking the derivative of (2.1) with respect to allocation of a resource,

$$\frac{\partial Q_{it}}{\partial Z_{irt}} = \sum_{x \in I} \left[\frac{\partial N_{xt}}{\partial Z_{irt}} P_{ixt} + \frac{\partial P_{ixt}}{\partial Z_{irt}} N_{xt} \right] \tag{2.5}$$

where

$$\frac{\partial N_{xt}}{\partial Z_{irt}} = \gamma_{irx} N_{xt} (1 - N_{xt}/N_x) \tag{2.6}$$

and

$$\frac{\partial P_{ixt}}{\partial Z_{irt}} = \beta_{irx} P_{ixt} \ (1 - P_{ixt}) \tag{2.7}$$

Here, the effect of brand i allocation on brand i sales is modeled as the sum of segment level sales effects: further, within each segment, the allocation potentially affects primary demand (2.6) and the share of brand i (2.7). In general, decreases in price and increases in other marketing variables (advertising, promotion, etc.) should have a positive effect on both primary demand and share; thus empirical estimates of γ_{irx} and β_{irx} are expected to be negative when r = price, and positive when r ≠ price. The self effect of resource allocation can be seen as the primary concern of the brand manager.

The concern of the category manager, however, extends over the entire product line. The category manager is not only interested in how allocations to a brand affect its own sales, but also how the sales of other items in the product line are affected. Inasmuch, the product line sales model (2.4) is representative of the objective of the category manager. Taking the derivative of (2.4) with respect to the allocation of a resource to a particular brand,

$$\frac{\partial Q_t}{\partial Z_{irt}} = \sum_{j \in A} \frac{\partial Q_{jt}}{\partial Z_{irt}} = \frac{\partial Q_{jt}}{\partial Z_{irt}} + \sum_{\substack{j \in A \\ i \neq j}} \frac{\partial Q_{jt}}{\partial Z_{irt}} \tag{2.8}$$

Here, the effect of brand i allocation on total product line sales is comprised of the self-sales effect (2.5) plus the sum of the effect of brand i allocation on the sales of other brands in the portfolio. For any other brand in the portfolio, the effect of brand i allocations can be written as

$$\frac{\partial Q_{jt}}{\partial Z_{irt}} = \sum_{x \in I \cap J} \left[\frac{\partial N_{xt}}{\partial Z_{irt}} P_{jxt} + \frac{\partial P_{jxt}}{\partial Z_{ixt}} N_{xt} \right] \tag{2.9}$$

where

$$\frac{\partial P_{jxt}}{\partial Z_{ixt}} = -\beta_{irx} P_{ixt} P_{jxt} \tag{2.10}$$

and other terms are defined as before. Note that the summation in 2.9 extends only over those segments which have both brand i and brand j in their consideration sets. Thus the size of the cross-brand effect in (2.8) determined by the size and sensitivity of those segments which consider multiple items from the firm's product line. If the firm marketed only one brand, or if there was no inter-product line competition (such that no consumer considered more than one brand from the firm's product line), there would be no cross-brand effect of allocation, i.e., (2.8) would reduce to (2.5), and the objectives of the brand managers are consistent with the objectives of the category manager.

The direction of the cross-brand effect is determined by the relative sizes of the primary demand effect and the share effect. For example, suppose that brands i and j are marketed by the same firm and a large number of consumers consider both brands. If allocations to brand i have a significant positive effect on primary demand, but little effect on share, there might be an observed positive cross-elasticity between i and j.

Most previous product line resource allocation models are defined in terms of cross elasticities (Urban 1969; Corstjens and Doyle 1981; Reibstien and Gatignon 1984; Zufryden 1986; Saghafi 1987). In nearly all cases, these cross elasticities are assumed constant. Elasticities in the proposed model have the following form:

$$e^{r}_{ii} = \frac{\partial Q_{it} Z_{irt}}{\partial Z_{irt} Q_{it}} = Z_{irt} \frac{\sum_{x \in I} N_{xt} P_{ixt} W^{i}_{irx}}{\sum_{x \in I} N_{xt} P_{ixt}} \tag{2.11}$$

where $W^{i}_{irx} = (\gamma_{irx}(1 - N_{xt}/N_{x}) + \beta_{irx}(1 - P_{ixt}))$

and

$$e^{r}_{ij} = \frac{\partial Q_{jt} Z_{irt}}{\partial Z_{irt} Q_{jt}} = Z_{irt} \frac{\sum_{x \in I \cap J} N_{xt} P_{jxt} W^{j}_{irx}}{\sum_{x \in I \cap J} N_{xt} P_{jxt}} \tag{2.12}$$

where $W^{j}_{ixt} = (\gamma_{irx}(1 - N_{xt}/N_{x}) - \beta_{ixt} P_{ixt})$

unlike previous product line resource allocation models, elasticities in the proposed model are dependent on the specific level of allocations

for each competing brand. Further, both self-elasticities (2.11) and cross-elasticities (2.12) have a common form, where the marginal elasticity is a weighted linear combination of sales within each consideration set segment. In this way, the structure of the SLM decomposes elasticity by segment. Note that the constant cross-elasticity models provide no information on segment effects; thus the SLM has the ability to obtain diagnostic (segment-level) information unavailable in traditional aggregate cross-elasticity models.

Optimality Conditions

To demonstrate the ability of the SLM to generate optimal values of marketing variables, consider the objective function of the profit-maximizing product line manager:

$$\max_{Z_{irt}} (\Pi_t) = \sum_{i \in A} Z_{ipt} Q_{it} - \sum_{\substack{r \\ r \neq p}} C_r(Z_{irt}) \tag{2.13}$$

where Π_t = Profit at t

Z_{irt} = the value of marketing variable r for brand i at t

$C_r(Z_{irt})$ = the unit cost of marketing variable Z_{irt}

The first order conditions for profit maximization (when r is not price) are then

$$\frac{\partial P_t}{\partial Z_{irt}} = \sum_{j \in A} Z_{jpt} \frac{\partial Q_{jt}}{\partial Z_{irt}} - C_r'(Z_{irt}) = 0 \text{ for all } Z_{irt}$$

$$\frac{\partial P_t}{\partial Z_{irt}} = Z_{ipt} \frac{\partial Q_{it}}{\partial Z_{irt}} + \sum_{\substack{j \in A \\ j \neq i}} Z_{jpt} \frac{\partial Q_{it}}{\partial Z_{irt}} - C_r'(Z_{irt}) \tag{2.14}$$

The first order conditions state that variables Z_{irt} should be increased until marginal costs equal the effect on marginal revenue for all brands in the product line. Contrast these conditions with the first order conditions of individual brand managers: the brand manager will continue spending on variable Z_{irt} until marginal cost equal the effect on marginal revenue for his brand alone. Zenor (1988) shows that a substantial penalty is accrued by the firm when cross-brand revenue effects are ignored in this fashion. In the special case where r = price, the first order conditions for profit maximization become

$$\frac{\partial P_t}{\partial Z_{ipt}} = Q_{it} + \sum_{j \in A} Z_{jpt} \frac{\partial Q_{jt}}{\partial Z_{ipt}} = 0 \text{ for all } i \in A \qquad (2.15)$$

As in the case for non-price variable, the first order conditions consider both the self-revenue and cross-revenue effect of pricing.

Phases in Model Estimation

In order to estimate the proposed segmented logit model on observed UPC scanner data, three phases are necessary. The first and second phases identify the consideration sets of individual consumers. The third phase estimates logit parameters for each segment.

The first stage in estimating the SLM involves derivation of an aggregate discrete market structure matrix, using typical market structuring techniques such as overlapping clustering, hierarchical clustering, or latent class analysis. The discrete structure is represented as an brand by partition rectangular matrix, where the partitions are interpreted as representing the consideration set of some subset of the consumer population. Each cell in the matrix is coded 1 if

the brand is contained within the partition, or 0 if the brand is not contained within the partition.

Once the discrete market structure has been identified, the next phase in model estimation involves the assignment of each consumer to the market partition that best resembles the consumer's observed array of purchases. That is, the set of consumers assigned to each partition are assumed to consider only partition brands during choice. Thus, partitions simultaneously refer to both subsets of brands <u>and</u> subsets of consumers.

While one might use the observed set of purchased brands to directly define the consideration sets of each household, the two-stage process described here is preferred since it is more consistent with the definition of consideration set. The frequency of choice in most UPC categories is typically very low; consumers with large (but latent) consideration sets may not have an observed purchase of a brand that they actually consider. Alternatively, the consumer might make "spurious" purchases of brands outside the consideration set. Thus, the observed purchase set of a household might either underrepresent or overrepresent their actual consideration sets. When households are assigned to predefined market partitions, we might infer that a consumer's consideration set consists of brands A, B, and C, even though the household was only observed to purchase brands A and B.

The final phase in model estimation is the separate maximum-likelihood estimation of logit parameters for each segment, using procedures similar to those used by Gensch (1984). The set of consumers assigned to each partition can be treated as a separate population for which the partition brands represent the set of brands considered.

The next three chapters of this dissertation perform the estimation steps on empirical UPC scanner data. In Chapter III, several well-known methods available for obtaining discrete market partitions are discussed and compared empirically (phase 1). In Chapter IV, a Bayesian method is used to assign each consumer to the partitions identified in Chapter III (phase 2). In Chapter V, maximum likelihood estimates of the parameters of (2.3) are obtained for each segment.

Data Description

Three different types of UPC code scanner files for the light duty detergent (LDD) product category, provided by SAMI/Burke, Inc. were used to estimate the proposed model. The first data set consisted of purchase records for 2312 panel households in the Quad Cities (Iowa-Illinois) metropolitan area. Among the variables included in this file were item purchased, amount purchased, retail store, and date of each purchase occasion. The second data set consisted of weekly UPC code scanner sales data for 14 usable stores, also in the Quad Cities area. In addition to standard sales data, this file also included both normal and special deal price. The third data set contained demographic information on the panel households.

The LDD product category has several appealing features as a candidate for the proposed analysis. First, it contains a small and stable number of brands, as no new brand introductions occurred in the LDD category during the data collection period. In many categories new brands and product line extensions are common; this, by definition, changes market structure. Since no product entry or removal occurred in the LDD category during the period of the observed data, there is at least partial confidence that the market structure is not being affected

in this way. Second, product differentiation is rather slight in the LDD category. In many other available categories, scanner data have recorded class and subclass; for example, in the coffee category, information is as to whether an item is regular or decaffeinated (class) and ground or instant (subclass). In the LDD category, additional classification beyond the category level is not provided. Thus there is an a priori reason to expect that the structure of the LDD market is not extensive in terms of the number of partitions. Third, it is unlikely that different household members have different brand preferences. When this situation exists (as is probably the case for categories such as breakfast cereal), observation of household-level purchase patterns can be extremely misleading as a surrogate measure of individual behavior. It is more likely that for dishwashing liquids, only one household member decides which item to purchase.

The household purchase data covered the period of April 15, 1985, to November 30, 1986, while the store sales data covered the period of April 15, 1985, to April 13, 1986. In order to match the two different periods, only those household purchases made before April 14, 1986, were examined. After deleting observations, the household file contained records of 13,280 purchase occasions.

Preliminary Data Analysis

A particularly difficult step in the analysis of scanner data is in determining the brands, or items, that will constitute the full set of alternatives. The LDD product category proved to be no exception. UPC codes exist for 46 distinct brand names (not counting the various names for store brands) in the LDD category; when package size and color

are considered, nearly 200 specific non-store brand items have UPC codes. When these are added to the hundreds of formulations of store brand and generic LDD's, the number of potential alternatives becomes enormous. To include all or even a moderate fraction of these items in modelling would be difficult if not impossible, given the current state of software development. For example, various market structuring routines (MAPCLUS, OVERCLUS, INDSCAL) either do not accommodate more than 20 to 50 items, or become exponentially more CPU time-intensive as the number of items increases. Further, the derived discrete market structure using this number of brands is likely to identify a large number of partitions. This increases the difficulty of assigning households and reduces the number of household assigned to any particular partition, compared with a less extensive market structure. Finally, estimation of multinomial logit choice models using maximum likelihood becomes exponentially more costly as the number of alternatives (and therefore parameters) grows. Thus, a necessary first step in data analysis was to reduce the number of alternative to a reasonable set.

The number of alternatives can be reduced in two ways. First, whole alternatives can be ignored. For example, all purchase records of 22-ounce Fels detergent might be deleted from the store and household data sets. Secondly, items can be aggregated to define the alternatives. For example, 12-ounce Ivory and 22-ounce Ivory might be treated as the same alternative. The former approach has the disadvantage of reducing the number of observations available for model estimation. The latter has the disadvantage of treating (potentially) different items as if they were identical. The set of alternatives used in this dissertation

were reduced using a combination of both approaches. First, market shares (defined in terms of dollar sales, weight-based volume, and individual unit sales) were analyzed after aggregating sizes within brand name. For store brands, data were further collapsed by aggregating across stores. All three measures of market share were computed from both the household panel data and store sales data. These market shares are presented in Table II-1.

Fortunately, a small number of brands dominated the LDD market; the top 10 brands accounted for over 95% of category market share, regardless of the type of measure. The three Procter and Gamble brands (Ivory, Joy, and Dawn) alone accounted for over 50% of category volume. This allows wholesale deletion of various brands whose total market shares are very small. For example, Trend had a total market share of .02% for all sizes. Deleting this brand, then, would leave remaining data analysis relatively unaffected. In a like fashion, only those brands with panel unit market shares greater than 0.7% were retained. This resulted in less than 1% of the household purchase records being thrown out, and reduced the number of brands to 11.

Each of the brands is marketed in a variety of package sizes (12, 22, 32, 48 and 64 ounce). These specific brand-size combinations will be henceforth referred to as "items." If buyer behavior (in terms of consideration sets) is significantly determined by size, then aggregation of items within brand name would result in model misspecification and lead to erroneous results. Consider the case where a large proportion of the consumer population is loyal to items of a particular size, but switches among brands: analysis of the brand (rather than item) level switching matrix for this market would show

Table II-1

Percentage Market Shares of Each Brand

Brand	Based on store data			Based on Panel Data		
	Units	Volume	Dollar Sales	Units	Volume	Dollar Sales
Dawn	21.79	20.07	22.74	20.46	19.07	21.41
Ivory	18.51	18.36	20.17	18.62	18.86	20.12
Joy	16.23	14.98	16.94	15.28	18.44	16.69
Sunlight	15.25	15.70	14.32	17.68	14.23	15.58
Palmolive	10.81	10.39	10.85	14.86	14.18	14.06
Store Brand	4.34	5.50	3.45	2.59	3.58	2.31
Ajax	3.04	3.02	3.37	3.19	3.24	3.49
Lux	2.72	2.95	2.42	2.61	2.72	2.23
Generic	2.30	2.92	1.06	.83	1.03	.41
Dove	1.98	2.35	2.14	1.30	1.51	1.36
Dermassage	1.69	1.93	1.71	1.78	1.99	1.79
Debbie	.96	1.20	.46	.52	.65	.26
Crystal White	.29	.54	.31	.24	.46	.26
Trend	.07	.06	.05	.02	.01	.01
Nice n Clean	.01	.00	.01	.01	.00	.01
Cinch	.00	.01	.01	.00	.00	.01
Class	.00	.01	.00	.00	.00	.00

little structure, or a structure determine mainly by the sizes offered by each brand.

A further point to consider when defining the set of alternatives is that non-advertising marketing activities often occur at a level below brand name. For example, price-off coupons will generally specify the size or sizes of the brand for which it may be redeemed. Certain package sizes of a brand might be given a price promotion while the other sizes are kept at regular price levels. Even at normal, non-promoted levels, unit price varies over sizes of a brand. This type of item-level marketing activity either creates size-dependent switching patterns, or signals that marketers believe that size is an important variable in determining choice.

Thus, the 11 significant brands revealed by the analysis of market share might be an inadequate set in determining the underlying market structure for the LDD product category. To decide whether the set of alternatives should be expanded to include sizes within brand, a second analysis was conducted, focusing on item-level marketing activity. In this analysis, the correlations of marketing variables (price, feature advertising, and display) between sizes of the same brand within stores were computed. The correlations revealed that for a given brand, there was little association between the marketing activity of specific sizes. In other words, knowing that the 22-ounce package of a particular brand was being promoted did not help predict whether any other size of that brand were also being promoted. Such a finding casts doubt on an analysis that disregards size information in defining alternatives. Inasmuch, the original set of 11 brands was expanded to a set of 29 size-specific items. This set of items is summarized in Table

II-2. All but three of the brands (Dove, Generic, and Store Brand) were disaggregated. For the remaining three, total brand sales were dominated by one size (32 ounce) while sales of other sizes were relatively insignificant. Thus, while each of these alternatives represents all sizes of a particular brand, each is closely associated with a specific size. These 29 items defined the set of alternatives on which all modeling took place.

Table II-2

UPC Items

Item	Manufacturer	Brand	Size
1.	Procter & Gamble	Ivory	12
2.		Ivory	22
3.		Ivory	32
4.		Ivory	48
5.	Procter & Gamble	Joy	12
6.		Joy	22
7.		Joy	32
8.		Joy	48
9.	Colgate-Palmolive	Ajax	22
10.		Ajax	32
11.	Colgate-Palmolive	Dermassage	22
12.		Dermassage	32
13.	Colgate-Palmolive	Palmolive	12
14.		Palmolive	22
15.		Palmolive	32
16.		Palmolive	48
17.	Lever Bros.	Dove	all
18.	Lever Bros.	Lux	22
19.		Lux	32
20.	Procter & Gamble	Dawn	12
21.		Dawn	22
22.		Dawn	32
23.		Dawn	48
24.	Lever Bros.	Sunlight	12
25.		Sunlight	22
26.		Sunlight	32
27.		Sunlight	48
28.	Various	Store	all
29.	Various	Generic	all

CHAPTER III

IDENTIFYING DISCRETE MARKET STRUCTURE

The first phase in estimating the proposed segmented logit model is the identification of the aggregate discrete market structure (brand by partition) matrix. This matrix will later serve as a template for assigning each household in the panel to a segment. Partitions here refer to subsets of all the available items, and segments refer to subsets of consumers. Since the desired segments are homogeneous with respect to consideration set, the partitions define subsets of both items and consumers. In this chapter, different analytical techniques for obtaining market partitions are discussed and tested on the empirical LDD data. This phase is necessary, as the identified partitions will act as a template for categorizing consumers into consideration set segments.

Clustering Methods

While there are a variety of methods for obtaining aggregate representations of market structure (i.e., market partitions and/or market segments), each can be seen as a type of cluster analysis. Clustering methods, in general, seek to group objects or stimuli such that the resulting groups are homogeneous within and heterogeneous between with regard to some criterion or criteria. Clustering methods

can be seen to differ in terms of the objects they classify, the type of data used, and the restrictions placed on group membership.

In marketing applications of clustering methods, the objects for which classification is sought are either brands or consumers. When the objects are consumers, the resultant groups are typically referred to as segments, and the approach is called market segmentation. When the objects are brands, the resultant groups are often referred to as market partitions. Most studies falling into the category of market structure analysis seek classification of brands.

The types of measures used in marketing applications of cluster analysis differ depending on the objects being classified. If the objects are consumers (as in segmentation studies), then a wide variety of multivariate data can used. The most typical types of multivariate measures on each consumer are demographic variables (such as age, geographical location, and race), socioeconomic variables (such as income, social class, and education), psychometric variables (such as lifestyles, activities, interests, and opinions), and marketing response variables (such as brand attitudes, attribute importance or conjoint part-worths, perceptions, and individual price elasticities). For marketers, marketing response variables are thought to be a better criterion for clustering consumers as they are more directly associated with behavior (Frank, Massy, and Wind 1972). If the objects are brands, the data can be multivariate measures of the amount of each attribute possessed by the brand. In market structuring studies, a more typical type of measure is an index of the similarity of dissimilarity of the brand with other brands. Similarity measures can be found through direct interrogation of consumer perceptions, or constructed from observed

individual behavioral data such as brand switching (Lehmann 1972; Rao and Sabavala 1981) or aggregate behavioral measures such as pairwise cross-elasticities (Carpenter et al. 1988; Cooper 1988). When the similarity measure is based on observed behavior, it is usually assumed that a consumer's brand switching or brand copurchase indicates that the consumer considers the brands to be substitutes.

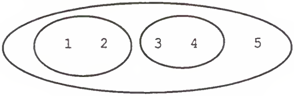
Finally, clustering methods differ in terms of the restriction that each places on group membership. Clustering methods can be seen as either disjoint, hierarchical, overlapping, or probabilistic (Everitt 1980). Disjoint clustering methods find groups of objects such that each object is assigned to one (and only one) group, and no overlap in membership is allowed. A example of a disjoint clustering representation is presented in Figure III-1(b). Disjoint clustering solutions can be found using a variety of methods, including k-means clustering (MacQueen 1967), truncated hierarchical clustering, or factor analysis with an oblique rotation.

One particular advantage of k-means clustering is that it can be used to jointly obtain market structure and segment membership (see subsequent discussion of "sticky" clusters). For example, Ramaswamy (1989) uses each household's vector of purchase probabilities (across brands) as input for a k-means clustering. The clustering technique seeks segments of households which have similar purchase probabilities or "share of requirements": the similarities between households are based on a least-squares criterion (Sarle 1982). After segmentation, the purchase probabilities within each segment give some indication of the most preferred brands for the households assigned to each segment. Thus,

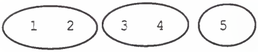
Figure III-1

Clustering Representations

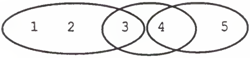
(a) Hierarchical Clustering



(b) Disjoint Clustering



(c) Overlapping Clustering



(d) Probabilistic Clustering

disjoint clustering can obtain both market partitions (brands preferred by each segment) and segment membership of each household.

Hierarchical clustering seeks groups such that one group may be completely contained within another group, but no other type of overlap is allowed. An example of a hierarchical cluster representation is presented in Figure III-1(a). Originally developed in the psychometric literature (Johnson 1967), hierarchical clustering has been frequently employed in market structure studies. For example, Rao and Sabavala (1981) use hierarchical clustering to derive a structure for the coffee market. The Hendry model (Kalwani and Morrison 1977) tests different a priori hierarchical representations account for switching within a market. Similar approaches using hierarchical clustering can be found in Grover and Dillon (1985).

A third type of clustering method is overlapping clustering. In overlapping clustering, objects can belong simultaneously to more than one group, and any type of overlap is allowed. An overlapping clustering representation is presented in Figure III-1(c). Overlapping clustering methods can be seen as structurally less restrictive than either hierarchical or disjoint clustering.

Empirical examples of overlapping clustering in the marketing literature are somewhat limited, owing to its recent introduction. Arabie, Carroll, DeSarbo, and Wind (1981) were the first to outline potential applications of overlapping clustering in marketing research. Further applications were provided in Srivastava, Alpert, and Shocker (1984). Applications of overlapping clustering techniques on empirical UPC include Grover and Srinivasan (1987), Hutchinson and Zenor (1987), and Parry and Gengler (1988). Both Grover and Srinivasan and Parry and

Gengler use latent class analysis (Goodman 1974) to obtain overlapping clusters of brands. Hutchinson and Zenor, like Arabie, et al. and Srivastava et al., use the MAPCLUS program (Arabie and Carroll 1980) to accomplish the same task. The basic difference between the MAPCLUS approach and the latent class analysis approach is in how degrees of freedom are "spent". The latent class analysis approach is somewhat more flexible in determining equilibrium brand shares within partitions, but is limited in the number of partitions that can be found. The MAPCLUS approach is more flexible in finding partitions, but makes limiting assumptions about brand share within partition. For example, Grover and Srinivasan obtain market structure for instant coffee consisting of only 5 partitions, but where brand share within partition is relatively flexible. Hutchinson and Zenor (1987) obtain a structure for the coffee market (ground and instant) consisting of 100 partitions, but where share within partition is assumed to follow a proportionality rule.

A final type of clustering method is probabilistic clustering. In probabilistic clustering, each object's group membership is a matter of degree; i.e., the membership of an object in a group is represented by a probability. The overlap between groups or objects is, by definition, rather arbitrary. Thus probabilistic clustering can be seen as having the most flexible structure of all clustering methods. Some popular scaling techniques in marketing research produce what might be loosely interpreted as probabalistic clusters or partitions. For example interobject distances computed from multidimensional scaling, distance-to-centroid measures from disjoint clustering, or factor loadings from factor analysis might all be thought of as an indices the strength of an object's association with a group.

Discussion

The various empirical methods discussed vary in many ways, and each has been used by a number of researchers to determine market structure. Unfortunately, there has been little research as to which method is most preferred. In the vast majority of market structuring research, only one type of model (disjoint, hierarchical, overlapping or probabalistic) has been applied. To date, Hutchinson and Zenor (1987) is the only paper to use a variety of structuring approaches to obtain market partitions. Hutchinson and Zenor applied both hierarchical and overlapping clustering on normalized brand switching data from the coffee category. The empirically revealed partitions from both methods were combined with managerially meaningful a priori market partitions. A weighted least squares regression approach was then used to estimate the size of each of the partitions.

While the results of Hutchinson and Zenor demonstrate that different methods can result in different partitions, it is unclear which method is preferred. Indeed, testing the validity of structuring/ clustering methods is theoretically very difficult, since the "true" structure is unobservable: without the true structure, different structural representations can only be compared in terms of fit to the original calibration data.

The objective of this dissertation with regard to structure is to obtain market partitions which are significant both in size and in interpretability. In order to find the "best" set of partitions, an approach similar to Hutchinson and Zenor (1987) will be applied here, using both a priori knowledge and a wide variety of market structuring methods to obtain a number of possible partitions. These partitions will

then be tested to obtain a final "hybrid" structure, where the final partitions may be identified by any of the structuring/clustering methods.

A Priori Partitions

As a first step in obtaining market partitions, managerial judgment can be used to define market partitions. In many categories, meaningful market partitions can be defined in terms of product attributes. For example, in the coffee category, many researchers have assumed "natural" partitions which consist of either decaffeinated or caffeinated brands (Kalwani and Morrison 1977; Grover and Dillon 1985; Hutchinson and Zenor 1987). The hypothesis of such a priori partitions is strengthened by the fact that scanner tapes for the coffee category have class and subclass codes indicating whether the brand is caffeinated or decaffeinated, and whether the brand is ground, freeze dried instant, or spray dried instant, as well as the size of each UPC item. Unfortunately, there are no strong a priori attribute-based brand partitions in the LDD product category. This is borne out by the fact that no a priori class or subclass codes are present on the LDD category tape: thus, the research supplier does not make any further subclassification below the category level. However, since the items used in this analysis are defined below the brand level, both size and brand can be used to define a number of a priori partition of items.

Four types of a priori partitions were assumed for the LDD category. First, 29 single-item loyal partitions are assumed, corresponding to the 29 items. These partitions represent the consideration sets of consumers who would only buy the item contained in

the partition. A second class of a priori partitions are single-brand loyal. These partitions represent the consideration sets of consumers who buy only one brand, but consider all sizes of that brand. There are 8 possible single-brand loyal partitions, corresponding all brands marketed in more than one size (Ivory, Joy, Sunlight, Palmolive, Lux, Ajax, Dawn and Dermassage). The third class of a priori partitions are single-size loyal. These partitions represent the consideration sets of consumers who buy only one size of LDD, but consider all possible brands marketed in that size. There are 4 such partitions corresponding to the 12, 22, 32 and 48-ounce sizes marketed within the LDD category. The final a priori partition is the "all brands" partition, which represents the consideration set of consumers who consider all possible items.

Revealed Partitions

Three different empirical clustering methods were employed to identify candidate partitions not represented by the a priori partitions. The empirical methods included versions of hierarchical, overlapping and disjoint clustering. Both hierarchical and overlapping clustering use indices of similarity between items as input data. Aggregate pairwise similarity matrices were obtained by first computing aggregate (across household) switching matrices for all households who made more than one switch during the 12-month period. The purchases of non-switching households were eliminated because the clustering methods are meant to identify the consideration sets of households who consider multiple items. Different switching matrices were obtained for three different time periods: first six months, second six months, and for the entire year. The split-half six-month time periods were used to test

whether market structure in the LDD category had changed significantly during the entire 12-month period. If the structure found in the first six-month period differs significantly from the structure found in the second six-month period, then the assumption of stable market structure (and thus stable consideration sets) becomes somewhat tenuous.

Since the clustering methods used in this analysis assume symmetric data, the item switching matrices were was then symmetrized using the following formula:

$$SF_{ij} = (F_{ij}+F_{ij})/2 \qquad (3.1)$$

where SF_{ij} = symmetrized switching frequency

F_{ij} = observed switching frequency

This symmetrization is equivalent to the one used by Lehmann (1972).

It is tempting at this point to conduct the cluster analyses using the raw, symmetrized switching frequencies or joint probabilities as a similarity measure (Lehmann 1972). However, the raw frequencies will tend to bias clustering solutions such that the clusters contain large market share brands. For example, there might a large number of switches between two dissimilar but high share items, and a small number of switches between two similar low market share items; in such a case, both overlapping and hierarchical clustering will tend to identify partitions containing only high market share items. To reduce the influence of market share, researchers will typically normalize the observed switching frequencies or probabilities. Different researchers have employed different normalization procedures and have justified them on both intuitive (Day, Shocker, and Srivastava 1979; Lehmann 1972; Rao and Sabavala 1981) and mathematical (Bass 1974; Hutchinson and Zenor 1987) grounds.

The simplest normalization technique is to divide each element in the symmetrized switching matrix by the product of the marginal sums, which is equivalent to normalizing by market share (Kalwani and Morrison 1977):

$$S(MS)_{ij} = SF_{ij}/(SF_{i.})(SF_{.j}) \qquad (3.2)$$

where $SF_{i.}$ = the marginal row sum for item i

$SF_{.j}$ = the marginal column sum for item j

$S(MS)_{ij}$ = switching normalized by market share

A second way of normalizing the symmetrized switching matrix is to divide each element by the product of the square root of the market shares (Bass 1974):

$$S(SRMS)_{ij} = SF_{ij}/((SF_{i.})^{.5}\ (SF_{.j})^{.5}) \qquad (3.3)$$

where $S(SRMS)_{ij}$ = switching normalized by square root of market share

A third method for normalizing the symmetrized switching frequencies is to divide each element by the product of the square root of the switching matrix major diagonal (Hutchinson and Zenor 1987):

$$S(SRMD)_{ij} = SF_{ij}/((SF_{ii})^{.5}\ (SF_{jj})^{.5}) \qquad (3.4)$$

where $S(SRMD)_{ij}$ = switching normalized by square root of major diagonal

All three normalization approaches reduce the biasing effect of the overall market share of each brand. For example, the aggregate frequency

of switching between two high market share (but dissimilar) items might be higher than the frequency of switching between two similar low market share items. The normalization techniques scale the switching frequencies so that the normalized switching between i and j represents the observed switching as a proportion of the switching expected under a null hypothesis of switching independence.

Hierarchical Clustering

The PROC CLUSTER procedure of SAS was used to obtain average-distance hierarchical clustering solutions for each of the normalized switching matrices. The resulting dendrograms obtained from each switching matrix are presented in Figures III-2 through III-10.

It can be seen from the dendrograms that brand and size account for a great deal of the structure found through hierarchical clustering. For medium size items (22 and 32 ounce sizes), brand name appears to explain most of the clustering. For example, in Figure III-10, the first 4 pairs of items joined are the medium sizes of Ivory, Palmolive, Dawn, and Sunlight, respectively. The importance of size in explaining the hierarchical clusters seems to be greater than brand for small and large sizes of LDD items. For example, in Figure III-9, the 12-ounce items cluster more strongly with each other than do items with common brand names. The same pattern seems to hold for the 48-ounce items.

The exact tree structure did exhibit some difference between time periods and between normalization technique. For example, the dendrogram obtained from the first six months of data normalized by

Figure III-2

Dendrogram of the Hierarchical Solution for the First Six Month Switching Matrix Normalized by the Product of Marginal Market Shares

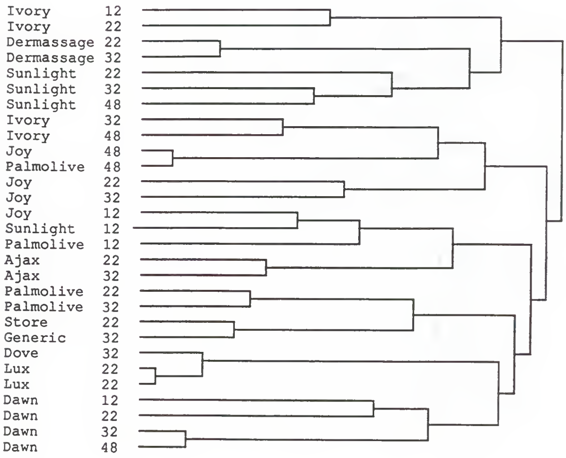


Figure III-3

Dendrogram of the Hierarchical Solution for the First Six Month Switching Matrix Normalized by the Square Root of Market Shares

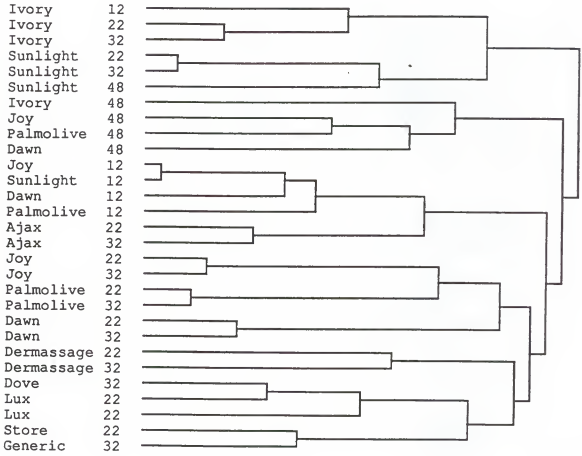


Figure III-4

Dendrogram of the Hierarchical Solution for the First Six Month Switching Matrix Normalized by the Square Root of Major Diagonals

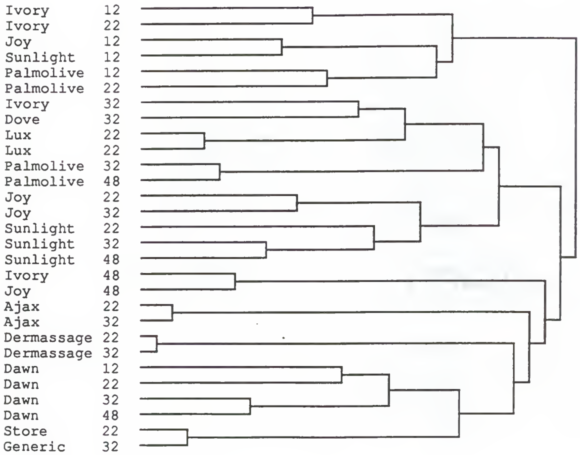


Figure III-5

Dendrogram of the Hierarchical Solution for the Second Six Month Switching Matrix Normalized by the Product of Marginal Market Shares

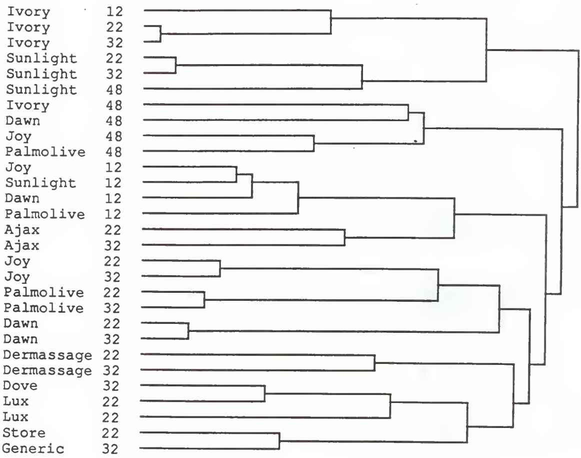


Figure III-6

Dendrogram of the Hierarchical Solution for the Second Six Month Switching Matrix Normalized by the Square Root of Market Shares

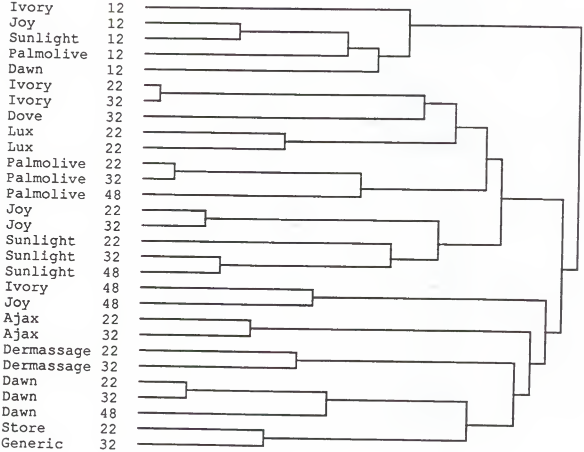


Figure III-7

Dendrogram of the Hierarchical Solution for the Second Six Month Switching Matrix Normalized by the Square Root of Major Diagonals

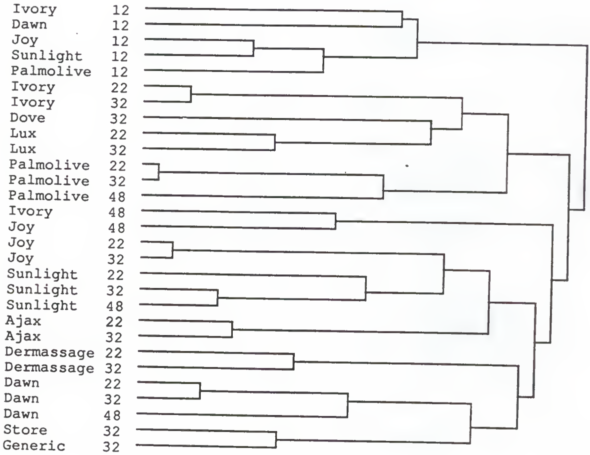


Figure III-8

Dendrogram of the Hierarchical Solution for the Twelve Month Switching Matrix Normalized by the Product of Marginal Market Shares

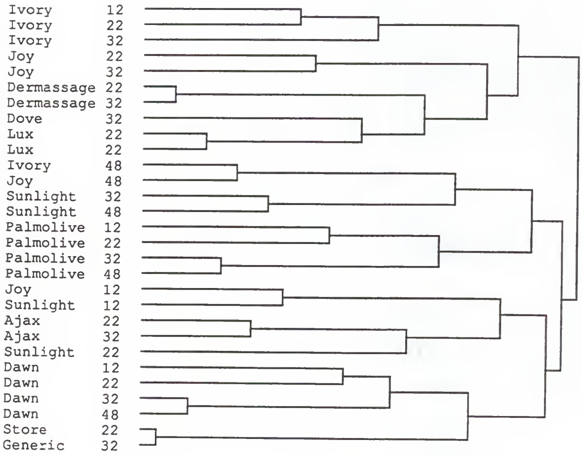


Figure III-9

Dendrogram of the Hierarchical Solution for the Twelve Month Switching Matrix Normalized by the Square Root of Market Shares

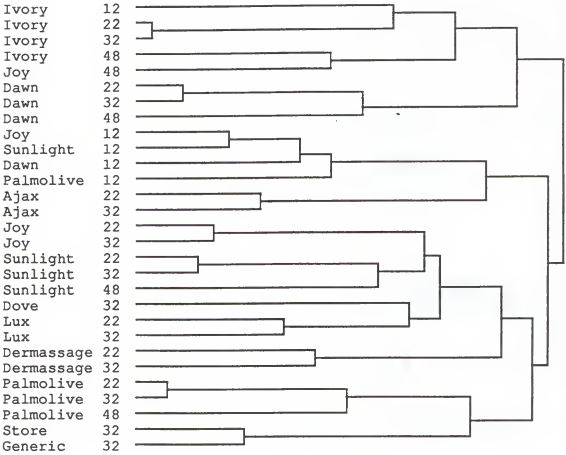
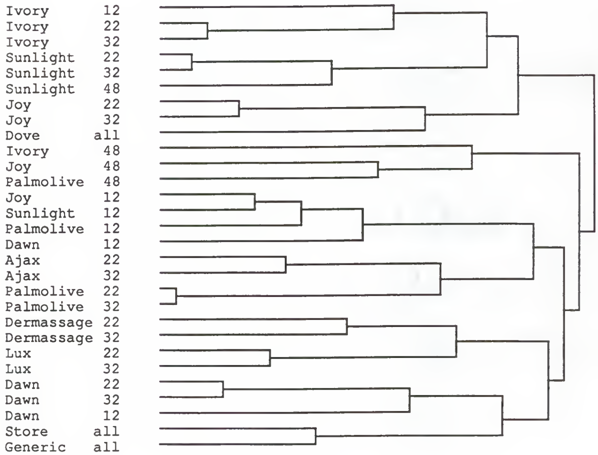


Figure III-10

Dendrogram of the Hierarchical Solution for the Twelve Month Switching Matrix Normalized by the Square Root of Major Diagonals



square root of major diagonal (Figure III-4) is visibly different from the dendrogram obtained from the second six months of data using the same normalization (Figure III-7). The dendrograms obtained by the various normalization techniques were also different, although the differences were less pronounced than the differences between time periods.

The different hierarchical structures for the two split-halves of the data might lead one to conclude that the structure of the market had changed over time. However, the relatively low purchase rate (approximately seven purchases per household per year) may have been a significant cause in the instability of clustering solutions. On average, household only made three or four item purchases per six-month period, and thus the average household only contributed two brand switches to each matrix. Even if a household had a relatively stable consideration set, the observed purchases or switches made within consecutive short time frames may look very different. Also, when the brand switching matrix is defined over a short period of time, there is an increasing danger that temporary market share shifts due to promotional activity will contaminate the results. For example, suppose that market structure is actually stable, but one brand is promoted in the first six-month period and another is promoted in the second six-month period. Naturally, the switching matrices observed from the two split halve will be different, leading to different derived market structures. Thus, changing market structure alone might not be sufficient to reject a null hypothesis of stable market structure.

Fortunately, the same general pattern of size and brand-based clusters appears consistently in each dendrogram, regardless of the time

frame or normalization technique. Thus, while their is some concern that market structure has changed, the empirical data exhibits a common theme regarding the hierarchical structure of the market. Further evidence for the similarity of hierarchical clustering solutions can be found by tabulating the number of exactly matching partitions among the various approaches. Table III-1 presents the number of exactly identical partitions between all possible pairs of the nine hierarchical solutions, nine overlapping solutions, and the a priori partitions. Note that 13 of the 28 hierarchical partitions identified using the marginal market share normalization on the first six-month switching matrix were also contained in the hierarchical solution for the second six-month switching matrix using the same normalization, representing nearly a 50% overlap in the structures between the two time periods. Both remaining normalization methods (square root of market share and square root of major diagonal) resulted in eleven exact partition matches (43% overlap) between the first six-month and second six-month switching matrices. Overall, the hierarchical solution for the first six-month switching matrix was more consistent with the hierarchical solution for the twelve-month switching matrix, regardless of normalization method.

The identified hierarchical clusters are also generally consistent with the hypothesized a priori partitions (i.e., they are base on either brand name or size), thus lending general credence to them. For example, the nine different hierarchical solutions identified 3.44 of the eight multi-item a priori partitions on average, with a maximum of five and a minimum of two. Unfortunately, because of the structural constraints of hierarchical clustering, the obtained

Table III-1

Agreement Between Cluster Solutions*

										Case		
Case	Method	Months	Norm	1	2	3	4	5	6	7	8	9
1.	Hiclus	1-6	MS	-	10	10	13	11	12	13	11	10
2.	Hiclus	1-6	SRMS	.36	-	27	7	11	11	8	15	16
3.	Hiclus	1-6	SRMD	.36	.96	-	7	11	11	8	15	16
4.	Hiclus	7-12	MS	.46	.25	.25	-	11	11	16	8	10
5.	Hiclus	7-12	SRMS	.39	.39	.39	.39	-	23	9	15	16
6.	Hiclus	7-12	SRMD	.43	.39	.39	.39	.82	-	10	14	16
7.	Hiclus	all	MS	.46	.29	.29	.57	.32	.36	-	8	10
8.	Hiclus	all	SRMS	.39	.54	.54	.29	.54	.50	.29	-	15
9.	Hiclus	all	SRMD	.36	.57	.57	.36	.57	.57	.36	.54	-
10.	Overclus	1-6	MS	.30	.20	.20	.20	.25	.30	.30	.20	.25
11.	Overclus	1-6	SRMS	.30	.30	.30	.25	.25	.25	.30	.25	.35
12.	Overclus	1-6	SRMD	.10	.25	.25	.05	.20	.20	.10	.20	.30
13.	Overclus	7-12	MS	.45	.20	.20	.55	.40	.40	.55	.25	.30
14.	Overclus	7-12	SRMS	.45	.35	.35	.45	.50	.50	.40	.50	.45
15.	Overclus	7-12	SRMD	.40	.35	.35	.40	.50	.50	.35	.45	.45
16.	Overclus	all	MS	.40	.25	.25	.35	.35	.40	.45	.20	.35
17.	Overclus	all	SRMS	.50	.35	.35	.45	.45	.45	.40	.35	.40
18.	Overclus	all	SRMD	.40	.35	.35	.45	.40	.40	.45	.40	.40
19.	A priori			.33	.17	.17	.33	.33	.33	.42	.25	.25

Case	Method	Months	Norm	10	11	12	13	14	15	16	17	18	19
1.	Hiclus	1-6	MS	6	6	2	9	9	8	8	10	8	4
2.	Hiclus	1-6	SRMS	4	6	5	4	7	7	5	7	7	2
3.	Hiclus	1-6	SRMD	4	6	5	4	7	7	5	7	7	2
4.	Hiclus	7-12	MS	4	5	1	11	9	8	7	9	9	4
5.	Hiclus	7-12	SRMS	5	5	4	8	10	10	7	9	8	4
6.	Hiclus	7-12	SRMD	6	5	4	8	10	10	8	9	8	4
7.	Hiclus	all	MS	6	6	2	11	8	7	9	8	9	5
8.	Hiclus	all	SRMS	4	5	4	5	10	9	4	7	8	3
9.	Hiclus	all	SRMD	5	7	6	6	9	9	7	8	8	3
10.	Overclus	1-6	MS	-	4	4	5	3	4	9	4	5	3
11.	Overclus	1-6	SRMS	.20	-	9	3	3	3	6	8	7	1
12.	Overclus	1-6	SRMD	.20	.45	-	1	3	4	4	4	4	0
13.	Overclus	7-12	MS	.25	.15	.05	-	9	9	10	7	9	5
14.	Overclus	7-12	SRMS	.15	.15	.15	.45	-	13	3	7	9	3
15.	Overclus	7-12	SRMD	.20	.15	.20	.45	.65	-	4	8	9	4
16.	Overclus	all	MS	.45	.30	.20	.50	.15	.20	-	9	5	5
17.	Overclus	all	SRMS	.20	.40	.20	.35	.35	.40	.45	-	8	4
18.	Overclus	all	SRMD	.25	.35	.20	.45	.45	.45	.25	.40	-	3
19.	A priori			.25	.08	.00	.42	.25	.33	.42	.33	.25	-

* Numbers above the diagonal = number of exact matches
Numbers below the diagonal = exact matches as a proportion of maximum possible

solutions are unable to represent a crossed structure, i.e., where the market is simultaneously partitioned by brand name and size.

Overlapping Clustering

A second analysis of the normalized item switching matrices was conducted using the PROC OVERCLUS procedure of SAS. PROC OVERCLUS is based on the additive clustering model of Shepard and Arabie (1975) and is similar to the ADCLUS (Shepard and Arabie 1979) and MAPCLUS (Arabie and Carroll 1980) algorithms. OVERCLUS produces overlapping clusters of objects (in this case, items). Unlike hierarchical clustering, which always obtains N-1 clusters, the number of groups in overlapping clustering must be specified: further, the number of groups specified will alter the solution. To obtain overlapping clustering solutions that were complete and parsimonious, the iteration history of the program was traced as new clusters were added. The first iteration in which all items were clustered was chosen as the final solution. For each of the nine switching matrices, a 20-cluster solution was sufficient for grouping all items. The overlapping clustering solutions are presented in Tables III-2 through III-10.

The overlapping clustering solutions can be seen as similar to the hierarchical clustering solutions in that combinations of brand name and/or size account for membership in most of the groups. In Table III-10, for example, group 10 consists of all available 12-ounce items and group 8 consists of all sizes of Ajax. While the overlapping cluster solutions were conceptually similar to each other, the number of exact matches between the solutions were rather small. Turning to Table III-1, it can be seen that the degree of redundancy among pairs of overlapping

Table III-2

Overlapping Cluster Solution, First 6-Month Switching Matrix Normalized by Marginal Market Share

OVERALL RMSE = 0.470   OVERALL R^2 = 0.846   INTERCEPT =0.363

GROUP	WEIGHT	ITEMS
1	4.591	Dove 32, Lux 22, Lux 32
2	1.523	Derm. 22, Derm. 32, Lux 32, Dawn 48
3	3.116	Palm. 22, Palm. 32, Palm. 48
4	5.681	Dawn 32, Dawn 48
5	5.785	Lux 22, Lux 32
6	7.915	Lux 22, Generic 32
7	1.629	Ivory 48, Joy 48, Palm. 48, Dawn 48, Sunl. 48
8	3.829	Joy 48, Lux 22
9	1.692	Ajax 22, Ajax 32, Derm. 22, Derm. 32
10	1.885	Ivory 32, Ivory 48, Joy 48
11	1.660	Dawn 12, Dawn 22, Dawn 32
12	1.150	Ivory 12, Joy 12, Palm. 12, Dawn 12, Sunl. 12
13	2.496	Sunl. 32, Sunl. 48
14	1.798	Ivory 12, Ivory 22, Ivory 32
15	4.558	Derm. 22, Lux 32
16	.4208	Joy 12, Joy 22, Ajax 22, Ajax 32, Palm. 12, Palm. 22, Dove 32, Lux 22, Sunl. 12, Sunl. 22, Sunl. 32, Store 32, Generic 32
17	.9530	Palm. 32, Palm. 48, Store 32, Generic 32
18	4.113	Joy 48, Palm. 48
19	2.649	Store 32, Generic 32
20	1.823	Joy 22, Joy 32, Joy 48

Table III-3

Overlapping Cluster Solution, Second 6-Month Switching Matrix Normalized by Marginal Market Share

OVERALL RMSE = .385   OVERALL R^2 = 0.8464   INTERCEPT = .292

GROUP	WEIGHT	ITEMS
1	9.209	Derm. 22, Derm. 32
2	5.283	Ajax 22, Ajax 32
3	5.211	Store 32, Generic 32
4	4.830	Lux 22, Lux 32
5	3.837	Palm. 22, Palm. 32
6	4.152	Ivory 48, Joy 48
7	1.208	Dawn 12, Dawn 22, Dawn 32, Dawn 48
8	2.055	Ivory 22, Ivory 32, Ivory 48
9	3.213	Sunl. 32, Sunl. 48
10	1.309	Ivory 12, Joy 12, Palm. 12, Dawn 12, Sunl. 12
11	1.097	Joy 22, Joy 32, Joy 48, Palmolive 48, Sunlight 48
12	1.146	Ajax 32, Dermassage 32, Dove 32, Lux 32
13	2.624	Ivory 12, Ivory 22
14	2.117	Dove 32, Generic 32
15	3.399	Dawn 32, Dawn 48
16	1.757	Palm. 12, Palm. 22
17	.4103	Ivory 22, Joy 12, Joy 22, Joy 32, Ajax 22, Ajax 32, Palm. 22, Lux 22, Dawn 22, Sunl. 12, Sunl. 22, Sunl. 32, Store 32, Generic 32
18	4.782	Palm. 32, Palm. 48
19	.5354	Ivory 48, Joy 48, Derm. 32, Palm. 48, Dove 32, Lux 22, Dawn 48, Sunl. 48, Store 32
20	.6966	Ivory 32, Ivory 48, Joy 32, Derm. 22, Dove 32, Lux 32

Table III-4

Overlapping Cluster Solution, 12-Month Switching Matrix Normalized by Marginal Market Share

OVERALL RMSE = 0.403 OVERALL R^2 = 0.842 INTERCEPT = .398

GROUP	WEIGHT	ITEMS
1	8.096	Store 32, Generic 32
2	7.077	Derm. 22, Derm. 32
3	4.248	Dawn 32, Dawn 48
4	2.672	Dove 32, Lux 22, Lux 32
5	2.466	Palm. 22, Palm. 32, Palm. 48
6	1.462	Ivory 48, Joy 48, Palm. 48, Dawn 48, Sunl. 48
7	4.064	Ajax 22, Ajax 32
8	3.149	Ivory 32, Ivory 48
9	2.700	Sunl. 32, Sunl. 48
10	1.195	Ivory 12, Joy 12, Palm. 12, Dawn 12, Sunl. 12
11	1.751	Ivory 12, Ivory 22, Ivory 32
12	1.337	Derm. 22, Lux 22, Lux 32, Store 32
13	1.168	Dawn 12, Dawn 22, Dawn 32, Dawn 48
14	1.277	Joy 48, Palm. 32, Palm. 48, Store 32
15	3.097	Ivory 48, Joy 48
16	1.846	Dove 32, Generic 32
17	1.108	Palm. 12, Palm. 22, Palm. 32
18	1.014	Ajax 32, Derm. 32, Dove 32, Lux 32
19	1.819	Joy 22, Joy 32, Joy 48
20	0.454	Joy 12, Joy 22, Ajax 22, Ajax 32, Palm. 22, Lux 22, Sunl. 12, Sunl. 22, Sunl. 32, Store 32, Generic 32

Table III-5

Overlapping Cluster Solution, First 6-Month Switching Matrix Normalized by Square Root of Marginal Market Shares

OVERALL RMSE = 0.011   OVERALL R^2 = 0.844   INTERCEPT = 0.005

GROUP	WEIGHT	ITEMS
1	.0777	Ivory 12, Ivory 22, Ivory 32
2	.0766	Sunl. 22, Sunl. 32, Sunl. 48
3	.0979	Dawn 12, Dawn 22
4	.0588	Palm. 22, Palm. 32, Palm. 48
5	.1178	Joy 22, Joy 32
6	.0229	Ivory 22, Joy 12, Joy 22, Ajax 22, Ajax 32, Derm. 32, Palm. 22, Dawn 22, Sunl. 12, Sunl. 22, Sunl. 32
7	.0469	Ivory 22, Ivory 32, Ivory 48
8	.0306	Ivory 12, Joy 12, Dawn 12, Sunl. 12, Sunl. 22
9	.1145	Dawn 22, Dawn 32
10	.0528	Palm. 12, Palm. 22, Palm. 32
11	.0483	Dove 32, Lux 22, Lux 32
12	.0985	Dawn 32, Dawn 48
13	.0174	Ivory 22, Ivory 32, Joy 22, Joy 32, Palm. 22, Palm. 32, Dove 32, Lux 32, Dawn 32, Sunl. 22, Sunl. 32, Store 32
14	.0314	Lux 22, Store 32, Generic 32
15	.0295	Ivory 12, Ivory 22, Joy 12, Joy 22
16	.0510	Joy 12, Palm. 12, Dawn 12, Sunl. 12
17	.0203	Ivory 32, Ivory 48, Joy 32, Joy 48, Palm. 48, Dawn 48, Sunl. 32, Sunl. 48
18	.0361	Derm. 22, Derm. 32, Lux 32
19	.0284	Dawn 12, Dawn 22, Dawn 32, Dawn 48
20	.0127	Joy 22, Ajax 22, Derm. 22, Palm. 12, Palm. 22, Dawn 12, Dawn 22, Sunl. 22, Store 32, Generic 32

Table III-6

Overlapping Cluster Solution, Second 6-Month Switching Matrix Normalized by Square Root of Marginal Market Shares

OVERALL RMSE = 0.011 OVERALL R^2 = 0.847 INTERCEPT = 0.006

GROUP	WEIGHT	ITEMS
1	.1370	Ivory 12, Ivory 22
2	.1520	Palm. 22, Palm. 32
3	.0843	Dawn 22, Dawn 32, Dawn 48
4	.1038	Joy 22, Joy 32
5	.0828	Sunl. 22, Sunl. 32, Sunl. 48
6	.0661	Ivory 22, Ivory 32, Ivory 48
7	.0173	Ivory 22, Joy 12, Joy 22, Ajax 22, Palm. 22, Lux 22, Dawn 12, Dawn 22, Sunl. 12, Sunl. 22, Sunl. 32, Store 32
8	.0842	Ajax 22, Ajax 32
9	.0872	Palm. 32, Palm. 48
10	.0834	Store 32, Generic 32
11	.0489	Dawn 12, Dawn 22, Dawn 32
12	.0663	Lux 22, Lux 32
13	.0768	Derm. 22, Derm. 32
14	.0247	Ivory 12, Joy 12, Joy 22, Palm. 12, Dawn 12, Dawn 22, Sunl. 12, Sunl. 22
15	.0110	Ivory 22, Ivory 32, Joy 22, Joy 32, Ajax 22, Ajax 32, Derm. 32, Palm. 22, Palm. 32, Dove 32, Lux 22, Lux 32, Dawn 22, Dawn 32, Sunl. 22, Sunl. 32, Store 32
16	.0767	Palm. 12, Palm. 22
17	.0382	Joy 12, Palm. 12, Sunl. 12
18	.0177	Ivory 32, Ivory 48, Joy 22, Joy 32, Joy 48, Palm. 48, Sunl. 32, Sunl. 48
19	.0885	Ivory 22, Ivory 32

Table III-7

Overlapping Cluster Solution, 12-Month Switching Matrix Normalized by Square Root of Marginal Market Shares

OVERALL RMSE = 0.010 OVERALL R^2 = 0.855 INTERCEPT = 0.007

GROUP	WEIGHT	ITEMS
1	.1341	Ivory 12, Ivory 22
2	.0654	Palm. 22, Palm. 32, Palm. 48
3	.1014	Dawn 22, Dawn 32
4	.0796	Sunl. 22, Sunl. 32, Sunl. 48
5	.1251	Joy 22, Joy 32
6	.0224	Ivory 22, Joy 12, Joy 22, Ajax 22, Ajax 32, Palm. 22, Dawn 22, Sunl. 12, Sunl. 22, Sunl. 32, Store 32
7	.1002	Store 32, Generic 32
8	.0821	Ivory 32, Ivory 48
9	.0327	Joy 12, Joy 22, Dawn 12, Sunl. 12, Sunl. 22
10	.0305	Dawn 12, Dawn 22, Dawn 32, Dawn 48
11	.0945	Dawn 32, Dawn 48
12	.0138	Ivory 22, Ivory 32, Joy 22, Joy 32, Derm. 32, Palm. 22, Palm. 32, Dove 32, Lux 22, Lux 32, Dawn 22, Dawn 32, Sunl. 22, Sunl. 32
13	.0552	Palm. 12, Palm. 22, Palm. 32
14	.0555	Derm. 22, Derm. 32
15	.0486	Ajax 22, Ajax 32
16	.0399	Ivory 12, Joy 12, Palm. 12, Dawn 12, Sunl. 12
17	.1463	Ivory 22, Ivory 32
18	.0312	Lux 22, Lux 32, Store 32
19	.0148	Ivory 32, Ivory 48, Joy 32, Joy 48, Palm. 48, Dawn 48, Sunl. 32, Sunl. 48, Store 32
20	.0940	Dawn 12, Dawn 22

Table III-8

Overlapping Cluster Solution, First 6-Month Switching Matrix Normalized by Square Root of Major Diagonal

OVERALL RMSE = 0.034 OVERALL R^2 = 0.835 INTERCEPT = 0.012

GROUP	WEIGHT	ITEMS
1	.0712	Ivory 22, Joy 12, Joy 22, Ajax 22, Ajax 32, Palm. 22, Sunl. 12, Sunl. 22, Sunl. 32
2	.1674	Palm. 12, Palm. 22, Palm. 32
3	.2631	Joy 22, Joy 32
4	.2097	Ivory 12, Ivory 22, Ivory 32
5	.1790	Dawn 12, Dawn 22, Dawn 32
6	.1986	Joy 12, Palm. 12, Dawn 12, Sunl. 12
7	.2135	Sunl. 22, Sunl. 32, Sunl. 48
8	.1756	Dove 32, Lux 22, Lux 32
9	.1119	Ivory 22, Ivory 32, Ivory 48
10	.1853	Palm. 22, Palm. 32, Palm. 48
11	.0449	Ivory 12, Ivory 22, Joy 12, Joy 22, Ajax 22, Dove 32, Dawn 12, Dawn 22, Sunl. 12, Sunl. 22
12	.1593	Dawn 22, Dawn 32
13	.3394	Dawn 32, Dawn 48
14	.0315	Ivory 32, Joy 22, Joy 32, Ajax 22, Ajax 32, Derm. 22, Derm. 32, Palm. 22, Palm. 32, Dove 32, Lux 32, Dawn 22, Dawn 32, Sunl. 22, Sunl. 32, Store 32
15	.0281	Joy 22, Ajax 22, Ajax 32, Palm. 12, Palm. 22, Lux 22, Dawn 12, Dawn 22, Sunlight 22, Store 32, Generic 32
16	.0532	Ivory 32, Ivory 48, Joy 32, Joy 48, Palm. 48, Dawn 48, Sunl. 32, Sunl. 48
17	.0650	Sunl. 12, Sunl. 22, Sunl. 32, Store 32, Generic 32
18	.0678	Joy 12, Joy 22, Joy 32, Joy 48, Sunl. 22
19	.0611	Ivory 22, Ivory 32, Joy 22, Joy 32, Dove 32, Sunl. 32

Table III-9

Overlapping Cluster Solution, Second 6-Month Switching Matrix Normalized by Square Root of Major Diagonal

OVERALL RMSE = 0.032   OVERALL R^2 = 0.851   INTERCEPT = 0.023

GROUP	WEIGHT	ITEMS
1	.4823	Palm. 22, Palm. 32
2	.3776	Joy 22, Joy 32
3	.3836	Ivory 12, Ivory 22
4	.2187	Dawn 22, Dawn 32, Dawn 48
5	.2481	Sunl. 22, Sunl. 32, Sunl. 48
6	.3572	Ajax 22, Ajax 32
7	.0888	Joy 12, Joy 22, Dawn 12, Dawn 22, Sunl. 12, Sunl. 22
8	.1507	Ivory 22, Ivory 32, Ivory 48
9	.2421	Palm. 32, Palm. 48
10	.2361	Lux 22, Lux 32
11	.2175	Store 32, Generic 32
12	.0592	Ivory 22, Joy 12, Joy 22, Ajax 22, Palm. 22, Lux 22, Sunl. 12, Sunl. 22, Sunl. 32, Store 32
13	.0399	Ivory 22, Ivory 32, Joy 22, Joy 32, Ajax 22, Ajax 32, Palm. 22, Palm. 32, Dove 32, Lux 22, Dawn 22, Dawn 32, Sunl. 22, Sunl. 32
14	.1118	Ivory 12, Joy 12, Palm. 12, Dawn 12, Sunl. 12
15	.1281	Dawn 12, Dawn 22, Dawn 32
16	.1395	Derm. 22, Derm. 32
17	.1152	Palm. 12, Palm. 22, Sunl. 22
18	.2280	Ivory 22, Ivory 32
19	.0487	Ivory 32, Ivory 48, Joy 22, Joy 32, Joy 48, Sunl. 32, Sunl. 48
20	.1379	Joy 12, Joy 22

Table III-10

Overlapping Cluster Solution, 12-Month Switching Matrix Normalized by Square Root of the Major Diagonal

OVERALL RMSE = 0.026 OVERALL R^2 = 0.8444 INTERCEPT = 0.011

GROUP	WEIGHT	ITEMS	INTERPRETATION
1	.3678	Palmolive 22, Palmolive 32	medium Palmolive
2	.1891	Sunlight 22, Sunlight 32, Sunlight 48	large Sunlight
3	.1871	Ivory 12, Ivory 22, Ivory 32	small Ivory
4	.1781	Dawn 12, Dawn 22	small Dawn
5	.0538	Ivory 22, Joy 12, Joy 22, Ajax 22, Palmolive 22, Dawn 12 Sunlight 12, Sunlight 22,Sunlight 32	small sizes of medium and high power brands
6	.2828	Joy 22, Joy 32	medium Joy
7	.2095	Lux 22, Lux 32	all Lux
8	.1773	Ajax 22, Ajax 32	all Ajax
9	.2265	Palmolive 32, Palmolive 48	large Palmolive
10	.0958	Ivory 12, Joy 12, Palmolive 12, Dawn 12, Sunlight 12	all 12 ounce sizes
11	.0907	Ivory 22, Ivory 32, Ivory 48	large Ivory
12	.2377	Palmolive 12, Palmolive 22	small Palmolive
13	.0314	Ivory 22, Ivory 32, Joy 22, Joy 32, Ajax 22, Ajax 32, Palmolive 22, Palmolive 32, Dove 32, Lux 22, Dawn 22, Dawn 32, Sunlight 22, Sunlight 32, store brand 32	medium size, nationally advertised
14	.2907	Dawn 22, Dawn 32	medium Dawn

Table III-10 (continued)

GROUP	WEIGHT	ITEMS	INTERPRETATION
15	.0408	Ivory 32, Ivory 48, Joy 32, Joy 48, Palmolive 48, Sunlight 32, Sunlight 48	large size, medium power
16	.0849	Joy 12, Joy 22, Palmolive 12, Sunlight 12, Sunlight 22	small lemon and Palmolive
17	.2304	Dawn 32, Dawn 48	large Dawn
18	.0278	Ivory 12, Ivory 22, Joy 12, Joy 22, Palmolive 22, Dove 32, Lux 22, Lux 32, Sunlight 22	small sizes of gentle and medium power
19	.1447	store brand 32, generic 32	price brands
20	.0271	Ajax 22, Ajax 32, Dermassage 22, Dermassage 32, Lux 22, Sunlight 12, Sunlight 22, Sunlight 32	small Ajax, Dermassage, Lux, and Sunlight

clustering solutions was moderate, ranging between 5% and 65%, with a mean of 27%. The redundancy between pairs of hierarchical clusters, on the other hand, ranged between 25% and 96% with a mean of 52%. Surprisingly, the overlapping clustering solutions exhibited a greater level of redundancy with the hierarchical solution than with each other. One possible reason for this pattern of redundancy is that the overlapping clustering partitions contained larger numbers of items than the hierarchical partitions. Due to the small number of items in hierarchical partitions representing lower branches in the tree, two hierarchical clusterings of similar switching matrices are likely to contain similar or identical partition. Since the overlapping clustering partitions consist of greater numbers of items, two overlapping clusterings of similar switching matrices may lead to similar partitions but not partitions which are identical. While overlapping clustering is less constrained than hierarchical clustering, this result implies that overlapping clustering solutions are somewhat more sensitive to small variations in switching or similarity measures.

As was the case for hierarchical clustering, these clusters are consistent with and tend to validate the a priori partitions. Among the nine overlapping clustering solutions, a minimum of zero and a maximum of five a priori partitions were identified, with a mean of 3.11.

Many of the groups identified by overlapping clustering consist of subsets of all sizes of particular brands. For example, group 3 in table III-10 consists of the three smallest sizes of Ivory liquid. Interestingly, many groups have a "chained" relationship with other groups. For example, group 12 (small Palmolive), group 1 (medium Palmolive), and group 9 (large Palmolive) are all subsets of the a

priori Palmolive-loyal partition, and are chained by size. These size-chained groups were also evident for Ivory (groups 3 and 11) and Dawn (groups 4, 14 and 17). Some of the remaining segments were chained by cleaning strength rather than size. For example, group 5 consists of smaller sizes of medium strength (Ivory, Joy, Palmolive and Sunlight) and high strength (Ajax and Dawn) brands, while group 18 consists of the smaller sizes of the same medium strength brands and gentle brands (Dove and Lux). One way that this result might be interpreted is that size is a continuous attribute, rather than a discrete feature; see, for example, Restle (1959). Thus, the observed chaining of groups may provide an indication that both size and strength may act as continuous rather than discrete variables.

While there is some evidence of strength-based partitions in the clustering solutions, the dominant structure variable appears to be size, within brand. This empirical structure differs significantly from the LDD-market structure assumed by Hauser and Shugan (1983). In their analysis, "mildness" and "efficacy" were postulated as the two attributes determining competition in the LDD market. The empirical results presented here indicate that package size plays a more crucial role than either of these variables.

Sticky Clustering

In addition to overlapping and hierarchical clustering, a final empirical clustering technique was applied to the LDD data to obtain market partitions. This technique was developed by the author and J. Wesley Hutchinson, and we refer to it as "sticky clustering". The algorithm for sticky clustering first sorts the households in descending order of purchase frequency. The observed item purchase probability

vector of the first household (i.e., the household with the highest number of purchases) defines a cluster seed. The second household then enters the algorithm. The chi square between the second household's purchase probability vector and the cluster seed is computed. If the chi square value is greater than a prespecified cutoff level, then the second household's purchase probability vector defines a new cluster seed: if not, the second household's probabilities are combined with the original cluster seed, thus defining a new cluster seed. The process is repeated for the remaining households until all households are clustered. If a household's purchase probability vector is not significantly different from more than one cluster, then a nearest-neighbor rule is used. In this way, each household either defines a new (different) cluster, or "sticks" to an existing cluster.

Conceptually, the sticky clustering method is very different from either overlapping clustering or hierarchical clustering, even thought the eventual output is similar. Both hierarchical and overlapping clustering are based on aggregate level item switching matrices, and thus can be thought of as "top down" in approach. Sticky clustering, on the other hand, is based on individual household purchase probability vectors, and thus can be characterized as "bottom up" in approach. The sticky clustering method is disjoint with respect to households (i.e., each household is assigned to one and only one partition), but overlapping with respect to items. Each item has some purchase probability within each partition.

The sticky clustering algorithm is similar in spirit to k-means clustering, but has a few notable differences. First, k-means clustering uses a clustering criterion based on squared distances between two

objects, where sticky clustering's criterion is based on the chi-square between two vectors of probabilities. Second, the analyst specifies a desired number of clusters in k-means clustering. In sticky clustering, clusters are added as necessary (i.e., when a household has a vector of probabilities significantly different from the current clusters). Finally, k-means clustering performs an additional iteration wherein distances are recomputed and objects are potentially reassigned to different clusters.

Each switching household's vector of purchase frequencies over the twelve-month period were converted to probabilities, and the households were subjected to the sticky clustering algorithm with a .05 significance level for new cluster addition. Unlike overlapping clustering and hierarchical clustering, analysis of the split-half data was not attempted with sticky clustering. This was necessitated by the low purchase rate in the category (approximately seven purchases per household per year). If the data had been split into two six-month samples, then the household purchase probability vectors would have been based on only three to four purchases, on average.

After assigning each household to one of the sticky clusters, item purchases were tallied within each identified cluster. The item by cluster matrix of purchase frequencies obtained by sticky clustering is presented in Table III-11. Unlike the hierarchical and overlapping clustering methods, which require the intermediate aggregation of the purchase data into a switching matrix, the sticky clustering solution is essentially a straightforward way of disaggregating the raw purchase data.

Table III-11

Results of Sticky Clustering (Larger Values Underlined)

Cluster

Brand		1	2	3	4	5	6	7	8	9	10	11	12	13
Ivory	12	0	0	4	13	0	1	10	0	1	3	0	1	53
Ivory	22	2	14	0	36	43	1	12	1	72	0	1	54	42
Ivory	32	0	2	0	5	28	3	1	0	228	0	1	10	1
Ivory	48	0	0	0	0	0	3	0	0	26	0	3	0	0
Joy	12	0	6	52	0	1	67	55	0	1	0	2	0	34
Joy	22	24	3	3	16	120	255	33	5	21	0	8	14	33
Joy	32	0	15	0	1	20	132	1	10	27	1	1	0	2
Joy	48	0	2	0	0	0	0	0	2	0	0	0	0	0
Ajax	22	7	3	1	2	9	2	5	1	0	1	21	0	2
Ajax	32	0	17	0	4	2	0	0	0	0	0	28	0	0
Derm.	22	0	0	0	0	0	0	0	23	0	0	0	0	0
Derm.	32	0	1	0	51	0	0	1	32	0	1	0	0	0
Palm.	12	2	0	34	28	0	1	75	0	1	1	0	0	0
Palm.	22	4	15	0	147	36	2	20	0	5	118	15	0	12
Palm.	32	0	1	0	22	5	0	0	1	0	72	0	0	0
Palm.	48	0	0	0	0	2	0	1	0	0	8	0	0	0
Dove	32	0	5	0	0	43	0	3	0	0	0	0	11	2
Lux	22	1	21	0	5	4	0	1	3	0	0	0	28	0
Lux	32	1	10	0	2	0	0	0	9	0	0	1	60	0
Dawn	12	0	0	1	0	0	0	0	0	0	0	1	0	0
Dawn	22	6	58	0	15	26	0	5	1	8	172	0	1	10
Dawn	32	0	12	0	4	9	0	0	0	0	100	0	0	3
Dawn	48	0	0	0	0	0	0	0	3	0	12	0	0	0
Sunl.	12	9	0	121	0	1	0	1	3	0	1	0	0	49
Sunl.	22	189	58	38	2	101	0	17	76	9	3	2	4	85
Sunl.	32	86	36	11	1	68	2	6	143	26	4	0	0	0
Sunl.	48	1	0	0	0	0	0	0	54	2	0	1	0	14
Store	32	8	50	0	0	0	0	0	0	0	3	83	1	0
Gen.	32	0	0	6	0	0	0	0	0	0	0	11	0	0
Total		340	329	271	354	518	469	247	367	427	500	179	184	342

Table III-11 (continued)

Cluster

Brand		14	15	16	17	18	19	20	21	22	23	24	25	26
Ivory	12	1	0	1	0	52	5	1	0	1	12	3	1	4
Ivory	22	17	0	22	7	202	38	0	0	39	10	0	16	22
Ivory	32	30	9	6	9	118	4	13	3	0	5	0	0	0
Ivory	48	0	1	35	2	0	0	48	0	1	5	0	0	3
Joy	12	0	8	1	3	0	39	1	0	0	6	1	11	20
Joy	22	14	9	9	9	2	23	14	1	13	12	0	28	18
Joy	32	45	19	0	6	2	5	16	6	14	0	4	3	1
Joy	48	0	42	3	1	0	0	28	0	0	0	1	0	3
Ajax	22	7	0	0	0	0	16	1	0	39	6	4	5	0
Ajax	32	2	0	3	0	0	3	1	3	3	9	0	0	2
Derm.	22	0	0	0	0	0	0	0	8	0	6	0	2	0
Derm.	32	3	0	0	0	2	0	0	8	9	1	0	0	1
Palm.	12	2	1	6	0	0	4	0	3	11	3	5	5	9
Palm.	22	1	3	40	20	2	0	0	47	23	0	18	32	8
Palm.	32	5	0	0	4	0	0	6	40	0	5	39	0	0
Palm.	48	1	6	22	0	2	0	7	28	0	0	45	0	0
Dove	32	8	0	0	0	8	0	2	0	5	3	2	4	4
Lux	22	4	0	0	0	0	0	2	0	6	0	0	0	3
Lux	32	3	0	0	1	7	0	3	1	1	0	3	0	4
Dawn	12	0	0	0	0	0	0	0	0	0	0	0	64	8
Dawn	22	45	3	0	34	1	48	0	1	0	15	0	186	0
Dawn	32	8	12	0	126	2	5	0	1	0	6	6	37	0
Dawn	48	12	1	4	109	0	0	12	0	0	2	5	1	2
Sunl.	12	3	1	1	1	0	9	0	0	25	0	1	4	15
Sunl.	22	18	0	15	0	3	3	3	0	49	19	5	24	21
Sunl.	32	97	33	31	5	2	0	6	13	13	15	0	9	2
Sunl.	48	10	53	8	0	0	0	24	1	3	4	0	0	1
Store	32	4	3	2	6	0	0	0	27	6	10	0	0	41
Gen.	32	0	0	4	0	0	0	0	2	0	16	1	0	0
Total		340	204	213	343	405	202	188	193	261	170	143	432	192

Table III-11 (continued)

Cluster

Brand		27	28	29	30	31	32	33	34	35	36	37	38	39
Ivory	12	1	8	17	0	76	12	7	0	0	2	0	3	0
Ivory	22	7	0	48	16	67	1	0	26	3	6	0	7	33
Ivory	32	16	0	0	14	39	2	15	0	6	0	0	2	7
Ivory	48	6	0	0	0	3	1	0	0	0	0	9	0	0
Joy	12	0	7	64	3	2	29	0	0	10	22	1	2	6
Joy	22	32	0	10	37	0	14	6	42	28	13	0	0	8
Joy	32	4	1	2	5	0	0	0	0	14	2	3	0	6
Joy	48	0	0	0	0	0	0	0	0	0	2	0	0	0
Ajax	22	0	3	13	2	2	0	4	17	42	13	1	0	6
Ajax	32	2	0	7	2	0	0	0	18	17	5	2	0	0
Derm.	22	5	0	1	1	1	0	0	0	5	0	0	0	1
Derm.	32	1	0	6	0	0	0	2	10	1	0	1	0	0
Palm.	12	6	2	29	0	1	0	0	4	0	14	0	0	0
Palm.	22	49	6	16	3	4	7	11	86	14	16	2	0	18
Palm.	32	11	1	6	12	0	0	0	21	1	2	1	0	3
Palm.	48	0	0	0	3	0	0	5	2	1	0	0	0	0
Dove	32	0	0	3	0	0	6	1	2	0	16	2	3	4
Lux	22	7	0	1	10	1	12	1	2	2	5	5	1	14
Lux	32	0	2	0	4	2	6	0	3	1	0	0	0	23
Dawn	12	9	151	27	4	5	28	7	5	10	37	0	4	3
Dawn	22	2	34	4	21	2	21	3	26	19	28	2	121	7
Dawn	32	0	1	0	0	3	3	3	1	4	1	10	30	9
Dawn	48	0	1	0	0	0	0	6	0	0	1	7	20	4
Sunl.	12	9	4	55	4	2	0	0	0	0	40	1	0	2
Sunl.	22	60	8	40	54	0	0	9	18	4	50	3	6	11
Sunl.	32	71	0	25	15	1	0	9	28	2	6	13	0	9
Sunl.	48	7	1	0	1	0	0	8	2	0	2	24	1	6
Store	32	4	3	3	5	0	7	55	2	22	1	18	6	0
Gen.	32	0	10	2	2	0	9	0	0	1	4	20	0	0
Total		309	243	379	218	211	158	152	315	207	288	125	206	180

Table III-11 (continued)

Cluster

Brand		40	41	42	43	44	45	46	47	48	49	50	51	52
Ivory	12	8	2	5	9	4	0	0	0	7	0	2	0	8
Ivory	22	56	7	2	15	2	5	3	6	0	7	0	5	2
Ivory	32	3	22	0	0	1	12	2	8	2	2	0	12	0
Ivory	48	0	48	0	1	6	0	0	0	0	0	0	0	0
Joy	12	2	1	14	0	0	11	0	0	3	1	0	2	5
Joy	22	28	5	0	0	6	1	1	7	4	1	0	3	17
Joy	32	0	0	0	0	0	0	1	50	0	1	0	1	2
Joy	48	0	0	0	0	8	0	1	1	0	1	0	0	0
Ajax	22	0	1	1	2	9	1	0	0	10	5	3	2	2
Ajax	32	0	0	1	0	3	3	3	0	3	0	1	5	0
Derm.	22	0	0	0	0	0	0	0	0	1	0	0	0	0
Derm.	32	0	0	24	2	0	2	3	0	14	0	0	0	0
Palm.	12	0	0	12	2	2	0	0	0	3	6	0	1	1
Palm.	22	1	0	1	21	12	9	5	0	2	23	0	5	0
Palm.	32	0	0	0	8	4	11	0	0	0	0	1	1	0
Palm.	48	0	0	0	0	6	2	0	0	0	0	3	1	0
Dove	32	0	5	0	1	1	1	0	3	5	1	0	1	1
Lux	22	0	0	0	0	6	0	1	0	0	0	0	0	0
Lux	32	0	3	0	4	1	3	2	0	4	2	0	2	0
Dawn	12	0	4	43	5	2	5	0	0	0	0	2	0	1
Dawn	22	0	3	8	12	19	1	0	0	3	1	1	0	0
Dawn	32	0	1	5	0	4	7	5	0	2	0	0	0	0
Dawn	48	0	0	2	0	5	0	0	0	2	0	0	0	0
Sunl.	12	0	2	6	5	1	7	0	0	1	0	1	3	3
Sunl.	22	3	2	5	6	16	13	1	0	13	19	4	1	2
Sunl.	32	4	1	0	0	22	11	52	0	1	8	13	1	0
Sunl.	48	0	0	1	0	15	0	14	0	0	4	13	6	0
Store	32	0	0	0	6	0	1	0	0	0	2	0	0	0
Gen.	32	1	0	1	0	0	0	0	0	1	9	2	0	0
Total		106	107	131	99	155	106	94	75	81	93	46	52	44

The solution obtained by the sticky clustering algorithm contained 52 switching clusters, which were interpreted as partitions. Most of the identified partitions were similar to at least one of the partitions identified by overlapping clustering and hierarchical clustering. For example, partition 6 is dominated by the three smallest sizes of Ivory. This partition is similar to partition 3 obtained from overlapping clustering of the twelve-month switching matrix normalized by the square root of the major diagonal (Table III-10) and the top branch of the dendrogram for hierarchical clustering of the same switching matrix (Figure III-10). Like hierarchical clustering and overlapping clustering, the sticky clustering results tended to corroborate the a priori structure, where market partitions were base on either brand name or package size. Partition 7, for example, was dominated by small sizes (12-ounce and 22-ounce) of branded LDD items, and partition 5 was dominated by medium sizes (22-ounce and 32-ounce) of branded LDD items; partitions 6, 18, 21, and 38 are dominated by Joy, Ivory, Palmolive, and Dawn, respectively. Further, the sticky clustering algorithm found many of the same type of chained partitions that were found with overlapping clustering.

A number of substantive disparities between the sticky cluster solution and the other solutions bears discussion. First, several of the partitions found from sticky clustering appear to be combinations of partitions found from the other methods. For example, partition 37 is dominated by store brand, generic, and large sizes of Sunlight. This partition is similar to the union of partitions 2 and 19 from the overlapping clustering solution (Table III-10). Second, several of the sticky clustering partitions were somewhat redundant. Note that

partitions 7 and 13 are both dominated by small sizes of branded items. Likewise, partitions 18 and 31 both represent small sizes of Ivory. While the chi square values between each pair of partitions are significant (118.3 for partitions 7 and 13, 49.7 for partitions 18 and 31), the items contained within the partitions are nearly identical. One explanation of these result is that k-means clustering tends to find clusters with equal numbers of observations (Sarle 1982). If the same effect holds for sticky clustering, then a large group of similar households might be split into separate groups, and small (but dissimilar) groups of households might be clustered together so that the resultant cluster sizes are approximately equal. As the sizes of the empirical partitions did not vary a great deal (coefficient of variation = .53), this explanation appears to be plausible.

The reader may note that sticky clustering jointly achieves phases 1 and 2 of model estimation for the SLM. Like k-means clustering, sticky clustering obtains the discrete market partition matrix and at the same time assigns households partitions, thus obtaining market segments. However, it is somewhat difficult to infer that the sticky clusters represent homogeneous consideration sets. Note that in Table III-11, there are few zero cells; in general, there are at least a few purchase of most items in each partition. Ideally, the sticky cluster solution would have clusters in which each item was "in" only a few partitions, and excluded from the remaining partitions, and that each partition contained only a small number of items. If the data had shown this type of result, there would have been greater confidence that each sticky cluster represented the homogeneous consideration set of a group

of consumers, and the consumer groups found by the sticky clustering might have been used directly as market segments.

Hybrid Partitions

The market structuring analyses described above yielded hundreds of potential market partitions. The next objective was to reduce the total set of partitions to a manageable number of hybrid partitions, such that the hybrid partitions represent the choice sets of a significant number of consumers.

The first decision was to include all the a priori single-item loyal partitions in the final hybrid structure. The preliminary data analysis indicated that there was a substantial number of households that did not switch at all between items. Further, the single item partitions have strong managerial significance, as they represent the consideration sets of single-item-loyal consumers (see Chapter II). Further, many of the a priori partitions were identified by either the hierarchical solutions, the overlapping solutions, or both.

The next decision was to pick the "best" hierarchical clustering representation from among Figures III-2 to III-10, and the "best" overlapping clustering solution from among Tables III-2 to III-10. As discussed in the section on hierarchical clustering, the split-half-year solution are more prone to contamination by temporary marketing activity. This possible contamination effect favors using the structure found using the longer time frame, i.e., the entire twelve months of data. The remaining consideration is which of the normalization methods is preferred. There have been arguments on statistical grounds (Hutchinson and Zenor 1987) that normalization by

the square root of the major diagonal is a preferred measure; thus, for both overlapping and hierarchical clustering, we focus only on those partitions found from the 12 month switching matrix, normalized by square root of the major diagonal.

To further pare the set of partitions, a pretest was conducted to estimate the sizes of the partitions. The method used in the pretest was the weighted least squares (WLS) regression method proposed by Hutchinson and Zenor (1987). This method involves regressing the observed switching frequencies on a design matrix generated by a discrete market structure. Under the assumptions that switching is a zero-order process and that within-partition choice probabilities are proportional to the square root of the major diagonal, Hutchinson and Zenor show that WLS parameter estimates for partitions are unbiased estimates of partition sizes. While the WLS method does not directly assign households to partitions, the partition size estimates obtained give an indication of whether an identified partition warrants removal. For example, if the WLS estimate for the size of an identified partition is insignificant, that partition might be excluded from the hybrid structure.

Three WLS regression runs were performed, using the discrete market structure matrices identified from (1) hierarchical clustering; (2) overlapping clustering; and (3) a priori analysis. The discrete market structure matrices obtained from each of the techniques are presented in Tables III-12, III-13, and III-14. These matrices were used to generate the design matrices for each WLS regression. Tables III-15, III-16, and III-17 present the WLS partition size estimates for the

multi-item partitions obtained from hierarchical, overlapping, and a priori analysis, respectively.

Table III-15 indicates that the partitions obtained from hierarchical clustering did a relatively poor job of explaining the brand switching matrix. Overall R^2 was relatively low (.61) and only 13 of 28 identified partitions were significantly greater than zero. Further, of the thirteen significant hierarchical partitions, eleven were identified either in the a priori analysis or with overlapping clustering. The remaining two significant partitions were supersets of partitions found with overlapping clustering. The partitions identified with overlapping clustering fared much better (Table III-16), as all 20 identified partitions were estimated as significantly greater than zero. Despite having eight fewer degrees of freedom, the overlapping cluster representation provided a better overall fit to the original switching matrix (adjusted R^2 = .79) than the hierarchical partitions. The thirteen identified a priori partitions (those partitions based on brand name and sizes) were again all significant or near significance, and the fit provided by the a priori partitions was nearly equal to the fit provided by the hierarchical partitions, despite having thirteen fewer degrees of freedom.

The uniformly poor fit of the hierarchical partitions to the observed data might be explained by the constraints that hierarchical clustering puts on structure representation. Note that hierarchical clustering is prevented from finding <u>crossed</u> structure, in which both brand name and item size are represented as partitions. A priori, such structure was expected in the LDD category, and the WLS size estimates indicate that the partitions representing crossed structure are indeed

Table III-12

Hierarchical Clustering Multi-Item Partitions

Partitions

Ivory	12	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	1	0	0	0	0
Ivory	22	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	1	0	0	0	0
Ivory	32	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	1	0	0	0	0
Ivory	48	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1
Joy	12	1	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	1	1
Joy	22	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0
Joy	32	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0
Joy	48	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	1
Ajax	22	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	1	1
Ajax	32	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	1	1
Derm.	22	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	1	1	1
Derm.	32	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	1	1	1
Palm.	12	1	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	1	1
Palm.	22	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	1	1
Palm.	32	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	1	1
Palm.	48	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	1
Dove	32	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0
Lux	22	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	1	1
Lux	32	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	1	1
Dawn	12	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	1	1
Dawn	22	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	1	1	1
Dawn	32	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	1	1	1
Dawn	48	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	1	1	1
Sunl.	12	1	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	1	1
Sunl.	22	1	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0
Sunl.	32	1	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0
Sunl.	48	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0
Store	32	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	1
Gen.	32	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	1

Table III-13

Overlapping Clustering Multi-Item Partitions

Partitions

Ivory	12	1	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0
Ivory	22	1	0	0	1	0	1	0	0	0	0	0	1	0	1	0	0	0	0	1	0	0
Ivory	32	1	0	0	1	0	0	0	0	0	0	0	1	0	1	0	1	0	0	0	0	0
Ivory	48	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0
Joy	12	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	1	0	1	0	0
Joy	22	1	0	0	0	0	1	1	0	0	0	0	0	0	1	0	0	1	0	1	0	0
Joy	32	1	0	0	0	0	0	1	0	0	0	0	0	0	1	0	1	0	0	0	0	0
Joy	48	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
Ajax	22	1	0	0	0	0	1	0	0	1	0	0	0	0	1	0	0	0	0	0	0	1
Ajax	32	1	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	1
Derm.	22	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
Derm.	32	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
Palm.	12	1	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	1	0	0	0	0
Palm.	22	1	1	0	0	0	1	0	0	0	0	0	0	1	1	0	0	0	0	1	0	0
Palm.	32	1	1	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0
Palm.	48	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0
Dove	32	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0
Lux	22	1	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	1	0	1
Lux	32	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0
Dawn	12	1	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
Dawn	22	1	0	0	0	1	1	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0
Dawn	32	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	0	0
Dawn	48	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
Sunl.	12	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	1	0	0	0	1
Sunl.	22	1	0	1	0	0	1	0	0	0	0	0	0	0	1	0	0	1	0	1	0	1
Sunl.	32	1	0	1	0	0	1	0	0	0	0	0	0	0	1	0	1	0	0	0	0	1
Sunl.	48	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
Store	32	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0
Gen.	32	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0

Table III-14

A Priori Multi-Item Partitions

Partitions

Ivory 12	1	0	0	0	0	0	0	0	1	0	0	0
Ivory 22	1	0	0	0	0	0	0	0	0	1	0	0
Ivory 32	1	0	0	0	0	0	0	0	0	0	1	0
Ivory 48	1	0	0	0	0	0	0	0	0	0	0	1
Joy 12	0	1	0	0	0	0	0	0	1	0	0	0
Joy 22	0	1	0	0	0	0	0	0	0	1	0	0
Joy 32	0	1	0	0	0	0	0	0	0	0	1	0
Joy 48	0	1	0	0	0	0	0	0	0	0	0	1
Ajax 22	0	0	1	0	0	0	0	0	0	1	0	0
Ajax 32	0	0	1	0	0	0	0	0	0	0	1	0
Derm. 22	0	0	0	1	0	0	0	0	0	1	0	0
Derm. 32	0	0	0	1	0	0	0	0	0	0	1	0
Palm. 12	0	0	0	0	1	0	0	0	1	0	0	0
Palm. 22	0	0	0	0	1	0	0	0	0	1	0	0
Palm. 32	0	0	0	0	1	0	0	0	0	0	1	0
Palm. 48	0	0	0	0	1	0	0	0	0	0	0	1
Dove 32	0	0	0	0	0	0	0	0	0	0	1	0
Lux 22	0	0	0	0	0	1	0	0	0	1	0	0
Lux 32	0	0	0	0	0	1	0	0	0	0	1	0
Dawn 12	0	0	0	0	0	0	1	0	1	0	0	0
Dawn 22	0	0	0	0	0	0	1	0	0	1	0	0
Dawn 32	0	0	0	0	0	0	1	0	0	0	1	0
Dawn 48	0	0	0	0	0	0	1	0	0	0	0	1
Sunl. 12	0	0	0	0	0	0	0	1	1	0	0	0
Sunl. 22	0	0	0	0	0	0	0	1	0	1	0	0
Sunl. 32	0	0	0	0	0	0	0	1	0	0	1	0
Sunl. 48	0	0	0	0	0	0	0	1	0	0	0	1
Store 32	0	0	0	0	0	0	0	0	0	0	1	0
Gen. 32	0	0	0	0	0	0	0	0	0	0	1	0

Table III-15

WLS Regression Size Estimates for the Hierarchical Clustering Switching Partitions

Adjusted R^2 =.6159

Partition	Parameter	Std. Error	t
1	699.1	91.1	7.67
2	261.5	62.4	4.18
3	319.2	82.1	3.89
4	279.8	80.1	3.49
5	256.9	70.8	3.63
6	269.1	65.6	4.10
7	90.6	50.2	1.81
8	37.1	21.2	1.75
9	40.9	23.9	1.71
0	13.8	54.6	0.25
11	58.1	26.8	2.17
12	232.2	67.2	3.46
13	21.3	17.7	1.21
14	315.4	68.8	4.59
15	7.7	13.7	0.56
16	234.2	68.5	3.41
17	174.7	52.3	3.34
18	16.4	41.0	0.40
19	43.3	34.6	1.25
20	10.7	12.2	0.88
21	22.4	15.9	1.40
22	-215	76.5	-2.8
23	12.9	18.7	0.69
24	830.2	111	7.50
25	220.2	52.8	4.17
26	-11.6	19.3	-.60
27	75.4	50.2	1.50
28	-212	55.4	-3.8

underlined values indicate significantly positive at .05

Table III-16

WLS Regression Size Estimates for the Overlapping Clustering Switching Partitions

Adjusted R^2 =.7887

Partition	Parameter	Std. Error	t
1	206.3	46.2	4.47
2	284.3	45.2	6.30
3	368.9	43.1	8.56
4	338.6	44.2	7.67
5	302.3	48.0	6.30
6	1442.4	84.9	17.0
7	308.9	44.6	6.92
8	33.8	15.6	2.17
9	45.5	17.5	2.60
10	54.4	17.9	3.04
11	402.3	43.1	9.34
12	220.1	36.9	5.96
13	177.8	34.3	5.19
14	860.5	67.9	12.7
15	338.6	47.7	7.10
16	46.0	15.6	2.94
17	189.9	36.1	5.27
18	125.8	27.2	4.62
19	417.8	51.7	8.08
20	59.5	19.8	3.01
21	63.8	20.0	3.19

underlined values indicate significantly positive at .05

Table III-17

WLS Regression Size Estimates for the A Priori Partitions

Adjusted R^2 =.5907

Partition	Parameter	Std. Error	t
1	400.1	64.1	6.25
2	206.7	41.1	5.03
3	157.7	36.1	4.37
4	51.6	24.3	2.12
5	24.0	17.9	1.34
6	143.6	34.6	4.15
7	39.3	21.7	1.81
8	408.2	57.1	7.15
9	447.9	59.4	7.55
10	480.4	59.6	8.07
11	618.7	67.0	9.24
12	50.1	27.0	1.85
13	60.8	22.2	2.73

significant. The results here indicate that market structuring methods that rely on a hierarchical representation of alternatives (Kalwani and Morrison 1977; Rao and Sabavala 1981; Grover and Dillon 1985) may tend to obscure the true structure of the market.

Since the hierarchical partitions were unsuccessful in recovering brand switching, and since the hierarchical partitions were generally insignificant in size, all of the hierarchical partitions were eliminated from further consideration. The remaining non-redundant partitions (those obtained through overlapping clustering or a priori) were concatenated to obtain a hybrid discrete market structure matrix of 58 partitions. The first 29 partitions are the single-item partitions corresponding to each item in the market. The remaining 29 partitions are all multi-item partitions derived from either the a priori partitions or from the overlapping cluster solutions. These hybrid partitions are presented in Table III-18. Thirteen of the 29 multi-item partitions were specified in the a priori analysis: one "all items" partition, eight brand-name-loyal partitions, and four size-loyal partitions. The remaining sixteen partitions were those partitions found through overlapping clustering. Verbal interpretations of the hybrid partitions are presented in Table III-19.

As a final diagnostic test, the hybrid structure was compared with six other market structure representations using the likelihood ratio chi square statistic G^2 (Bishop, Feinberg, and Holland 1975). The first comparison market structure contained 58 discrete partitions. The first 30 of the partitions were identical to the first 30 partitions in the hybrid structure: 29 single-item partitions and one all-item partition. The remaining 29 multi-item partitions were randomly

Table III-18

Final Multi-Item Hybrid Partitions

Partitions

Ivory	12	1 1 0 0 0 0 0 0 0	1 0 0 0	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
Ivory	22	1 1 0 0 0 0 0 0 0	0 1 0 0	1 1 0 0 0 0 0 0 0 0 0 1 0 0 1 0
Ivory	32	1 1 0 0 0 0 0 0 0	0 0 1 0	1 1 0 0 0 0 0 0 0 0 0 1 1 0 1 0
Ivory	48	1 1 0 0 0 0 0 0 0	0 0 0 1	0 1 0 0 0 0 0 0 0 0 0 0 1 0 0 0
Joy	12	1 0 1 0 0 0 0 0 0	1 0 0 0	0 0 0 0 0 0 0 0 0 0 1 0 0 1 0 0
Joy	22	1 0 1 0 0 0 0 0 0	0 1 0 0	0 0 1 0 0 0 0 0 0 0 1 1 0 1 1 0
Joy	32	1 0 1 0 0 0 0 0 0	0 0 1 0	0 0 1 0 0 0 0 0 0 0 0 1 1 0 1 0
Joy	48	1 0 1 0 0 0 0 0 0	0 0 0 1	0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0
Ajax	22	1 0 0 1 0 0 0 0 0	0 1 0 0	0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0
Ajax	32	1 0 0 1 0 0 0 0 0	0 0 1 0	0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0
Derm.	22	1 0 0 0 1 0 0 0 0	0 1 0 0	0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0
Derm.	32	1 0 0 0 1 0 0 0 0	0 0 1 0	0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0
Palm.	12	1 0 0 0 0 1 0 0 0	1 0 0 0	0 0 0 1 0 0 0 0 0 0 1 0 0 0 0 0
Palm.	22	1 0 0 0 0 1 0 0 0	0 1 0 0	0 0 0 1 1 0 0 0 0 0 1 1 0 0 1 0
Palm.	32	1 0 0 0 0 1 0 0 0	0 0 1 0	0 0 0 0 1 1 0 0 0 0 0 1 1 0 1 0
Palm.	48	1 0 0 0 0 1 0 0 0	0 0 0 1	0 0 0 0 0 1 0 0 0 0 0 0 1 0 0 0
Dove	32	1 0 0 0 0 0 0 0 0	0 0 1 0	0 0 0 0 0 0 0 0 0 0 0 1 0 0 1 0
Lux	22	1 0 0 0 0 0 1 0 0	0 1 0 0	0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0
Lux	32	1 0 0 0 0 0 1 0 0	0 0 1 0	0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0
Dawn	12	1 0 0 0 0 0 0 1 0	1 0 0 0	0 0 0 0 0 0 1 0 0 0 1 0 0 0 0 0
Dawn	22	1 0 0 0 0 0 0 1 0	0 1 0 0	0 0 0 0 0 0 1 1 0 0 1 1 0 0 0 0
Dawn	32	1 0 0 0 0 0 0 1 0	0 0 1 0	0 0 0 0 0 0 0 1 1 0 0 1 0 0 0 0
Dawn	48	1 0 0 0 0 0 0 1 0	0 0 0 1	0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
Sunl.	12	1 0 0 0 0 0 0 0 1	1 0 0 0	0 0 0 0 0 0 0 0 0 0 1 0 0 1 0 0
Sunl.	22	1 0 0 0 0 0 0 0 1	0 1 0 0	0 0 0 0 0 0 0 0 0 0 1 1 0 1 1 0
Sunl.	32	1 0 0 0 0 0 0 0 1	0 0 1 0	0 0 0 0 0 0 0 0 0 1 0 1 1 0 1 0
Sunl.	48	1 0 0 0 0 0 0 0 1	0 0 0 1	0 0 0 0 0 0 0 0 0 1 0 0 1 0 0 0
Store	32	1 0 0 0 0 0 0 0 0	0 0 1 0	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
Gen.	32	1 0 0 0 0 0 0 0 0	0 0 1 0	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1

Table III-19

Interpretation of Multi-Item Segments in Hybrid Structure

A priori Partitions

30	All available brands
31	Ivory, all sizes
32	Joy, all sizes
33	Ajax, all sizes
34	Dermassage, all sizes
35	Palmolive, all sizes
36	Lux, all sizes
37	Dawn, all sizes
38	Sunlight, all sizes
39	All 12 ounce sizes
40	All 22 ounce sizes
41	All 32 ounce sizes
42	All 48 ounce sizes

Overlapping Clustering Partitions

43	Small size Ivory (<48 ounce)
44	Large size Ivory (>12 ounce)
45	Medium size Joy (22 and 32 ounce)
46	Small size Palmolive (12 and 22 ounce)
47	Medium size Palmolive (22 and 32 ounce)
48	Large size Palmolive (32 and 48 ounce)
49	Small size Dawn (12 and 22 ounce)
50	Medium size Dawn (22 and 32 ounce)
51	Large size Dawn (32 and 48 ounce)
52	Large size Sunlight (32 and 48 ounce)
53	Small size, medium effectiveness
54	Medium size, nationally advertised
55	Large size, medium strength
56	Small size, lemon
57	Medium size, gentle and medium effectiveness
58	Price brands

generated. This structure was designated as the "random structure". Four of the comparison structures consisted of subsets of the 58 partitions in the hybrid structure. One subset, designated as the "reduced hybrid" was identical to the hybrid structure, except that the 14 smallest multi-item partitions (as estimated by WLS) were eliminated. The second subset structure consisted of only those partitions from the a priori analysis. The third subset structure consisted of only the single-item partitions and the all-item partition. The fourth comparison structure consisted only of the all-item partition, and is equivalent to an assumption of no structure. The final comparison structure was the 82 partition solution from sticky clustering (29 single-item partitions, one all-item partition, and 52 multi-item partitions).

For each market structure representation, the likelihood ratio chi square statistic G^2 was computed between each household's observed purchase probability vector and the expected purchase probability vector for each partition:

$$(G^2)_{mhx} = 2 \ \Sigma_i \, p_{ih} \log(p_{ih}/p_{mix}) \tag{3.5}$$

where p_{ih} = the observed probability of brand i being purchased by household h

p_{mix} = the expected probability of brand i purchase in partition x in structure representation m

After identifying the partition that minimized G^2 for each household, the minimum G^2 value for each household were summed to obtain G^2 for each of the market structure representations.

$$(G^2)_m = \sum_h \min_x [(G^2)_{mhx}] \tag{3.6}$$

Expected within-partition purchase probabilities (p_{mix}) were computed under different assumptions. For discrete structures containing multi-item partitions (hybrid, random, reduced hybrid,and a priori only), p_{mix} was assumed proportional to the square root of the major diagonal of the switching matrix, which is consistent with the normalization method used in obtaining the partitions. For the remaining discrete structures (single item and all item, and no structure), p_{mix} was computed as proportional to the marginal market shares, which is consistent with the models underlying these structures. For the sticky cluster structure, p_{mix} was obtained directly from the observed within-partition purchase probabilities of Table III-11. The G^2 statistics for each structure are presented in Table III-20.

The results of the analysis lend some credence to the identified hybrid structure as a good representation. G^2 for the hybrid structure was substantially lower than G^2 for a random structure containing the same number of partitions. The reduced version of the hybrid structure also outperformed the random structure while containing 14 fewer partitions. Elimination of partitions from the hybrid structure appears to cause a substantial increase in G^2; even after eliminating the smallest 14 partitions, G^2 increased by approximately 10%. All of the structural representations showed great improvements in fit compared to no structure.

The lowest G^2 statistic for any of the representations was for the sticky cluster solution. This is not surprising, as the sticky cluster representation has many more degrees of freedom than the discrete market structure representations. First, the sticky cluster solution contained the greatest number of partitions (82). Secondly,

within-partition purchase probabilities (p_{mix}) were computed directly from the observed data, essentially leaving them unconstrained. In contrast, p_{mix} for the discrete representations were computed using a proportionality rule.

Table III-20

Likelihood Ratio Chi Square Statistics For Various Market Structures

Market Structure	#of partitions	G^2
Hybrid Structure	58	2430.4
Random	58	2957.1
Reduced Hybrid	44	2687.9
A Priori Only	42	2924.6
Single-Item and All Items	30	3839.2
Sticky Clusters	82	2198.7
No Structure	1	10101.3

CHAPTER IV

ASSIGNMENT OF HOUSEHOLDS TO PARTITIONS

Bayesian Assignment

Using the derived discrete market structure and marginal market shares for each item, a fairly straightforward Bayesian approach can be used to assign each household to one of the revealed partitions. This approach was suggested first by Grover and Srinivasan (1987), but this dissertation is the first to employ the method on empirical panel data. To motivate the Bayesian approach, define the following terms:

n_{hi} = number of purchases of brand i by household h

L_{hx} = likelihood of household h purchase pattern being generated from partition x

q_{ix} = marginal purchase probability of brand i within partition x

α_x = prior probability of a household belonging to partition x

then

$$L_{hx} = (\sum_i n_{hi})!(\prod_i (q_{ix})^{n_{hi}})/\prod_i (n_{hi})! \qquad (4.1)$$

since the first term in the equation is a constant for each household, it can be rewritten as

$$L_{hx} = (k_h)\Pi(q_{ix})^{n_{hi}} \qquad (4.2)$$

using the familiar formula for Bayes' theorem, the posterior probability of household h belonging to partition x is

$$\pi_{hx} = (k_h \alpha_x \prod_i (q_{ix})^{n_{hi}})/(k_h \sum_y \alpha_y \prod_i (q_{iy})^{n_{hi}}) \tag{4.3}$$

where X = the number of partitions. If equal prior probabilities are assumed for each partition (i.e. $\alpha_x = 1/X$ for all x), then (3) can be simplified further to

$$\pi_{hx} = (\prod_i (q_{ix})^{n_{hi}})/(\sum_y \prod_i (q_{iy})^{n_{hi}}) \tag{4.4}$$

Thus, given a household's vector of purchases, and marginal purchase probabilities within segment, the relatively simple equation (4.4) can be used to generate posterior probabilities for partition membership.

While the assignment procedure is simple in concept, there are pragmatic problems in implementing it on empirical data. First, it can be very expensive computationally, depending on the number of identified partitions and the number of households one wishes to assign. For the data investigated in this dissertation, 134,096 posterior probabilities of the form (4.4) were computed. In general, the number of posterior probabilities required by the Bayesian approach equals the number of panel households times the number of identified partitions.

Computing Within-Segment Probabilities

A second pragmatic problem is how to compute initial within-partition purchase probabilities (q_{ix}). Certain estimation techniques such as Latent Class Analysis (Goodman 1968; Grover and Srinivasan 1987;

Parry and Gengler 1988) or disjoint clustering jointly estimate structure as well as within-partition purchase probabilities, and the purchase probabilities can be used directly. The revealed structure approach employed here only identifies discrete partitions, and the within-partition purchase probabilities must be reconstructed according to a model. One such model is that within-partition probabilities are proportional to the marginal market shares of the brands within the partition. A second model is that the within-partition probabilities are proportional to the square root of market shares (Bass 1974); a third model is that the within-partition probabilities are proportional to the square root of the major diagonal of the switching matrix (Hutchinson and Zenor 1987). Since the switching matrix was normalized using either the square root of the market shares or the square root of the major diagonal, the two latter models for within-partition probabilities were employed in the Bayesian assignment model. The values of the initial matrix of q_{ix}'s were computed by the following equation:

$$Q = (\mathrm{diag}(F'DF)^{-1} (DF)')' \tag{4.5}$$

where D = a diagonal matrix of normalization constants: either the square roots of market shares or the square roots of the major diagonal

F = the brand by partition feature matrix

Relaxation

A third pragmatic difficulty arise out of the form of the likelihood function. The Bayesian model imposes a rather severe likelihood penalty for household purchases outside the set of brands contained in a given partition. In essence, if a household made 10

purchases of brand i and one purchase of brand j, then the model would compute as zero that household's probability of being from the "brand loyal to i" partition, since $q_{ij} = 0$ for all $j \neq i$. This property of the likelihood function suggests that the sizes of brand loyal partitions will be systematically underestimated and that the size of the "all-brands" partition will be overestimated. In order to deal with this problem, the initial q_{ix} probabilities were relaxed such that there was a small but positive probability of making a non-partition purchase. In one relaxation, the probability of non-partition purchases was set at .01; in another, the probability of non-partition purchases was set at .10. In both cases, the probability for non-partition purchases was distributed proportionally to all non-partition items. For example, without relaxation, the probability of purchase of 12 ounce Ivory Liquid in partition 1 was 1; the probability of purchase for all other items was zero. After relaxation (using .01), there was a .99 probability of purchase of 12 ounce Ivory, while the remaining .01 of total purchase probability was distributed proportionally across the remaining 28 brands. The same relaxation of probabilities was applied to all 58 partitions. This adjustment mitigated against overassigning households to the "all brands" partition (29). The initial q_{ix}'s were adjusted according to the following equation:

$$Q_r = (1-k)Q + k(Q_c) \qquad (4.6)$$

where $Q_c = (\mathrm{diag}(F_c'DF_c)^{-1}(DF_c)')'$

F_c = the complement of the discrete feature matrix F

k = .01 (low probability of non-partition purchase)
.10 (high probability of non-partition purchase)

A further concern in the assignment of households to partitions is whether to use a proportional assignment rule (i.e., assign households to partitions with a probability equal to the posterior probabilities from equation 4) or a deterministic assignment rule (assign households to the partition that has the maximum computed posterior probability). An advantage of the proportional assignment rule is that it takes into account situations where the posterior probabilities of two or more partitions are close. For example, suppose that a household's posterior probabilities for partitions 4 and 31 are computed as .52 and .48, respectively. In such a case, it would seem reasonable to consider this household's partition membership equiprobable for the two partitions. A probabilistic assignment rule would do precisely that, while a deterministic rule would assign the entire household to partition 4. The drawback of proportional assignment is that it is not particularly amenable to choice modelling. For instance, a proportional assignment rule may be used to assign 48% of the household described above to partition 31 and 52% to partition 4. It might be assumed that 48% of the household's purchases were made when the household had the consideration set represented by partition 31, or it might be assumed that 48% of <u>each</u> of the household's purchases "came from" partition 31. If the former is assumed, a type of "coin flip" is required to assign each of the household's purchases to one of the partitions in order to model choice at the partition level. If the second assumption is adopted, then the posterior probabilities are required to attach a weight to each of the household's purchase records for further estimation. Either of these additional assignment procedures represent a substantial addition to the already considerable

computational cost entailed in the assignment of households. These procedures are outlined in Figures IV-1 and IV-2. Deterministic assignment eliminates many of these difficulties, by assigning all of the household's purchases to a single group. In essence, deterministic assignment acts as a method for the discrete segmentation of households. Disaggregated demand functions can be estimated by treating each segment, or group of households, as an independent submarket. The procedure for deterministic assignment is outlined in Figure IV-3.

Thus, a key concern is if the Bayesian approach yields differing marginal partition sizes depending on whether deterministic or probabilistic assignment is used. If the sizes of partitions estimated by probabilistic assignment differ greatly from those estimated by deterministic assignment, there is evidence that the deterministic assignment procedure does not adequately reflect the uncertainty regarding partition membership. If this is the case, then it is prudent to employ the probabilistic assignment rule and use one of the additional assignment procedures (Figures IV-1 and IV-2) before proceeding to demand function estimation. If, on the other hand, both the deterministic and the probabilistic methods yield similar marginal partition sizes, then there is little uncertainty as to which partition each household belongs. Thus, deterministic assignment would be seen as roughly equivalent to probabilistic and no additional assignment procedures would be necessary (e.g., Figure IV-3). Demand function estimation could proceed directly.

These issues relating to Bayesian assignment were investigated empirically by trying several different assignment procedures. First, two different methods of constructing initial estimates of within-

Figure IV-1

Probabilistic Assignment Procedure 1

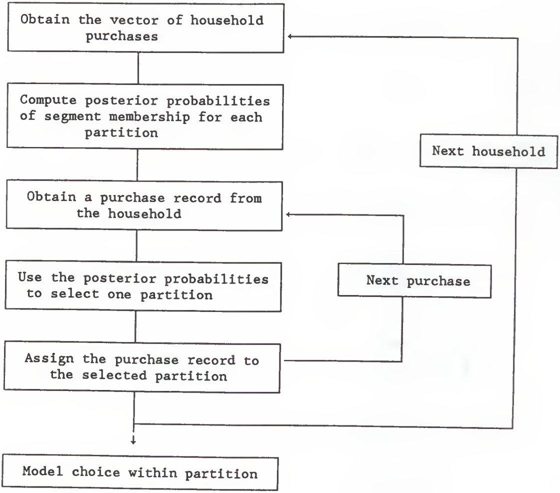


Figure IV-2

Probabilistic Assignment Procedure 2

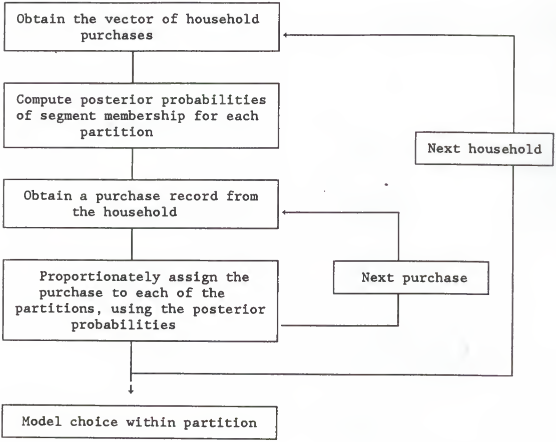
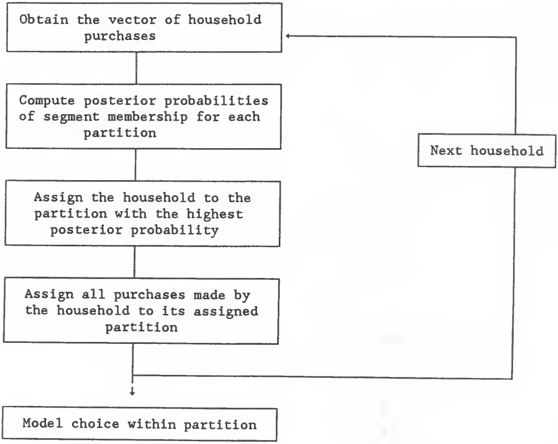


Figure IV-3

Discrete Assignment Procedure



partition purchase probability (q_{ix}) were used: in one, within-partition probabilities were initially set proportional to the square roots of market shares of each brand. In the other, within-partition probabilities were set proportional to the square root of the major diagonal of the 12-month switching matrix. Second, two different relaxations of within-partition purchase probabilities were used: one allowed a 10% probability of non-partition item purchases (high probability condition) while the other allowed a 1% probability of non-partition item purchases (low probability condition). Finally, both deterministic and probabilistic household assignment methods were used. Together, these generated eight different Bayesian assignment procedures. For each of these eight procedures, 134,096 posterior probabilities (2312 household times 58 partitions) were computed. In each of the eight procedures, household-level brand-by-partition sales matrices were constructed by the following equation:

$$\mathbf{S}_i = \mathbf{N'P} \tag{4.7}$$

where $\mathbf{N}_i$ = 1·29 vector of n_{hi}

$\mathbf{P}_i$ = 1·58 vector of π_{hx}

When deterministic assignment is used, P is a vector of 57 zeros and a single 1. When probabilistic assignment is used, P is a vector of the computed posterior probabilities. After summing over all households, the aggregate matrix S is an estimate of the sales of each brand in each partition. The S matrices computed from each of the six procedures are presented in Tables IV-1 through IV-8.

A very different method by which segment sizes might be obtained is a procedure suggested by Hutchinson and Zenor (1987). They use a weighted least squares (WLS) regression technique using the

observed pairwise switching frequencies as dependent measures, and generate a design matrix using the discrete market partitions. They show that the WLS parameter estimates are interpretable as estimated segment sizes. An advantage of the approach used by Hutchinson and Zenor is that panel data is only necessary to obtain the aggregate switching matrix. Unfortunately, the method does not segment individual households. The aggregate method proposed by Hutchinson and Zenor might also be seen as having restrictive assumptions, in that within-segment purchase probabilities are assumed to be proportional to the square root of the major diagonal of the switching matrix. For purposes of comparison, Hutchinson and Zenor's WLS regression method was replicated on the LDD data to obtain final estimates of segment sizes.

By summing over the rows in S, aggregate estimates of the size of each segment (in terms of unit purchases) can be obtained. The aggregate segment sizes estimated by the (household-level) Bayesian methods and by the (aggregate level) WLS method are presented in Table IV-9. To test if there are systematic differences between assignment methods, correlations and chi-square measures between each of the estimates are presented in Table IV-10.

To obtain a objective measure of fit for each of the assignment procedures, the estimated segment sizes and within-segment probabilities from each was used to generate predicted switching matrices, under the assumption of a zero-order switching process. Additional predicted switching matrices were generated using the WLS regression estimates and the sticky cluster structure described in Chapter III. Each of the predicted switching matrices were then compared to the observed

switching matrix. Chi-squares and correlations between the different measures are presented in Table IV-11.

Discussion

Referring to Tables IV-1 through IV-8, note that the most significant factor distinguishing among the eight Bayesian procedures is the level of relaxation of within-partition choice probabilities. When these probabilities were relaxed to allow 10% of purchases to fall outside the set of brands within the partition, the estimated size of the all-brands partition (30) dropped considerably, compared to the 1% relaxation. For example, when the q_{ix}'s were set proportional to the square roots of the market shares, proportional assignment estimated 2704.5 purchases from partition 30, when the 1% relaxation was used. However, when the 10% relaxation was used, the estimate of the size of partition 30 dropped to 487.52. When the q_{ix}'s were set proportional to the square root of the diagonal of the switching matrix, the estimated size of partition 30 dropped from 2445.74 to 389.27. In the deterministic assignment condition using the square root of the major diagonal, the estimated size of segment 30 dropped from 2512 to 320 when the relaxation increased from 1% to 10% . Such an effect was not completely unexpected. Consider what would have happened had the within-partition q_{ix}'s not been relaxed at all: in the brand loyal partitions, q_{ii}'s would have been set at 1 while q_{ij}'s would have been set at 0. A household which made 10 purchases of brand i and one purchase of brand j would have a 0 posterior probability of belonging to the brand-i-loyal partition. Only partitions containing both brand i and brand j would have positive posterior probabilities. Hence, slight relaxation tends to favor multi-brand partitions in general and the all-brands partition in

Table IV-1

Results of Bayesian Assignment
10% Relaxation, q_{ix} Based on Square Root of Market Share

Segment

Item	1	2	3	4	5	6	7	8
1	92.78	4.56	0.15	0.04	2.85	0.81	0.16	1.01
2	6.73	179.25	6.30	4.87	4.57	2.85	3.87	0.06
3	0.83	1.62	220.30	8.22	0.29	0.35	8.53	0.56
4	0.04	0.23	4.66	143.03	0.04	0.05	0.02	0.03
5	0.11	3.22	0.16	0.05	116.59	2.48	0.40	2.50
6	0.69	5.31	1.73	0.20	2.31	110.36	8.07	1.87
7	0.19	0.32	2.46	0.02	0.07	0.88	125.33	5.81
8	0.00	0.00	0.05	0.60	0.00	0.01	1.04	74.97
9	0.30	1.03	0.04	0.35	0.02	0.44	0.18	0.90
10	0.10	0.69	1.11	0.05	0.00	0.14	0.10	0.00
11	0.00	0.00	0.00	0.00	0.95	0.02	0.05	0.00
12	0.01	1.17	0.13	0.01	1.00	0.01	0.01	0.01
13	0.00	0.26	0.00	0.00	1.20	0.11	0.00	0.00
14	0.36	1.48	0.64	0.03	0.83	2.19	0.04	2.03
15	0.05	1.31	0.01	0.00	0.70	0.01	0.11	0.90
16	0.00	0.00	0.00	0.45	0.00	0.01	0.00	0.05
17	0.03	0.41	2.83	0.21	0.85	0.79	1.05	0.01
18	0.31	0.86	0.06	0.00	0.02	0.16	0.02	0.00
19	0.27	0.07	2.78	1.02	0.02	0.01	0.00	0.00
20	1.60	1.66	0.99	1.07	1.62	0.28	0.13	0.05
21	1.25	0.95	0.94	0.16	1.52	0.65	0.67	0.08
22	0.04	0.08	0.10	1.04	0.04	0.19	0.47	0.26
23	0.01	0.02	0.02	0.45	0.01	0.02	0.02	0.02
24	0.08	1.61	0.02	0.05	2.63	0.05	0.06	0.01
25	0.22	2.32	0.23	0.40	1.57	0.85	1.48	0.11
26	0.06	0.66	1.80	0.21	0.97	1.07	3.64	0.34
27	0.35	0.20	0.57	0.31	0.23	0.26	0.06	0.15
28	0.51	0.37	3.70	0.36	1.25	0.66	0.60	0.76
29	0.01	0.01	0.01	0.02	0.96	0.02	0.01	0.00

	9	10	11	13	14	15	16	17	18
1	1.65	0.02	0.02	0.04	2.02	0.14	0.81	0.01	0.98
2	0.91	0.19	0.06	1.36	0.46	3.18	0.41	0.07	0.59
3	0.09	1.03	0.98	0.97	0.05	0.64	3.01	1.02	2.13
4	0.77	1.86	0.02	0.19	0.02	0.02	0.02	0.06	0.51
5	1.57	0.03	0.03	0.08	0.04	0.91	0.03	0.99	0.04
6	1.52	0.19	0.05	3.15	0.05	0.47	0.05	0.08	3.56
7	1.26	0.04	0.31	0.02	0.02	0.15	0.25	0.02	1.92
8	0.00	0.00	0.00	0.08	0.00	0.00	0.00	0.03	0.00
9	43.66	0.96	1.14	1.82	0.01	0.28	0.60	0.82	0.43
10	0.21	39.17	0.28	1.39	0.11	0.05	0.09	0.00	1.10
11	0.05	0.00	34.44	0.02	0.01	0.00	0.71	0.00	0.00
12	0.03	0.01	4.75	100.30	0.02	0.12	0.01	0.01	0.01
13	0.65	0.21	0.08	0.07	47.04	0.17	0.08	0.00	0.00

Table IV-1 (continued)

Segment

Item	9	10	11	13	14	15	16	17	18
14	0.28	0.88	0.04	8.31	5.22	67.63	4.81	4.25	0.14
15	0.03	0.00	0.01	2.39	0.08	0.37	46.65	0.24	0.00
16	0.00	0.00	0.05	0.00	1.00	0.03	0.06	50.28	0.01
17	1.33	0.13	0.07	0.19	0.01	0.03	0.01	0.01	69.22
18	0.02	0.00	0.25	0.00	0.00	0.33	0.10	0.21	0.00
19	0.05	0.00	0.06	0.01	0.01	0.38	0.00	0.00	0.48
20	1.81	0.19	2.06	0.22	1.30	0.10	1.04	0.10	0.11
21	3.71	0.11	1.71	3.10	0.10	0.90	0.08	0.14	1.82
22	0.89	0.36	0.07	0.94	0.06	0.91	0.57	0.99	1.07
23	0.01	0.01	2.01	0.09	0.01	0.02	0.30	0.81	1.03
24	0.36	0.01	0.01	0.66	0.03	0.04	0.13	0.01	0.04
25	1.69	0.34	0.94	4.17	0.55	1.20	0.20	0.57	2.98
26	2.38	0.16	1.03	1.45	0.08	0.83	0.85	1.17	0.35
27	0.04	0.02	0.03	0.32	0.02	0.08	0.02	0.93	1.53
28	3.07	0.37	0.05	0.03	0.95	2.88	1.08	0.14	0.06
29	0.39	0.00	0.00	0.00	0.01	0.10	0.02	0.00	0.03

	19	20	21	22	23	23	24	25
1	0.10	0.69	1.86	0.68	0.03	0.01	0.08	1.05
2	1.03	0.38	1.07	2.84	2.28	0.07	0.71	2.23
3	0.55	0.34	0.19	1.49	1.97	2.37	0.06	0.59
4	0.02	0.04	0.02	0.05	0.02	0.02	0.02	0.14
5	0.10	0.05	1.48	1.45	0.88	0.02	1.35	0.85
6	2.50	0.07	1.82	4.21	1.20	0.84	0.30	0.96
7	0.10	0.02	0.04	0.27	1.56	0.81	1.07	0.26
8	0.00	0.00	0.00	0.00	0.00	0.17	0.00	0.00
9	0.02	0.22	1.61	0.05	0.67	0.00	1.16	1.94
10	0.00	0.00	0.02	0.04	1.26	0.00	0.03	0.14
11	0.01	0.03	0.00	0.76	0.74	0.00	0.00	0.01
12	0.01	0.95	0.01	0.44	0.13	0.17	0.48	0.04
13	0.00	0.00	0.32	0.64	0.01	0.00	1.41	1.12
14	1.42	0.05	1.31	1.91	1.17	0.03	0.18	1.31
15	0.22	0.02	0.06	0.05	0.50	0.19	0.43	0.04
16	0.41	1.72	0.01	0.01	0.00	0.00	0.00	0.10
17	0.01	1.25	0.02	0.13	1.52	0.01	0.01	0.69
18	14.59	0.24	0.03	0.27	0.05	0.08	0.11	0.25
19	0.00	31.61	0.01	0.13	0.00	0.00	0.00	0.14
20	1.42	1.13	220.01	5.77	4.82	0.40	1.96	1.57
21	0.78	0.31	8.07	200.03	9.95	3.09	0.08	1.62
22	0.91	0.04	1.48	3.03	122.34	2.16	0.05	1.27
23	0.01	0.01	0.07	1.16	0.39	93.16	0.01	0.02
24	0.36	0.01	3.26	0.08	0.63	0.01	52.56	1.38
25	1.63	0.34	4.50	3.86	0.63	0.11	7.56	130.96
26	0.04	0.04	0.74	0.30	1.46	0.06	0.19	4.49
27	0.26	0.02	0.14	0.02	0.27	0.10	0.49	0.27
28	0.37	0.04	3.15	5.61	0.28	0.48	0.03	4.23
29	0.01	0.02	0.09	0.09	0.01	0.01	0.69	0.17

Table IV-1 (continued)

Segment

Item	26	27	28	29	30	31	32	33
1	0.04	0.93	2.11	0.47	13.03	34.75	1.00	0.20
2	2.34	0.14	3.24	0.08	39.53	91.98	3.74	0.41
3	0.97	0.79	0.05	0.52	11.07	66.92	0.94	0.18
4	0.94	0.21	1.62	0.06	5.25	25.47	0.31	0.61
5	0.20	0.89	1.20	0.02	23.87	1.72	67.80	2.91
6	2.31	1.54	4.12	1.17	34.05	2.36	111.59	9.63
7	0.19	0.70	0.80	1.02	10.32	0.75	36.63	1.32
8	0.07	0.00	0.80	0.00	3.17	0.35	9.47	0.00
9	2.18	0.01	1.65	1.08	21.82	0.58	0.57	45.94
10	0.77	0.55	0.11	0.00	14.46	0.87	0.74	27.90
11	0.47	0.00	0.53	0.00	1.58	0.56	0.04	1.85
12	0.05	1.18	0.01	0.01	6.93	0.56	0.04	2.33
13	0.81	0.00	0.00	1.33	12.43	0.73	0.53	1.32
14	4.33	0.05	1.96	5.63	35.19	2.31	1.25	7.21
15	0.28	0.39	0.53	0.55	4.79	0.36	0.31	0.31
16	0.11	0.93	1.34	0.00	4.32	0.01	0.32	0.00
17	0.08	0.97	1.22	2.03	6.51	1.10	0.23	1.50
18	0.06	1.00	1.16	1.65	9.59	0.58	0.24	0.47
19	0.67	0.15	0.85	0.47	5.75	0.64	0.07	0.02
20	0.43	0.46	1.86	3.28	27.14	1.83	1.99	0.49
21	2.68	0.12	1.19	1.82	28.92	1.93	3.00	2.76
22	0.31	0.49	0.91	0.06	12.38	0.70	0.47	0.62
23	0.06	1.87	3.77	0.02	9.11	0.32	1.02	0.01
24	1.12	1.38	1.33	0.56	15.67	1.26	0.61	0.26
25	6.20	1.40	4.52	3.79	44.69	1.64	3.55	2.13
26	99.65	5.29	5.41	1.38	28.16	2.46	1.72	1.44
27	0.54	70.99	1.81	0.01	14.85	0.76	0.37	0.03
28	0.85	0.86	188.08	3.16	31.83	0.91	1.67	0.98
29	0.00	0.03	0.25	61.21	10.98	0.46	0.02	0.21

	34	35	36	37	38	39	40	41	42
1	0.03	0.18	0.36	1.95	1.56	65.54	7.63	0.88	0.21
2	0.58	4.66	0.49	3.10	7.73	26.74	99.01	4.78	1.55
3	1.13	1.30	0.35	1.88	2.93	3.65	3.95	32.04	3.05
4	0.13	0.07	0.04	0.83	0.23	3.23	2.31	0.99	43.44
5	0.09	1.34	3.07	1.94	6.92	138.45	5.29	2.59	0.13
6	0.85	2.67	2.79	3.19	19.55	9.60	92.80	6.10	3.02
7	0.08	2.34	1.06	1.03	5.46	0.86	3.10	23.30	1.21
8	0.04	0.01	0.01	0.00	0.46	0.00	0.82	2.00	18.15
9	1.10	0.52	0.13	0.99	5.01	7.98	26.71	7.50	0.13
10	0.84	1.04	0.00	0.43	2.78	2.99	1.01	17.16	0.95
11	6.21	0.03	1.02	0.35	0.07	0.06	4.63	0.24	0.00
12	38.73	0.84	2.71	1.76	5.97	2.00	1.31	18.73	0.15
13	1.68	21.44	0.00	1.03	0.41	83.76	4.70	0.13	0.00
14	3.15	107.05	2.88	2.17	13.69	6.57	77.45	7.17	1.63
15	1.18	39.88	0.58	0.81	1.72	0.11	2.20	12.99	0.07
16	0.00	31.79	0.41	0.02	0.69	0.05	1.46	1.22	13.75

Table IV-1 (continued)

Segment

Item	34	35	36	37	38	39	40	41	42
17	1.11	0.09	3.34	1.30	3.68	2.36	2.96	13.82	1.97
18	0.13	0.17	37.46	2.45	4.11	3.67	18.70	5.07	0.32
19	0.01	0.37	75.73	0.04	3.38	1.16	4.38	16.00	1.75
20	0.99	0.99	1.45	81.28	9.11	179.19	7.66	1.52	0.37
21	1.83	4.71	0.38	115.12	13.12	17.75	88.72	4.68	2.89
22	0.43	0.57	1.18	62.10	2.00	2.02	2.11	23.86	1.83
23	0.06	0.17	0.01	41.84	0.28	0.04	0.79	3.32	22.98
24	0.36	0.76	0.78	1.38	50.86	126.84	6.67	0.61	0.22
25	2.01	3.71	2.11	3.27	266.07	24.76	106.76	5.91	4.67
26	0.63	3.36	1.05	3.08	217.42	10.66	12.89	43.15	6.05
27	0.25	0.06	0.11	2.00	59.33	1.13	1.74	2.77	34.90
27	0.03	6.37	0.14	6.00	1.74	7.40	5.63	29.74	2.02
29	0.00	1.27	0.02	0.84	1.76	4.03	0.87	4.14	0.18

	43	44	45	46	47	48	49	50
1	91.31	0.81	0.35	0.12	2.11	0.26	1.92	0.86
2	228.42	89.69	3.26	3.26	3.70	0.25	0.59	1.95
3	114.81	105.57	3.26	0.24	0.59	0.11	0.39	0.59
4	0.56	32.78	0.05	0.02	0.98	0.04	0.04	0.04
5	3.79	0.70	1.24	3.42	0.58	0.05	2.83	2.39
6	3.48	2.60	207.59	1.32	1.31	0.13	8.47	4.67
7	1.59	0.78	115.32	0.09	2.39	1.70	0.23	1.51
8	0.03	0.51	0.13	0.00	0.00	0.02	0.00	0.00
9	0.84	1.02	2.87	0.17	0.63	1.24	0.13	1.09
10	1.80	0.52	0.07	0.05	2.11	0.06	0.05	0.92
11	0.69	0.72	0.03	0.00	0.02	0.20	0.86	0.14
12	1.09	0.08	0.01	1.0	1.05	0.78	0.16	0.29
13	4.64	0.11	0.03	34.4	0.85	0.04	0.26	0.06
14	7.02	2.10	1.14	84.9	180.27	8.35	1.83	5.66
15	0.62	0.58	0.14	0.3	78.70	38.70	0.60	2.82
16	0.00	0.01	0.01	0.0	0.25	41.59	0.02	0.01
17	2.61	1.30	1.23	0.0	1.14	0.01	0.11	2.54
18	1.91	0.60	0.88	1.1	0.22	0.15	0.05	2.87
19	1.22	0.25	0.01	0.2	2.88	0.00	0.03	2.71
20	3.74	1.75	0.73	0.2	0.14	0.09	127.01	4.35
21	5.44	2.09	0.63	0.4	6.71	0.13	159.90	219.62
22	0.81	0.61	0.49	0.6	1.57	0.24	1.53	106.44
23	0.03	0.49	0.02	0.02	0.07	1.13	0.56	2.47
24	1.69	1.48	0.24	0.30	1.15	0.07	1.09	1.52
25	3.34	1.34	4.05	5.26	4.99	0.50	3.92	2.79
26	3.59	2.66	4.22	1.13	5.19	1.48	0.69	2.66
27	1.16	0.97	0.26	0.06	0.06	0.09	0.10	0.05
28	0.97	0.72	0.56	6.03	6.15	1.07	3.06	4.95
29	0.87	0.18	0.02	0.21	0.15	0.78	0.14	0.17

Table IV-1 (continued)

	Segment							
Item	51	52	53	54	55	56	57	58
1	1.98	0.59	12.81	9.82	1.15	4.11	4.57	1.51
2	3.45	2.33	18.84	89.27	6.85	10.88	108.05	2.48
3	1.69	2.21	1.73	43.20	43.00	1.09	60.26	0.08
4	1.06	1.43	1.41	1.79	10.50	0.10	0.88	0.46
5	0.17	1.30	75.33	9.19	1.47	66.39	8.40	0.63
6	1.84	2.58	68.46	97.67	10.18	52.02	117.36	2.11
7	3.08	3.78	6.44	32.28	27.32	0.91	43.26	0.69
8	1.02	0.99	0.70	1.27	9.27	1.78	1.52	0.21
9	0.23	1.36	45.25	38.40	0.55	0.98	7.32	2.11
10	0.32	2.17	11.07	32.16	1.32	0.53	5.85	0.08
11	0.09	0.25	0.59	0.42	0.05	0.18	4.75	0.34
12	0.28	0.39	1.29	2.72	1.27	0.46	26.65	0.01
13	0.01	0.12	40.49	5.76	0.76	2.72	6.00	0.86
14	2.03	6.67	73.09	86.65	2.68	5.62	88.89	8.58
15	0.83	1.73	3.90	22.88	11.79	0.61	20.43	0.60
16	0.00	1.58	0.72	5.41	7.77	0.10	2.10	0.57
17	0.80	0.86	3.90	16.58	1.44	0.52	25.17	0.50
18	0.15	0.95	5.27	6.94	0.37	1.66	21.23	2.51
19	0.00	1.96	4.24	5.46	0.52	1.14	23.47	0.20
20	2.04	0.53	93.42	11.63	0.55	10.22	6.26	2.60
21	8.73	3.55	88.97	90.71	2.92	2.50	15.63	1.23
22	84.22	1.37	6.70	30.57	3.49	0.34	3.14	2.17
23	79.04	0.91	1.33	4.35	2.69	0.02	1.50	1.73
24	0.19	0.58	59.19	5.60	1.71	56.92	3.30	0.94
25	1.58	7.63	130.05	108.11	6.37	86.71	129.97	5.49
26	2.99	142.43	18.53	102.04	58.05	16.59	133.48	1.80
27	0.83	76.74	4.13	10.99	18.73	1.45	7.51	0.22
28	2.15	2.77	9.55	10.70	0.96	4.22	9.97	101.45
29	0.01	0.01	3.33	5.33	0.14	1.43	1.56	24.43

Table IV-2

Results of Bayesian Assignment
1% Relaxation, q_{ix} Based on Square Root of Market Share

Segment

Item	1	2	3	4	5	6	7	8	9
1	86.35	2.07	0.01	0.00	0.34	0.09	0.00	1.00	0.98
2	5.07	136.79	2.47	2.66	0.15	1.02	0.26	0.00	0.61
3	0.05	0.10	165.38	4.35	0.14	0.04	1.81	0.05	0.00
4	0.00	0.02	1.82	122.32	0.00	0.00	0.00	0.00	0.52
5	0.00	2.33	0.08	0.00	84.17	0.33	0.00	1.18	0.02
6	0.21	0.11	0.38	0.00	0.30	85.28	3.06	1.24	0.04
7	0.00	0.03	0.47	0.00	0.00	0.06	82.04	1.87	0.15
8	0.00	0.00	0.00	0.01	0.00	0.00	0.05	61.91	0.00
9	0.00	0.11	0.00	0.24	0.00	0.00	0.05	0.33	32.11
10	0.00	0.14	0.00	0.00	0.00	0.00	0.00	0.00	0.00
11	0.00	0.00	0.00	0.00	0.67	0.00	0.00	0.00	0.00
12	0.00	0.29	0.00	0.00	0.67	0.00	0.00	0.00	0.00
13	0.00	0.06	0.00	0.00	0.04	0.00	0.00	0.00	0.00
14	0.17	0.12	0.16	0.00	0.40	0.31	0.00	1.29	0.00
15	0.00	0.23	0.00	0.00	0.07	0.00	0.01	0.33	0.00
16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
17	0.00	0.00	1.00	0.13	0.77	0.04	0.05	0.00	0.21
18	0.03	0.26	0.02	0.00	0.00	0.01	0.00	0.00	0.00
19	0.00	0.02	0.68	0.99	0.00	0.00	0.00	0.00	0.00
20	0.05	0.42	0.34	1.00	0.09	0.05	0.00	0.00	0.83
21	0.19	0.28	0.26	0.01	0.24	0.09	0.15	0.00	2.91
22	0.00	0.00	0.00	1.00	0.00	0.03	0.01	0.00	0.82
23	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.00	0.00
24	0.00	0.99	0.00	0.00	0.48	0.00	0.00	0.00	0.10
25	0.01	1.06	0.01	0.18	0.08	0.07	0.14	0.01	0.58
26	0.00	0.02	0.01	0.02	0.09	0.20	1.93	0.05	1.11
27	0.00	0.00	0.00	0.03	0.12	0.06	0.00	0.00	0.00
28	0.04	0.13	0.00	0.04	0.07	0.21	0.03	0.17	0.34
29	0.00	0.00	0.00	0.00	0.15	0.00	0.00	0.00	0.21

	10	11	12	13	14	15	16	17	18
1	0.00	0.00	0.00	2.00	0.06	0.26	0.00	0.31	0.00
2	0.00	0.00	1.00	0.24	0.58	0.08	0.00	0.00	0.01
3	0.51	0.19	0.91	0.00	0.00	2.29	0.96	1.31	0.21
4	1.02	0.00	0.00	0.00	0.00	0.00	0.00	0.33	0.00
5	0.00	0.00	0.00	0.00	0.65	0.00	0.97	0.00	0.00
6	0.00	0.00	0.13	0.00	0.02	0.00	0.00	3.10	0.23
7	0.00	0.01	0.00	0.00	0.00	0.02	0.00	1.03	0.00
8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
9	0.51	0.00	1.05	0.00	0.00	0.05	0.41	0.15	0.00
10	34.11	0.00	0.16	0.00	0.00	0.00	0.00	1.00	0.00
11	0.00	16.26	0.00	0.00	0.00	0.26	0.00	0.00	0.00
12	0.00	1.09	78.84	0.00	0.00	0.00	0.00	0.00	0.00
13	0.01	0.00	0.00	34.53	0.01	0.00	0.00	0.00	0.00
14	0.81	0.00	0.51	2.41	54.25	2.10	2.54	0.00	0.83

Table IV-2 (continued)

Segment

Item	10	11	12	13	14	15	16	17	18
15	0.00	0.00	0.00	0.00	0.02	35.01	0.02	0.00	0.01
16	0.00	0.00	0.00	0.21	0.00	0.00	42.48	0.00	0.33
17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	64.18	0.00
18	0.00	0.00	0.00	0.00	0.00	0.00	0.17	0.00	8.11
19	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
20	0.01	2.00	0.00	1.01	0.00	0.76	0.00	0.00	0.00
21	0.00	0.39	0.71	0.00	0.03	0.00	0.01	1.19	0.16
22	0.01	0.00	0.77	0.00	0.43	0.01	0.95	1.00	0.01
23	0.00	0.48	0.00	0.00	0.0	0.21	0.27	1.00	0.00
24	0.00	0.00	0.65	0.00	0.0	0.00	0.00	0.00	0.01
25	0.01	0.19	2.36	0.09	0.0	0.01	0.28	2.03	0.36
26	0.00	0.20	0.00	0.00	0.1	0.07	0.45	0.05	0.00
27	0.00	0.00	0.16	0.00	0.0	0.00	0.63	1.31	0.01
28	0.07	0.00	0.00	0.89	2.5	0.81	0.00	0.00	0.01
29	0.00	0.00	0.00	0.00	0.0	0.00	0.00	0.00	0.00

	19	20	21	22	23	24	25	26
1	0.66	0.17	0.07	0.00	0.00	0.00	0.84	0.00
2	0.11	0.97	0.63	0.52	0.00	0.01	0.86	1.03
3	0.00	0.08	0.50	0.57	1.84	0.00	0.00	0.03
4	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.46
5	0.00	0.05	0.35	0.33	0.00	0.01	0.13	0.01
6	0.00	0.21	1.61	0.88	0.76	0.00	0.05	0.28
7	0.00	0.00	0.05	0.02	0.76	0.85	0.02	0.00
8	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00
9	0.00	0.49	0.00	0.51	0.00	0.69	0.87	0.64
10	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00
11	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.23
12	0.77	0.00	0.32	0.02	0.00	0.05	0.00	0.00
13	0.00	0.01	0.00	0.00	0.00	0.84	1.02	0.45
14	0.00	0.99	0.48	0.14	0.00	0.00	0.14	1.32
15	0.00	0.00	0.00	0.05	0.13	0.04	0.00	0.01
16	1.54	0.00	0.00	0.00	0.00	0.00	0.05	0.01
17	0.78	0.00	0.02	0.98	0.00	0.00	0.14	0.01
18	0.02	0.00	0.01	0.00	0.00	0.00	0.03	0.00
19	28.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00
20	0.86	154.46	1.54	1.63	0.02	0.97	1.04	0.03
21	0.00	1.05	126.13	4.13	1.08	0.00	0.40	1.13
22	0.00	0.12	0.10	85.71	0.42	0.00	1.06	0.04
23	0.00	0.00	0.08	0.03	85.38	0.00	0.00	0.00
24	0.00	1.90	0.00	0.06	0.00	34.56	1.01	0.47
25	0.01	1.51	1.10	0.15	0.01	2.58	109.82	0.85
26	0.00	0.01	0.03	0.26	0.00	0.00	1.19	65.99
27	0.00	0.06	0.00	0.00	0.01	0.06	0.01	0.04
28	0.00	3.00	4.93	0.03	0.18	0.00	3.91	0.03
29	0.00	0.05	0.04	0.00	0.00	0.04	0.00	0.00

Table IV-2 (continued)

Segment

Item	27	28	29	30	31	32	33	34
1	0.37	1.48	0.00	95.65	32.68	0.05	0.02	0.00
2	0.00	1.75	0.00	206.85	88.05	2.49	0.20	0.14
3	0.00	0.00	0.00	61.25	67.64	0.41	0.49	0.24
4	0.01	0.00	0.00	32.20	28.26	0.02	1.27	0.00
5	0.28	0.06	0.00	153.63	0.41	65.89	1.63	0.00
6	0.19	1.95	0.87	182.75	0.64	119.24	1.91	0.08
7	0.21	0.07	1.00	60.05	0.42	41.21	0.45	0.00
8	0.00	0.00	0.00	19.71	0.27	18.15	0.00	0.00
9	0.00	0.63	0.28	103.06	0.22	0.23	25.94	0.14
10	0.09	0.02	0.00	59.59	0.56	0.02	24.32	0.10
11	0.00	0.07	0.00	10.29	0.57	0.01	0.00	18.50
12	0.71	0.00	0.00	44.39	0.17	0.01	0.23	32.72
13	0.00	0.00	0.00	68.34	0.07	0.13	0.90	0.00
14	0.00	1.09	0.35	173.98	1.06	2.47	2.59	0.48
15	0.00	0.16	0.00	45.08	0.18	0.03	0.00	0.00
16	0.00	0.00	0.00	25.29	0.00	0.18	0.00	0.00
17	0.26	0.05	1.73	37.24	0.42	0.42	0.55	0.99
18	0.55	0.14	0.00	55.24	0.53	0.10	0.00	0.00
19	0.00	0.83	0.00	39.03	0.34	0.00	0.00	0.00
20	0.20	0.74	2.39	150.94	0.29	0.96	0.16	0.00
21	0.00	0.57	1.39	202.48	0.77	1.39	0.13	1.33
22	0.01	0.02	0.00	72.29	0.25	0.03	0.16	0.29
23	0.62	0.63	0.00	47.23	0.00	0.99	0.00	1.51
24	0.47	1.26	0.00	96.65	0.40	0.24	0.06	0.34
25	0.13	2.95	2.02	236.77	0.69	0.62	1.20	1.64
26	1.22	3.97	0.00	169.44	1.34	0.13	0.37	0.01
27	47.08	1.79	0.00	73.94	0.22	0.08	0.00	0.13
28	0.07	163.80	1.21	140.78	0.70	1.74	0.14	0.00
29	0.00	0.02	49.17	40.28	0.24	0.00	0.12	0.00

	35	36	37	38	39	40	41	42
1	0.10	0.31	1.41	1.38	44.87	2.35	0.92	0.00
2	3.03	0.08	2.54	3.58	5.91	85.41	1.49	1.02
3	0.83	0.07	1.58	1.69	2.07	1.24	46.42	2.12
4	0.01	0.00	0.10	0.47	3.00	0.13	1.29	39.38
5	0.24	2.98	1.83	3.44	106.45	1.78	0.22	0.05
6	0.04	1.05	1.14	5.82	2.64	68.49	0.83	1.50
7	2.02	1.00	0.06	3.05	0.51	0.73	19.14	0.27
8	0.00	0.00	0.00	0.00	0.00	0.05	1.15	14.44
9	0.65	0.00	0.07	2.86	3.47	20.34	1.10	0.08
10	0.12	0.00	0.25	0.63	0.00	0.46	10.69	0.99
11	0.00	0.00	0.23	0.03	0.00	6.01	0.21	0.00
12	1.10	0.12	0.07	2.24	0.91	0.87	8.64	0.49
13	28.05	0.00	0.00	0.43	64.16	1.59	0.00	0.00
14	103.60	2.08	0.59	6.20	0.30	59.14	2.94	0.25
15	46.62	0.00	0.67	0.79	0.00	0.26	10.13	0.04
16	41.51	0.35	0.00	0.25	0.00	0.12	1.09	12.50

Table IV-2 (continued)

Segment

Item	35	36	37	38	39	40	41	42
17	0.00	3.10	0.07	2.38	0.09	0.15	12.62	0.20
18	0.06	34.32	0.04	2.99	0.46	20.18	0.15	0.47
19	0.11	72.01	0.00	2.47	0.00	1.75	11.15	1.11
20	0.01	1.14	93.59	0.37	151.17	1.66	1.18	0.06
21	3.05	0.05	124.04	7.28	4.62	71.66	0.50	0.06
22	0.20	1.00	77.31	0.92	0.06	0.33	25.16	0.02
23	0.43	0.00	44.48	0.02	0.00	0.01	0.81	21.24
24	0.46	0.00	0.87	48.94	92.16	6.13	0.30	0.01
25	1.74	0.12	1.50	220.94	10.16	81.52	1.35	1.12
26	1.77	0.99	2.14	201.33	6.42	1.32	37.72	1.95
27	0.09	0.00	1.02	54.71	0.09	0.85	2.38	29.33
28	3.77	0.00	2.81	1.38	1.85	1.08	25.99	1.72
29	1.10	0.00	0.10	0.23	3.91	0.18	4.83	0.00

	43	44	45	46	47	48	49	50
1	81.45	0.07	0.06	0.05	2.01	0.07	1.95	0.22
2	213.76	80.53	1.94	1.55	0.64	0.04	0.68	0.20
3	111.06	102.26	2.04	0.00	0.01	0.05	0.70	0.56
4	0.05	39.28	0.00	0.00	0.18	0.00	0.00	0.00
5	1.88	0.17	0.67	2.35	0.35	0.02	1.54	2.18
6	0.96	0.24	195.69	0.99	0.02	0.00	5.14	1.40
7	1.20	0.45	119.79	0.00	1.38	0.27	0.03	0.07
8	0.00	0.40	0.01	0.00	0.00	0.00	0.00	0.00
9	0.46	0.49	2.49	0.00	0.19	1.03	0.05	0.98
10	1.22	0.42	0.00	0.00	0.98	0.00	0.00	0.74
11	0.70	0.72	0.00	0.00	0.00	0.07	0.50	0.01
12	0.55	0.01	0.00	0.21	0.18	0.00	0.11	0.19
13	0.98	0.03	0.00	34.45	0.11	0.00	0.00	0.00
14	4.12	0.53	0.01	75.22	153.52	2.23	0.51	3.95
15	0.35	0.33	0.01	0.03	71.73	25.69	0.52	2.37
16	0.00	0.00	0.00	0.02	0.01	33.20	0.00	0.00
17	1.39	0.51	0.65	0.00	0.18	0.00	0.01	0.27
18	1.86	0.47	0.60	0.96	0.00	0.08	0.00	0.06
19	0.85	0.14	0.00	0.00	0.96	0.00	0.00	0.02
20	0.73	0.08	0.11	0.00	0.00	0.01	136.72	1.15
21	1.72	1.30	0.10	0.02	5.38	0.00	159.52	209.41
22	0.57	0.00	0.02	0.29	0.39	0.05	0.22	107.91
23	0.00	0.00	0.00	0.00	0.04	0.73	0.06	1.13
24	0.99	0.04	0.01	0.00	0.82	0.00	0.97	1.14
25	2.26	0.11	1.31	3.83	2.99	0.14	1.54	0.19
26	2.13	0.46	1.12	0.08	2.35	0.14	0.02	0.07
27	0.26	0.92	0.05	0.00	0.00	0.03	0.03	0.00
28	0.75	0.58	0.18	3.81	5.03	0.09	0.60	3.26
29	0.76	0.00	0.00	0.04	0.04	0.26	0.05	0.03

Table IV-2 (continued)

	Segment							
Item	51	52	53	54	55	56	57	58
1	1.96	0.01	3.59	2.61	0.77	3.56	2.13	0.44
2	0.88	1.23	3.71	101.78	3.22	2.06	119.02	0.84
3	0.54	1.03	0.13	50.01	44.30	0.06	88.57	0.00
4	0.89	0.60	0.12	0.35	14.50	0.03	1.09	0.00
5	0.06	1.41	70.53	2.92	0.48	53.40	4.26	0.03
6	1.35	0.36	63.89	124.78	3.99	32.93	121.23	0.73
7	1.56	0.61	2.30	35.49	35.01	0.15	58.84	0.03
8	1.00	0.05	0.03	0.13	11.99	0.00	0.49	0.00
9	0.17	0.24	29.59	48.56	0.48	0.06	3.25	1.45
10	0.00	0.97	1.63	35.42	0.45	0.01	2.70	0.01
11	0.00	0.12	1.01	0.09	0.01	0.01	8.20	0.04
12	0.01	0.05	1.07	1.10	0.17	0.21	52.22	0.00
13	0.00	0.06	38.23	1.28	0.05	0.30	4.67	0.00
14	0.98	2.96	60.69	101.31	0.22	0.89	111.96	6.53
15	0.07	0.20	0.37	31.39	12.20	0.46	21.09	0.10
16	0.00	0.16	0.08	3.83	7.43	0.02	0.17	0.00
17	0.12	0.94	1.17	20.22	1.48	0.08	28.14	0.03
18	0.00	0.61	2.40	2.25	0.03	0.08	17.44	1.09
19	0.00	1.22	1.03	3.41	0.03	0.32	26.13	0.16
20	0.40	0.08	114.19	5.49	0.28	2.10	2.03	1.38
21	3.02	1.04	86.58	105.41	0.76	0.49	7.64	0.44
22	68.91	0.85	3.19	39.73	1.50	0.04	0.38	1.14
23	75.42	0.64	0.79	1.16	1.57	0.00	0.45	0.35
24	0.02	0.09	74.49	2.74	0.07	37.00	3.23	0.64
25	0.19	0.51	120.93	133.81	1.35	56.05	141.49	3.99
26	1.71	99.86	7.71	132.86	42.54	4.74	167.08	0.98
27	0.04	62.22	3.05	2.78	32.76	0.41	4.83	0.20
28	1.12	1.58	4.41	2.17	0.99	1.15	2.81	90.75
29	0.00	0.00	2.04	1.02	0.00	0.33	0.14	22.41

Table IV-3

Results of Bayesian Assignment
10% Relaxation, q_{ix} Based on Square Root of Major Diagonal

Segment

Item	1	2	3	4	5	6	7	8	9
1	88.46	5.58	0.14	0.03	2.46	0.99	0.15	1.01	1.17
2	5.73	194.54	6.61	4.30	3.82	3.34	3.68	0.05	0.76
3	0.34	2.18	232.33	6.69	0.26	0.42	7.97	0.32	0.06
4	0.04	0.27	5.09	139.86	0.05	0.06	0.02	0.02	0.61
5	0.05	4.16	0.19	0.04	110.88	3.19	0.25	1.52	1.51
6	0.45	5.87	1.92	0.29	1.60	126.76	5.57	1.53	1.75
7	0.17	0.40	2.99	0.02	0.07	2.00	112.79	3.74	1.17
8	0.00	0.00	0.08	1.00	0.00	0.02	1.05	67.52	0.00
9	0.16	1.23	0.04	0.41	0.02	0.42	0.19	0.75	41.72
10	0.03	0.82	1.93	0.19	0.00	0.25	0.13	0.00	0.45
11	0.00	0.00	0.00	0.00	0.94	0.02	0.05	0.00	0.01
12	0.01	1.55	0.31	0.01	0.99	0.02	0.01	0.01	0.03
13	0.00	0.37	0.00	0.00	1.13	0.14	0.00	0.00	0.50
14	0.28	1.91	0.68	0.03	0.75	2.30	0.03	1.84	0.11
15	0.05	1.55	0.01	0.01	0.69	0.01	0.12	0.74	0.02
16	0.00	0.00	0.00	0.85	0.00	0.02	0.00	0.04	0.00
17	0.04	0.62	3.81	0.24	0.83	1.26	1.05	0.01	0.86
18	0.17	0.99	0.10	0.00	0.02	0.28	0.02	0.00	0.01
19	0.12	0.12	3.89	1.02	0.02	0.01	0.01	0.01	0.02
20	1.00	2.03	1.04	1.04	1.45	0.28	0.10	0.04	1.64
21	0.79	1.05	0.95	0.12	1.38	0.64	0.54	0.06	3.21
22	0.04	0.09	0.15	1.04	0.04	0.23	0.37	0.13	0.90
23	0.02	0.02	0.02	0.92	0.02	0.02	0.02	0.02	0.02
24	0.06	1.83	0.02	0.02	2.14	0.05	0.06	0.01	0.24
25	0.18	2.51	0.24	0.34	0.95	1.24	1.20	0.09	1.04
26	0.05	0.91	2.27	0.18	0.91	1.27	3.48	0.23	2.10
27	0.28	0.28	0.98	0.49	0.24	0.40	0.07	0.08	0.02
28	0.39	0.55	4.03	0.53	1.32	0.95	0.50	0.45	2.59
29	0.02	0.01	0.02	0.02	0.96	0.03	0.01	0.01	0.30

	10	11	12	13	14	15	16	17	18
1	0.01	0.02	0.02	2.02	0.17	0.69	0.01	0.79	0.06
2	0.11	0.05	1.27	0.33	4.25	0.26	0.06	0.08	0.44
3	0.98	0.72	1.04	0.04	0.89	3.00	1.00	1.79	0.35
4	1.83	0.02	0.14	0.02	0.02	0.02	0.06	0.38	0.02
5	0.04	0.02	0.06	0.03	1.25	0.03	0.99	0.03	0.06
6	0.12	0.04	2.95	0.04	0.54	0.04	0.07	3.34	1.06
7	0.03	0.14	0.02	0.02	0.22	0.18	0.02	1.52	0.06
8	0.00	0.00	0.04	0.00	0.00	0.00	0.03	0.00	0.00
9	0.97	0.05	2.24	0.01	0.32	0.41	0.64	0.28	0.01
10	38.15	0.01	1.27	0.09	0.11	0.03	0.00	1.05	0.00
11	0.00	21.24	0.09	0.01	0.00	0.62	0.00	0.00	0.00
12	0.02	1.99	107.51	0.03	0.23	0.01	0.01	0.01	0.01
13	0.13	0.00	0.05	40.31	0.43	0.06	0.00	0.00	0.00

Table IV-3 (continued)

Segment

	10	11	12	13	14	15	16	17	18
14	0.83	0.03	7.06	3.73	89.34	2.49	3.56	0.08	0.88
15	0.01	0.01	2.19	0.07	0.94	38.16	0.32	0.01	0.12
16	0.00	0.00	0.00	0.80	0.05	0.08	47.57	0.01	0.31
17	0.08	0.01	0.21	0.01	0.03	0.01	0.02	65.73	0.01
18	0.00	0.02	0.00	0.00	0.40	0.08	0.22	0.00	9.74
19	0.01	0.01	0.01	0.01	0.58	0.01	0.01	0.08	0.01
20	0.11	2.02	0.17	1.15	0.10	1.03	0.07	0.08	0.57
21	0.08	0.80	3.29	0.06	0.93	0.06	0.10	1.46	0.34
22	0.19	0.04	1.07	0.05	1.17	0.38	0.98	1.06	0.39
23	0.02	0.92	0.06	0.02	0.02	0.24	0.77	1.03	0.02
24	0.01	0.01	0.78	0.02	0.04	0.08	0.01	0.01	0.18
25	0.22	0.43	3.93	0.40	1.44	0.14	0.43	2.56	0.82
26	0.09	0.74	0.75	0.05	1.15	0.55	0.95	0.23	0.04
27	0.02	0.02	0.28	0.02	0.11	0.02	0.86	1.39	0.14
28	0.28	0.04	0.03	0.86	3.98	0.83	0.11	0.05	0.21
29	0.01	0.01	0.01	0.01	0.17	0.02	0.01	0.03	0.01

	19	20	21	22	23	24	25	26
1	0.74	3.15	1.18	0.04	0.01	0.06	1.16	0.05
2	0.29	1.08	4.43	1.71	0.06	0.45	2.68	2.61
3	0.24	0.20	1.95	1.64	2.13	0.05	1.15	1.47
4	0.04	0.02	0.05	0.02	0.03	0.02	0.18	1.10
5	0.03	2.13	2.09	0.64	0.02	0.28	0.99	0.28
6	0.04	1.82	4.93	1.15	0.80	0.11	1.26	2.52
7	0.02	0.04	0.35	1.51	0.77	1.02	0.29	0.43
8	0.00	0.00	0.00	0.00	0.23	0.00	0.01	0.20
9	0.10	1.85	0.07	0.64	0.01	0.99	2.04	2.47
10	0.00	0.03	0.05	1.27	0.00	0.01	0.18	0.94
11	0.00	0.00	0.91	0.71	0.00	0.00	0.01	0.55
12	0.94	0.01	0.54	0.15	0.02	0.31	0.04	0.35
13	0.00	2.16	0.74	0.01	0.00	0.97	1.16	0.91
14	0.04	1.23	2.27	0.87	0.03	0.09	1.40	4.87
15	0.01	0.05	0.07	0.32	0.19	0.25	0.04	0.39
16	1.85	0.02	0.02	0.00	0.00	0.00	0.16	0.21
17	1.48	0.03	0.19	1.35	0.02	0.01	0.92	0.12
18	1.02	0.07	0.62	0.06	0.01	0.10	0.36	0.07
19	40.40	0.01	0.30	0.01	0.01	0.01	0.25	0.77
20	1.06	228.00	6.35	3.83	0.15	1.46	1.60	0.48
21	0.15	6.00	250.93	6.03	2.09	0.06	1.83	2.91
22	0.04	1.51	5.43	104.58	1.82	0.05	1.33	0.37
23	0.02	0.08	1.73	0.49	94.01	0.02	0.02	0.14
24	0.01	4.10	0.15	0.49	0.01	43.65	1.60	1.27
25	0.19	4.55	5.56	0.52	0.10	5.83	143.77	7.41
26	0.04	0.81	0.37	1.38	0.06	0.12	6.71	116.02
27	0.02	0.19	0.03	0.21	0.14	0.32	0.35	1.13
28	0.04	3.19	6.28	0.30	0.30	0.04	4.45	1.14
29	0.03	0.17	0.17	0.02	0.01	0.41	0.24	0.01

Table IV-3 (continued)

Segment

Item	27	28	29	30	31	32	33	34
1	0.73	2.33	0.25	9.94	29.05	0.81	0.08	0.02
2	0.10	3.35	0.06	26.30	69.87	3.06	0.29	0.25
3	0.50	0.04	0.28	8.68	54.10	0.83	0.17	0.91
4	0.26	1.87	0.04	5.26	25.37	0.45	0.53	0.08
5	0.55	1.31	0.02	18.29	1.70	63.37	2.34	0.06
6	1.05	4.14	1.07	22.76	2.12	96.83	6.78	0.39
7	0.62	0.98	1.02	8.40	0.70	35.42	0.94	0.06
8	0.00	0.93	0.00	3.60	0.41	15.61	0.00	0.01
9	0.01	1.76	0.43	16.13	0.53	0.35	35.57	0.52
10	0.26	0.17	0.00	9.47	0.70	0.59	24.62	0.47
11	0.00	0.60	0.00	1.65	0.41	0.04	0.12	16.19
12	1.19	0.01	0.01	6.60	0.50	0.05	0.80	26.31
13	0.00	0.01	0.42	9.27	0.41	0.38	1.18	1.16
14	0.03	3.23	1.5	25.89	1.72	1.04	5.29	1.57
15	0.03	0.62	0.12	4.66	0.29	0.40	0.06	0.63
16	0.22	1.56	0.00	5.29	0.01	0.49	0.00	0.00
17	0.71	1.41	1.99	5.85	0.89	0.17	1.21	1.00
18	1.02	1.35	0.33	9.65	0.49	0.19	0.04	0.03
19	0.17	0.98	0.25	4.94	0.61	0.06	0.01	0.01
20	0.35	2.37	2.6	22.32	1.52	1.77	0.36	0.70
21	0.08	1.17	1.5	20.00	1.43	2.71	1.62	1.60
22	0.28	1.03	0.0	11.03	0.62	0.43	0.29	0.27
23	1.56	4.34	0.0	8.67	0.31	1.02	0.02	1.12
24	1.02	1.52	0.2	12.25	1.07	0.54	0.15	0.25
25	0.68	4.86	2.6	32.21	1.32	2.36	1.49	1.51
26	2.51	6.13	0.4	23.67	2.20	1.41	0.76	0.24
27	57.00	1.97	0.2	13.50	0.62	0.45	0.02	0.19
28	0.47	216.42	1.6	29.83	0.92	1.90	0.75	0.03
29	0.03	1.26	51.2	13.01	0.45	0.02	0.17	0.01

	35	36	37	38	39	40	41	42
1	0.14	0.30	1.82	1.67	60.51	9.75	1.09	0.12
2	3.87	0.30	3.30	6.70	24.39	114.66	2.46	0.49
3	1.19	0.25	2.51	2.58	3.27	4.59	15.51	2.45
4	0.04	0.03	1.77	0.25	3.22	2.81	0.57	34.34
5	0.64	3.04	1.90	6.44	123.29	6.98	2.20	0.11
6	1.44	1.44	2.84	16.29	6.34	110.81	4.18	1.84
7	2.39	1.04	1.23	5.03	0.86	3.59	18.18	0.78
8	0.01	0.00	0.01	0.52	0.00	1.15	1.57	15.17
9	0.63	0.05	1.03	4.62	6.88	38.66	6.80	0.15
10	0.47	0.00	0.55	2.79	2.78	3.91	13.41	0.82
11	0.06	0.00	0.34	0.06	0.07	8.77	0.17	0.00
12	1.43	0.10	2.10	6.21	1.99	2.98	16.73	0.11
13	17.88	0.00	1.04	0.35	76.46	6.95	0.12	0.00
14	81.09	2.27	2.49	12.61	3.82	93.25	5.74	0.48
15	41.28	0.06	1.25	1.73	0.11	2.78	11.35	0.08
16	40.01	0.28	0.02	0.83	0.06	1.81	1.23	11.13

Table IV-3 (continued)

Segment

	35	36	37	38	39	40	41	42
17	0.04	2.87	1.26	3.34	1.90	3.71	10.66	1.78
18	0.11	34.68	3.16	4.00	3.03	24.94	4.15	0.14
19	0.13	60.53	0.04	3.31	1.04	5.83	13.42	1.61
20	0.80	1.25	68.02	7.65	144.98	9.15	1.22	0.23
21	3.58	0.21	103.77	12.11	14.62	92.91	3.64	1.66
22	0.39	1.13	72.00	2.01	1.48	2.34	20.31	1.33
23	0.43	0.02	52.13	0.31	0.04	1.23	3.08	20.37
24	0.55	0.17	1.28	50.62	114.91	7.95	0.46	0.12
25	2.67	0.63	3.69	246.99	18.65	124.98	4.23	2.82
26	3.14	1.04	3.28	214.95	9.84	15.95	31.60	3.75
27	0.11	0.07	2.19	62.94	1.15	2.49	1.54	28.98
28	5.59	0.11	6.28	1.66	6.79	7.96	26.39	1.42
29	1.59	0.02	0.94	1.74	3.75	1.20	4.56	0.10

	43	44	45	46	47	48	49	50
1	101.83	0.92	0.33	0.13	2.23	0.22	2.75	0.79
2	240.89	94.03	2.79	2.89	3.14	0.16	0.67	2.39
3	123.61	116.80	3.94	0.15	0.43	0.10	0.39	0.87
4	0.69	44.33	0.06	0.02	1.05	0.04	0.05	0.05
5	4.71	0.84	1.09	3.50	0.46	0.05	3.61	2.15
6	3.87	2.70	204.14	1.26	1.19	0.07	9.45	4.33
7	1.91	1.04	129.45	0.08	2.61	0.57	0.28	1.84
8	0.04	0.76	0.23	0.00	0.03	0.02	0.00	0.00
9	0.85	1.14	2.95	0.12	0.64	1.12	0.17	1.14
10	1.99	0.54	0.09	0.05	2.35	0.02	0.08	0.99
11	0.76	0.81	0.03	0.00	0.06	0.17	1.02	0.13
12	1.22	0.10	0.02	1.36	1.13	0.06	0.17	0.29
13	5.02	0.14	0.02	43.13	1.57	0.03	0.29	0.02
14	7.74	2.26	1.11	85.43	188.06	3.54	2.39	5.60
15	0.62	0.59	0.19	0.69	93.27	27.05	0.73	3.36
16	0.00	0.02	0.01	0.20	0.55	34.17	0.02	0.01
17	2.84	1.42	0.92	0.03	1.10	0.02	0.20	3.23
18	2.11	0.61	0.95	1.13	0.17	0.16	0.13	2.92
19	1.59	0.34	0.01	0.16	2.97	0.01	0.04	3.59
20	4.40	2.17	0.78	0.33	0.17	0.07	158.19	4.29
21	5.90	2.29	0.55	0.31	7.25	0.08	180.27	188.36
22	0.89	0.72	0.44	0.63	1.64	0.19	2.53	124.89
23	0.04	0.61	0.03	0.02	0.09	0.84	0.74	2.38
24	2.14	1.77	0.27	0.22	1.28	0.05	1.22	1.62
25	3.49	1.37	3.89	5.16	5.02	0.31	4.62	2.75
26	3.78	3.00	4.18	1.19	5.36	0.56	0.99	3.04
27	1.26	1.03	0.35	0.06	0.06	0.07	0.14	0.07
28	1.19	0.88	0.75	6.48	6.61	0.53	3.72	4.52
29	1.02	0.25	0.03	0.68	0.31	0.40	0.23	0.21

Table IV-3 (continued)

	Segment							
Item	51	52	53	54	55	56	57	58
1	1.93	0.81	15.14	10.50	0.76	5.42	4.05	1.14
2	3.01	2.33	21.48	104.18	3.80	11.01	92.63	1.83
3	1.59	2.88	1.95	53.68	30.56	0.99	63.57	0.08
4	0.94	1.54	1.36	2.09	9.47	0.10	0.86	0.22
5	0.12	1.61	94.46	9.99	1.19	76.68	7.89	0.42
6	1.59	2.58	76.46	115.32	7.69	55.25	106.46	1.49
7	2.96	4.85	7.33	42.49	20.22	1.01	46.29	0.54
8	1.05	1.30	0.80	1.56	10.98	1.93	1.76	0.09
9	0.20	0.83	51.21	44.31	0.71	0.86	6.44	1.88
10	0.27	2.57	13.77	39.53	1.00	0.53	5.85	0.09
11	0.06	0.22	0.61	0.41	0.05	0.16	6.33	0.27
12	0.10	1.02	1.37	4.14	1.11	0.54	35.80	0.01
13	0.00	0.06	49.40	6.74	0.33	2.45	5.63	1.24
14	1.36	5.98	80.31	103.37	2.16	5.13	80.66	7.86
15	0.29	2.09	3.94	29.66	10.83	0.75	20.26	0.58
16	0.00	2.55	0.68	6.23	8.70	0.12	2.16	0.36
17	0.66	1.22	4.49	20.15	1.44	0.68	27.21	0.36
18	0.05	0.93	5.93	7.29	0.31	1.81	23.09	2.47
19	0.01	1.87	4.97	5.71	0.45	1.13	29.10	0.11
20	0.98	0.48	112.31	12.64	0.42	9.30	5.90	2.11
21	4.59	3.01	89.89	94.37	2.49	2.21	11.74	0.94
22	71.78	1.66	7.63	35.91	3.52	0.31	2.57	2.01
23	74.40	1.66	1.35	4.88	2.24	0.03	1.97	1.10
24	0.16	0.45	79.87	6.51	1.21	59.90	3.46	0.77
25	0.70	6.02	145.55	129.62	4.54	85.51	117.58	4.96
26	2.70	133.49	20.48	132.26	42.27	16.58	132.56	1.13
27	0.68	89.75	6.41	13.09	17.66	1.41	8.30	0.08
28	1.82	3.28	11.68	12.30	0.91	4.45	10.42	72.45
29	0.01	0.01	4.28	7.66	0.13	1.70	1.30	26.83

Table IV-4

Results of Bayesian Assignment
1% Relaxation, q_{ix} Based on Square Root of Major Diagonal

Segment

Item	1	2	3	4	5	6	7	8	9
1	84.69	2.70	0.01	0.00	0.30	0.13	0.00	0.99	0.99
2	4.90	148.13	2.48	2.09	0.13	1.08	0.07	0.00	0.33
3	0.02	0.15	170.28	2.93	0.13	0.04	0.76	0.03	0.00
4	0.00	0.02	2.08	113.95	0.00	0.00	0.00	0.00	0.37
5	0.00	2.55	0.09	0.00	82.35	1.86	0.00	1.05	0.00
6	0.16	0.21	0.43	0.00	0.22	102.43	1.36	1.07	0.00
7	0.00	0.04	0.65	0.00	0.00	0.11	69.19	0.97	0.08
8	0.00	0.00	0.01	0.03	0.00	0.00	0.00	58.08	0.00
9	0.00	0.18	0.00	0.25	0.00	0.00	0.05	0.13	25.38
10	0.00	0.30	0.00	0.00	0.00	0.00	0.00	0.00	0.00
11	0.00	0.00	0.00	0.00	0.64	0.00	0.00	0.00	0.00
12	0.00	0.58	0.00	0.00	0.64	0.00	0.00	0.00	0.00
13	0.00	0.10	0.00	0.00	0.04	0.00	0.00	0.00	0.00
14	0.14	0.21	0.18	0.00	0.28	1.80	0.00	1.00	0.00
15	0.00	0.35	0.00	0.00	0.07	0.00	0.01	0.13	0.00
16	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00
17	0.00	0.00	1.02	0.14	0.74	0.06	0.00	0.00	0.09
18	0.01	0.33	0.03	0.00	0.00	0.02	0.00	0.00	0.00
19	0.00	0.03	0.71	0.99	0.00	0.00	0.00	0.00	0.00
20	0.03	0.86	0.48	0.96	0.08	0.05	0.00	0.00	0.73
21	0.15	0.56	0.38	0.00	0.20	0.09	0.11	0.00	0.76
22	0.00	0.00	0.00	0.96	0.00	0.04	0.01	0.00	0.73
23	0.00	0.00	0.00	0.09	0.00	0.00	0.00	0.00	0.00
24	0.00	1.13	0.00	0.00	0.36	0.00	0.00	0.00	0.05
25	0.01	1.15	0.01	0.15	0.05	0.08	0.08	0.00	0.27
26	0.00	0.03	0.13	0.02	0.08	0.23	1.87	0.03	0.76
27	0.00	0.01	0.00	0.07	0.12	0.09	0.00	0.00	0.00
28	0.03	0.18	0.17	0.07	0.06	0.28	0.03	0.05	0.15
29	0.00	0.00	0.00	0.00	0.13	0.00	0.00	0.00	0.17

	10	11	12	13	14	15	16	17	18
1	0.00	0.00	0.00	2.00	0.08	0.12	0.00	0.12	0.00
2	0.00	0.00	1.04	0.17	0.80	0.04	0.00	0.00	0.00
3	0.47	0.02	0.99	0.00	0.00	0.63	0.84	1.12	0.09
4	0.93	0.00	0.00	0.00	0.00	0.00	0.00	0.22	0.00
5	0.00	0.00	0.00	0.00	0.91	0.00	0.97	0.00	0.00
6	0.00	0.00	0.08	0.00	0.02	0.00	0.00	1.95	0.06
7	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.44	0.00
8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
9	0.46	0.00	1.06	0.00	0.00	0.01	0.16	0.08	0.00
10	33.06	0.00	0.10	0.00	0.00	0.00	0.00	0.73	0.00
11	0.00	11.07	0.00	0.00	0.00	0.12	0.00	0.00	0.00
12	0.00	0.08	87.63	0.00	0.00	0.00	0.00	0.00	0.00
13	0.00	0.00	0.00	22.83	0.03	0.00	0.00	0.00	0.00

Table IV-4 (continued)

Segment

Item	10	11	12	13	14	15	16	17	18
14	0.66	0.00	0.43	0.14	68.36	0.73	1.60	0.00	0.46
15	0.00	0.00	0.00	0.00	0.07	23.15	0.02	0.00	0.00
16	0.00	0.00	0.00	0.01	0.00	0.00	38.10	0.00	0.21
17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	57.02	0.00
18	0.00	0.00	0.00	0.00	0.00	0.00	0.15	0.00	6.23
19	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
20	0.00	1.97	0.00	1.01	0.00	0.21	0.00	0.00	0.00
21	0.00	0.10	0.72	0.00	0.03	0.00	0.00	0.82	0.05
22	0.00	0.00	0.91	0.00	0.54	0.00	0.94	1.00	0.00
23	0.00	0.00	0.00	0.00	0.00	0.15	0.22	0.73	0.00
24	0.00	0.00	0.76	0.00	0.00	0.00	0.00	0.00	0.00
25	0.01	0.04	2.72	0.05	0.11	0.01	0.19	0.88	0.12
26	0.00	0.02	0.00	0.00	0.17	0.02	0.27	0.03	0.00
27	0.00	0.00	0.14	0.00	0.00	0.00	0.38	0.64	0.00
28	0.05	0.00	0.00	0.79	3.57	0.20	0.00	0.00	0.00
29	0.00	0.00	0.00	0.00	0.08	0.00	0.00	0.00	0.00

	19	20	21	22	23	24	25	26
1	0.69	0.60	0.11	0.00	0.00	0.00	0.91	0.00
2	0.06	0.99	0.78	0.42	0.00	0.01	0.91	1.12
3	0.00	0.09	0.62	0.18	1.75	0.00	0.01	0.05
4	0.00	0.00	0.00	0.00	0.00	0.00	0.09	0.64
5	0.00	0.42	0.94	0.16	0.00	0.00	0.18	0.02
6	0.00	0.16	1.93	0.84	0.71	0.00	0.06	0.29
7	0.00	0.00	0.06	0.01	0.71	0.67	0.02	0.00
8	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.01
9	0.00	0.87	0.00	0.41	0.00	0.41	0.90	0.71
10	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00
11	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.26
12	0.45	0.00	0.43	0.02	0.00	0.03	0.00	0.00
13	0.00	0.02	0.00	0.00	0.00	0.66	1.02	0.62
14	0.00	1.00	0.65	0.11	0.00	0.00	0.14	1.46
15	0.00	0.00	0.00	0.01	0.12	0.02	0.00	0.02
16	0.90	0.00	0.00	0.00	0.00	0.00	0.08	0.01
17	0.49	0.00	0.03	0.46	0.00	0.00	0.18	0.01
18	0.15	0.00	0.02	0.00	0.00	0.00	0.04	0.00
19	29.31	0.00	0.00	0.00	0.00	0.00	0.00	0.00
20	0.93	161.10	1.68	0.92	0.01	0.68	1.06	0.03
21	0.00	0.59	160.98	0.84	0.45	0.00	0.42	1.23
22	0.00	0.14	1.65	65.76	0.31	0.00	1.07	0.05
23	0.00	0.00	0.57	0.03	81.57	0.00	0.00	0.00
24	0.00	2.50	0.00	0.05	0.00	29.80	1.02	0.64
25	0.00	1.61	1.17	0.11	0.00	1.86	113.89	1.06
26	0.00	0.02	0.03	0.26	0.00	0.00	1.30	74.17
27	0.00	0.09	0.00	0.00	0.01	0.03	0.01	0.08
28	0.00	3.00	5.06	0.03	0.16	0.00	4.01	0.06
29	0.00	0.08	0.07	0.00	0.00	0.01	0.00	0.00

Table IV-4 (continued)

Segment

Item	27	28	29	30	31	32	33	34
1	0.17	1.60	0.00	90.20	27.62	0.05	0.01	0.00
2	0.00	1.90	0.00	183.39	68.52	1.98	0.10	0.05
3	0.00	0.00	0.00	56.37	55.67	0.30	0.53	0.04
4	0.02	0.00	0.00	32.49	27.53	0.03	1.27	0.00
5	0.18	0.06	0.00	139.42	0.30	58.02	0.59	0.00
6	0.15	2.02	0.43	155.85	0.65	88.60	0.06	0.04
7	0.16	0.11	1.00	55.18	0.32	37.86	0.04	0.00
8	0.00	0.00	0.00	20.21	0.30	21.25	0.00	0.00
9	0.00	0.69	0.20	95.31	0.18	0.11	18.99	0.03
10	0.03	0.02	0.00	56.46	0.54	0.02	20.73	0.04
11	0.00	0.06	0.00	8.45	0.41	0.01	0.00	23.01
12	0.81	0.00	0.00	43.66	0.08	0.01	0.04	23.42
13	0.00	0.00	0.00	63.72	0.04	0.07	0.59	0.00
14	0.00	1.35	0.21	149.58	0.74	1.25	0.31	0.23
15	0.00	0.20	0.00	41.01	0.15	0.04	0.00	0.00
16	0.00	0.00	0.00	26.39	0.00	0.23	0.00	0.00
17	0.18	0.05	0.86	34.97	0.34	0.11	0.10	0.99
18	0.67	0.13	0.00	51.77	0.42	0.05	0.00	0.00
19	0.00	0.94	0.00	37.12	0.26	0.00	0.00	0.00
20	0.16	0.90	2.23	133.26	0.23	0.41	0.12	0.02
21	0.00	0.63	1.23	171.38	0.59	0.27	0.03	1.32
22	0.00	0.03	0.00	66.48	0.21	0.03	0.12	0.14
23	0.71	0.68	0.00	45.38	0.00	0.99	0.00	2.00
24	0.21	1.44	0.00	87.18	0.34	0.23	0.03	0.23
25	0.07	3.16	2.01	202.62	0.53	0.20	1.08	1.35
26	0.39	4.51	0.00	149.01	1.30	0.10	0.22	0.00
27	41.08	1.94	0.00	70.07	0.24	0.09	0.00	0.10
28	0.03	180.22	0.82	138.29	0.63	1.83	0.07	0.00
29	0.00	0.17	43.64	40.40	0.16	0.00	0.09	0.00

	35	36	37	38	39	40	41	42
1	0.05	0.25	1.19	1.56	43.50	3.83	0.80	0.00
2	2.64	0.04	2.67	3.46	5.59	95.69	1.23	0.97
3	0.87	0.04	1.77	1.58	2.07	1.42	42.38	1.78
4	0.00	0.00	0.57	0.38	3.00	0.16	1.20	36.23
5	0.09	2.60	1.92	3.49	99.34	2.21	0.18	0.04
6	0.01	0.97	1.06	5.30	2.34	75.97	0.83	1.33
7	2.05	1.00	0.10	3.06	0.65	1.02	16.97	0.13
8	0.00	0.00	0.00	0.00	0.00	0.14	1.06	13.65
9	0.92	0.00	0.09	2.85	2.99	29.16	0.94	0.07
10	0.04	0.00	0.30	0.58	0.00	0.84	9.20	0.99
11	0.00	0.00	0.20	0.02	0.00	8.22	0.12	0.00
12	1.08	0.03	0.08	2.03	0.86	1.44	8.39	0.34
13	24.43	0.00	0.02	0.34	62.07	2.28	0.00	0.00
14	80.77	1.97	0.90	6.30	0.10	68.89	2.52	0.25
15	42.83	0.00	0.95	0.84	0.00	0.32	9.09	0.03
16	48.34	0.16	0.00	0.31	0.00	0.24	1.27	11.58

Table IV-4 (continued)

Segment

Item	35	36	37	38	39	40	41	42
17	0.00	2.65	0.19	2.48	0.08	0.23	12.76	0.13
18	0.03	33.35	0.13	2.94	0.38	24.30	0.12	0.33
19	0.03	67.23	0.00	2.59	0.00	3.21	11.06	1.05
20	0.01	1.02	81.63	0.46	135.56	2.32	0.95	0.04
21	1.56	0.02	113.40	7.02	4.00	81.15	0.37	0.02
22	0.12	1.00	86.85	0.93	0.02	0.63	21.79	0.01
23	0.88	0.00	52.23	0.03	0.00	0.01	0.69	20.36
24	0.37	0.00	0.79	49.23	89.25	6.65	0.24	0.00
25	1.31	0.05	1.99	214.52	7.99	91.77	0.80	1.09
26	1.53	0.70	2.26	198.26	6.16	1.78	32.31	1.35
27	0.34	0.00	1.05	56.99	0.09	1.20	2.07	23.64
28	2.83	0.00	2.92	1.40	1.79	1.76	24.72	1.27
29	1.22	0.00	0.18	0.26	3.89	0.24	6.26	0.00

	43	44	45	46	47	48	49	50
1	89.42	0.09	0.06	0.06	2.07	0.03	2.61	0.26
2	224.94	85.12	2.26	1.52	0.54	0.02	0.69	0.19
3	118.31	112.45	2.90	0.00	0.00	0.03	0.71	0.96
4	0.07	51.95	0.00	0.00	0.18	0.00	0.00	0.00
5	2.05	0.16	0.99	2.30	0.25	0.02	1.89	1.74
6	1.18	0.30	203.97	1.00	0.01	0.00	7.11	1.49
7	1.37	0.61	135.58	0.00	1.50	0.07	0.03	0.07
8	0.00	0.54	0.02	0.00	0.00	0.00	0.00	0.00
9	0.49	0.54	2.54	0.00	0.17	0.95	0.06	1.03
10	1.61	0.46	0.00	0.00	1.02	0.00	0.00	0.78
11	0.77	0.80	0.00	0.00	0.01	0.03	0.68	0.01
12	0.41	0.01	0.00	0.2	0.18	0.00	0.13	0.19
13	1.02	0.04	0.00	48.6	0.63	0.00	0.02	0.00
14	4.54	0.61	0.01	77.3	163.02	0.63	0.71	4.03
15	0.38	0.37	0.01	0.0	85.56	21.11	0.68	2.53
16	0.00	0.00	0.00	0.2	0.03	28.83	0.00	0.00
17	1.64	0.56	0.96	0.0	0.18	0.00	0.02	0.89
18	2.03	0.49	0.58	0.9	0.00	0.08	0.00	0.28
19	0.93	0.15	0.00	0.0	0.99	0.00	0.00	0.13
20	0.70	0.13	0.12	0.0	0.00	0.00	162.34	1.26
21	1.77	1.36	0.08	0.0	5.63	0.00	178.10	187.10
22	0.69	0.04	0.02	0.2	0.34	0.05	0.68	127.84
23	0.00	0.00	0.00	0.00	0.05	0.26	0.08	0.72
24	1.12	0.03	0.01	0.00	0.94	0.00	1.05	1.19
25	2.42	0.12	1.66	3.90	2.91	0.10	1.84	0.16
26	2.35	0.64	1.16	0.07	2.45	0.08	0.03	0.09
27	0.46	0.97	0.06	0.00	0.00	0.02	0.04	0.00
28	0.83	0.71	0.22	3.76	5.79	0.02	1.65	3.40
29	0.84	0.00	0.00	0.05	0.06	0.06	0.08	0.05

Table IV-4 (continued)

	Segment							
	51	52	53	54	55	56	57	58
1	1.92	0.01	5.16	3.72	0.45	3.93	2.46	0.24
2	0.44	1.15	6.82	119.41	0.76	2.76	108.78	0.58
3	0.34	1.03	0.16	61.74	34.78	0.06	89.25	0.00
4	0.42	0.76	0.15	0.48	13.41	0.03	1.16	0.00
5	0.02	1.50	91.54	4.28	0.42	59.90	4.72	0.02
6	1.29	0.40	77.31	145.15	2.74	35.95	114.64	0.55
7	1.24	0.94	3.28	46.56	21.07	0.19	69.57	0.03
8	1.00	0.13	0.08	0.26	11.85	0.01	1.20	0.00
9	0.12	0.17	37.60	54.52	0.49	0.06	3.07	1.37
10	0.00	1.07	3.10	43.10	0.29	0.01	2.36	0.01
11	0.00	0.10	0.99	0.09	0.01	0.02	8.69	0.03
12	0.01	0.06	1.12	1.18	0.27	0.26	54.55	0.00
13	0.00	0.03	44.55	2.03	0.01	0.44	4.50	0.00
14	0.84	2.88	65.38	124.46	0.19	0.72	105.49	6.21
15	0.06	0.50	0.85	38.47	15.14	0.54	20.97	0.09
16	0.00	0.70	0.13	4.71	8.03	0.03	0.26	0.00
17	0.08	1.03	1.84	27.38	0.80	0.12	31.88	0.03
18	0.00	0.62	3.38	2.61	0.03	0.09	17.94	1.06
19	0.00	1.22	1.72	4.59	0.02	0.47	28.01	0.05
20	0.16	0.08	126.23	6.98	0.79	2.28	2.33	1.20
21	1.23	0.95	90.40	115.93	0.62	0.56	7.09	0.36
22	58.85	0.86	3.99	46.92	1.28	0.04	0.28	1.13
23	72.54	0.87	0.93	2.54	1.53	0.00	0.70	0.25
24	0.01	0.06	84.02	3.86	0.03	42.30	3.21	0.44
25	0.09	0.73	136.48	157.49	0.93	57.88	132.69	3.15
26	1.46	98.07	10.78	165.43	35.91	5.63	161.71	0.52
27	0.03	72.42	3.72	5.67	31.53	0.38	5.84	0.05
28	1.09	2.12	5.56	2.97	0.71	1.32	3.12	73.91
29	0.00	0.00	2.52	1.81	0.00	0.42	0.13	24.73

Table IV-5

Results of Bayesian Assignment: Deterministic Assignment Based on Square Root of Market Share, 1% Relaxation

Segment

Item	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	88	2	0	0	0	0	0	1	1	0	0	0	2	0	0	0	0	0	1	0
2	5	158	3	2	0	1	0	0	1	0	0	1	0	1	0	0	0	0	0	1
3.	0	0	191	4	0	0	2	0	0	1	0	1	0	0	3	1	1	0	0	0
4	0	0	2	125	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0
5	0	3	0	0	88	0	0	1	0	0	0	0	0	2	0	1	0	0	0	0
6	0	0	0	0	0	107	4	1	0	0	0	0	0	0	0	0	3	0	0	0
7	0	0	1	0	0	0	96	1	0	0	0	0	0	0	0	0	1	0	0	0
8	0	0	0	0	0	0	0	59	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	37	1	0	1	0	0	0	0	0	0	0	1
10	0	0	0	0	0	0	0	0	0	41	0	0	0	0	0	0	1	0	0	0
11	0	0	0	0	1	0	0	0	0	0	11	0	0	0	0	0	0	0	0	0
12	0	0	0	0	1	0	0	0	0	0	0	100	0	0	0	0	0	0	1	0
13	0	0	0	0	0	0	0	0	0	0	0	0	33	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	1	0	1	0	1	2	95	2	2	0	1	0	1
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	42	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	44	0	1	2	0
17	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	63	0	1	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	0	0
19	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	37	0
20	0	0	0	1	0	0	0	0	1	0	2	0	1	0	1	0	0	0	1	165
21	0	0	0	0	0	0	0	0	3	0	0	1	0	0	0	0	1	0	0	0
22	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0	1	1	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
24	0	1	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	2
25	0	1	0	0	0	0	0	0	1	0	0	3	0	0	0	0	2	0	0	1
26	0	0	0	0	0	0	3	0	1	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
28	0	0	0	0	0	0	0	0	0	0	0	0	1	5	1	0	0	0	0	3
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
1	0	0	0	0	1	0	0	2	0	103	9	0	0	0	0	0	1	2	45
2	0	1	0	0	1	1	0	2	0	221	17	2	0	0	3	0	3	3	5
3	0	1	2	0	0	0	0	0	0	64	21	0	0	0	1	0	2	2	2
4	0	0	0	0	0	1	0	0	0	35	13	0	0	0	0	0	0	0	3
5	0	0	0	0	0	0	0	0	0	162	0	71	1	0	0	3	2	3	108
6	2	1	1	0	0	0	0	2	1	192	0	104	0	0	0	1	1	5	2
7	0	0	1	1	0	0	0	0	1	67	0	29	0	0	2	1	0	3	0
8	0	0	0	0	0	0	0	0	0	21	0	21	0	0	0	0	0	0	0
9	0	1	0	1	1	1	0	1	1	107	0	0	16	0	1	0	0	3	3
10	0	0	0	0	0	0	0	0	0	63	0	0	17	0	0	0	0	1	0
11	0	0	0	0	0	0	0	0	0	13	0	0	0	23	0	0	0	0	0
12	1	0	0	0	0	0	1	0	0	47	0	0	0	12	1	0	0	1	0
13	0	0	0	1	1	1	0	0	0	72	0	0	1	0	25	0	0	0	63

Table IV-5 (continued)

Segment

Item	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
14	1	0	0	0	0	1	0	1	1	178	0	3	3	0	75	2	0	6	0
15	0	0	0	0	0	0	0	0	0	48	0	0	0	0	34	0	0	1	0
16	0	0	0	0	0	0	0	0	0	28	0	0	0	0	42	0	0	0	0
17	0	1	0	0	0	0	0	0	2	43	0	0	0	1	0	3	0	2	0
18	0	0	0	0	0	0	1	0	0	56	0	0	0	0	0	33	0	3	0
19	0	0	0	0	0	0	0	1	0	38	0	0	0	0	0	64	0	2	0
20	1	1	0	1	1	0	0	1	3	157	0	1	0	0	0	1	76	0	143
21	170	6	1	0	0	1	0	1	2	211	0	2	0	1	3	0	99	8	4
22	0	115	0	0	1	0	0	0	0	73	0	0	0	0	0	1	76	1	0
23	0	0	94	0	0	0	1	1	0	48	0	1	0	2	0	0	46	0	0
24	0	0	0	37	1	1	0	2	0	103	0	0	0	0	0	0	1	55	89
25	1	0	0	2	133	1	0	3	2	245	0	0	1	1	1	0	1	222	11
26	0	0	0	0	1	94	1	5	0	177	1	0	0	0	1	1	2	193	7
27	0	0	0	0	0	0	52	2	0	82	0	0	0	0	0	0	1	54	0
28	5	0	0	0	4	0	0	198	1	150	0	2	0	0	1	0	3	1	2
29	0	0	0	0	0	0	0	0	53	41	0	0	0	0	1	0	0	0	4

	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58
1	1	1	0	102	0	0	0	2	0	3	0	2	0	4	0	0	3	2	0
2	92	1	1	301	42	2	2	0	0	1	0	0	1	2	76	3	1	133	0
3	1	41	2	178	66	2	0	0	0	1	0	0	1	0	33	43	0	102	0
4	0	1	38	0	58	0	0	0	0	0	0	1	1	0	0	9	0	1	0
5	2	0	0	2	0	1	2	0	0	2	2	0	2	65	1	0	45	3	0
6	75	0	2	1	0	202	1	0	0	4	1	1	0	59	104	4	24	136	0
7	0	15	0	1	0	123	0	2	0	0	0	1	0	3	25	40	0	61	0
8	0	1	14	0	1	0	0	0	0	0	0	1	0	0	0	12	0	0	0
9	22	1	0	1	0	3	0	0	1	0	1	0	0	27	51	0	0	3	1
10	0	9	1	1	0	0	0	1	0	0	1	0	1	1	38	0	0	2	0
11	4	0	0	1	1	0	0	0	0	1	0	0	0	1	0	0	0	9	0
12	0	8	0	1	0	0	0	0	0	0	0	0	0	1	1	0	0	55	0
13	1	0	0	1	0	0	39	0	0	0	0	0	0	38	0	0	0	5	0
14	74	2	0	5	0	0	69	154	2	1	4	1	3	49	80	0	1	124	6
15	0	8	0	1	0	0	0	84	21	1	3	0	0	0	29	11	0	24	0
16	0	1	13	0	0	0	0	0	30	0	0	0	0	0	3	7	0	0	0
17	0	12	0	2	0	1	0	0	0	0	0	0	1	1	18	1	0	30	0
18	25	0	0	3	0	1	1	0	0	0	0	0	1	2	2	0	0	14	1
19	2	11	1	1	0	0	0	1	0	0	0	0	2	0	5	0	0	26	0
20	1	1	0	1	0	0	0	0	0	159	0	0	0	109	4	0	1	2	1
21	83	0	0	3	1	0	0	7	0	170	187	2	1	72	97	0	0	6	0
22	0	26	0	1	0	0	0	0	0	0	100	53	1	3	38	1	0	0	1
23	0	1	18	0	0	0	0	0	1	0	1	69	1	1	0	2	0	0	0
24	6	0	0	1	0	0	0	2	0	1	1	0	0	76	1	0	26	3	0
25	99	0	1	3	0	2	5	3	0	2	0	0	0	104	111	1	43	149	4
26	0	33	2	3	0	1	0	3	0	0	0	2	87	6	103	37	3	199	0
27	1	2	25	0	1	0	0	0	0	0	0	0	63	3	1	30	0	3	0
28	0	24	2	1	1	0	3	5	0	0	3	1	3	3	1	0	1	3	56
29	0	5	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	22

Table IV-6

Results of Bayesian Assignment: Deterministic Assignment Based on Square Root of Major Diagonal, 1% Relaxation

Segment

Item	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	88	3	0	0	0	0	0	1	1	0	0	0	2	0	0	0	0	0	1	0
2	5	166	3	2	0	1	0	0	0	0	0	1	0	1	0	0	0	0	0	1
3	0	0	191	2	0	0	1	0	0	0	0	1	0	0	0	1	1	0	0	0
4	0	0	2	116	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	3	0	0	84	2	0	1	0	0	0	0	0	2	0	1	0	0	0	0
6	0	0	0	0	0	122	1	1	0	0	0	0	0	0	0	0	3	0	0	0
7	0	0	1	0	0	0	77	1	0	0	0	0	0	0	0	0	1	0	0	0
8	0	0	0	0	0	0	0	59	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	27	0	0	1	0	0	0	0	0	0	0	1
10	0	0	0	0	0	0	0	0	0	31	0	0	0	0	0	0	1	0	0	0
11	0	0	0	0	1	0	0	0	0	0	11	0	0	0	0	0	0	0	0	0
12	0	1	0	0	1	0	0	0	0	0	0	100	0	0	0	0	0	0	1	0
13	0	0	0	0	0	0	0	0	0	0	0	0	23	0	0	0	0	0	0	0
14	0	0	0	0	0	2	0	1	0	1	0	1	0	95	0	2	0	1	0	1
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	24	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	44	0	0	2	0
17	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	63	0	1	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8	0	0
19	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	37	0
20	0	1	0	1	0	0	0	0	1	0	2	0	1	0	0	0	0	0	1	165
21	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0
22	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0	1	1	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
24	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	2
25	0	1	0	0	0	0	0	0	0	0	0	3	0	0	0	0	2	0	0	1
26	0	0	0	0	0	0	3	0	1	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
28	0	0	0	0	0	0	0	0	0	0	0	0	1	5	0	0	0	0	0	3
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
1	0	0	0	0	1	0	0	2	0	94	9	0	0	0	0	0	1	2	43
2	0	1	0	0	1	1	0	2	0	192	17	2	0	0	3	0	3	3	5
3	0	0	2	0	0	0	0	0	0	61	21	0	1	0	1	0	2	2	2
4	0	0	0	0	0	1	0	0	0	36	13	0	2	0	0	0	1	0	3
5	1	0	0	0	0	0	0	0	0	150	0	63	1	0	0	3	2	3	94
6	2	1	1	0	0	0	0	2	0	163	0	81	0	0	0	1	1	5	2
7	0	0	1	1	0	0	0	0	1	58	0	29	0	0	2	1	0	3	0
8	0	0	0	0	0	0	0	0	0	22	0	21	0	0	0	0	0	0	0
9	0	1	0	0	1	1	0	1	0	95	0	0	15	0	1	0	0	3	3
10	0	0	0	0	0	0	0	0	0	58	0	0	24	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	7	0	0	0	23	0	0	0	0	0
12	1	0	0	0	0	0	1	0	0	45	0	0	0	12	1	0	0	1	0
13	0	0	0	1	1	1	0	0	0	65	0	0	1	0	24	0	0	0	60

Table IV-6 (continued)

Segment

Item	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
14	1	0	0	0	0	1	0	1	0	149	0	1	0	0	61	2	1	6	0
15	0	0	0	0	0	0	0	0	0	45	0	0	0	0	34	0	1	1	0
16	0	0	0	0	0	0	0	0	0	31	0	0	0	0	47	0	0	0	0
17	0	0	0	0	0	0	0	0	0	37	0	0	0	1	0	3	0	3	0
18	0	0	0	0	0	0	1	0	0	53	0	0	0	0	0	33	0	3	0
19	0	0	0	0	0	0	0	1	0	34	0	0	0	0	0	64	0	3	0
20	1	1	0	1	1	0	0	1	2	139	0	0	0	0	0	1	76	0	139
21	173	0	0	0	0	1	0	1	1	178	0	0	0	1	0	0	104	7	4
22	0	84	0	0	1	0	0	0	0	65	0	0	0	0	0	1	87	1	0
23	0	0	91	0	0	0	1	1	0	44	0	1	0	2	1	0	56	0	0
24	0	0	0	35	1	1	0	2	0	87	0	0	0	0	0	0	1	57	83
25	1	0	0	2	133	1	0	3	2	199	0	0	1	1	1	0	2	221	6
26	0	0	0	0	1	94	0	5	0	144	1	0	0	0	1	1	2	189	6
27	0	0	0	0	0	0	46	2	0	75	0	0	0	0	0	0	1	54	0
28	5	0	0	0	4	0	0	198	1	145	0	2	0	0	1	0	3	1	2
29	0	0	0	0	0	0	0	0	45	41	0	0	0	0	1	0	0	0	4

	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58
1	3	1	0	104	0	0	0	2	0	3	0	2	0	4	3	0	5	3	0
2	109	1	1	295	42	2	1	0	0	1	0	0	1	4	79	0	2	142	0
3	1	41	2	178	68	3	0	0	0	1	1	0	1	0	35	37	0	112	0
4	0	1	34	0	67	0	0	0	0	0	0	0	1	0	0	12	0	1	0
5	2	0	0	2	0	1	2	0	0	2	2	0	2	87	1	0	57	4	0
6	83	1	1	1	0	205	1	0	0	9	1	1	0	69	112	2	25	144	0
7	0	15	0	1	0	142	0	2	0	0	0	1	1	3	30	19	0	85	0
8	0	1	13	0	1	0	0	0	0	0	0	1	0	0	0	11	0	1	0
9	40	1	0	1	0	3	0	0	1	0	1	0	0	32	54	0	0	3	1
10	1	9	1	3	0	0	0	1	0	0	1	0	1	2	43	0	0	2	0
11	10	0	0	1	1	0	0	0	0	1	0	0	0	1	0	0	0	9	0
12	2	8	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	55	0
13	3	0	0	1	0	0	48	1	0	0	0	0	0	45	0	0	1	6	0
14	87	2	0	5	0	0	68	168	0	1	4	1	3	53	98	0	0	129	6
15	0	8	0	1	0	0	0	97	17	1	3	0	0	0	32	17	0	26	0
16	0	1	11	0	0	0	0	0	24	0	0	0	0	0	4	7	0	0	0
17	0	14	0	2	0	1	0	0	0	0	1	0	1	2	21	0	0	32	0
18	27	0	0	3	0	1	1	0	0	0	0	0	1	4	2	0	0	14	1
19	4	11	1	1	0	0	0	1	0	0	0	0	2	0	5	0	0	27	0
20	2	1	0	0	0	0	0	0	0	170	0	0	0	119	5	1	2	2	1
21	92	0	0	2	1	0	0	7	0	184	193	0	1	72	112	0	0	6	0
22	0	26	0	1	0	0	0	0	0	0	131	45	1	4	42	1	0	0	1
23	0	1	18	0	0	0	0	0	0	0	1	65	1	1	1	2	0	0	0
24	8	0	0	1	0	0	0	2	0	1	1	0	0	85	3	0	35	4	0
25	110	0	1	3	0	2	4	3	0	2	0	0	0	128	121	1	45	155	4
26	1	33	1	3	0	1	0	3	0	0	0	2	91	10	124	31	5	213	0
27	2	2	21	0	1	0	0	0	0	0	0	0	71	4	4	32	0	5	0
28	2	24	1	1	1	0	3	6	0	2	3	1	3	5	0	0	1	4	56
29	0	9	0	1	0	0	0	0	0	0	0	0	0	3	2	0	0	0	22

Table IV-7

Results of Bayesian Assignment: Deterministic Assignment Based on Square Root of Market Share, 10% Relaxation

Segment

Item	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	91	5	0	0	2	1	0	1	1	0	0	0	2	0	1	0	1	0	1
2	5	218	6	6	6	2	3	0	1	0	0	1	1	6	1	0	0	0	1
3	·0	1	255	8	1	0	8	0	0	1	1	1	0	1	3	1	2	1	0
4	0	0	5	157	0	0	0	0	2	2	0	0	0	0	0	0	1	0	0
5	0	4	0	0	119	3	0	1	1	0	0	0	0	2	0	1	0	0	0
6	1	7	3	0	1	134	5	1	0	0	0	3	0	1	0	0	4	0	0
7	0	0	2	0	0	0	119	4	2	0	0	0	0	0	0	0	2	0	0
8	0	0	0	1	0	0	1	68	0	0	0	0	0	0	0	0	0	0	0
9	0	1	0	0	0	0	0	1	49	1	0	3	0	0	1	1	1	0	0
10	0	1	2	0	0	0	0	0	0	41	0	2	0	0	0	0	1	0	0
11	0	0	0	0	1	0	0	0	0	0	13	0	0	0	1	0	0	0	0
12	0	1	0	0	1	0	0	0	0	0	0	119	0	0	0	0	0	0	1
13	0	0	0	0	1	0	0	0	1	0	0	0	44	0	0	0	0	0	0
14	1	2	1	0	1	2	0	2	0	1	0	6	4	116	2	4	0	1	0
15	0	2	0	0	1	0	0	1	0	0	0	2	0	0	48	0	0	0	0
16	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	56	0	1	2
17	0	1	4	0	1	1	1	0	1	0	0	0	0	0	0	0	72	0	1
18	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	10	1
19	0	0	4	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	49
20	0	2	2	1	1	0	0	0	2	0	2	0	1	0	1	0	0	0	1
21	1	1	1	0	1	1	0	0	3	0	0	4	0	0	0	0	2	0	0
22	0	0	0	1	0	0	0	0	1	0	0	1	0	2	0	1	1	0	0
23	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0
24	0	2	0	0	2	0	0	0	0	0	0	1	0	0	0	0	0	0	0
25	0	3	0	1	0	0	1	0	1	0	0	4	0	1	0	1	3	0	0
26	0	1	2	0	1	1	4	0	3	0	1	0	0	2	1	1	0	0	0
27	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	2	0	0
28	0	0	4	0	3	0	0	0	4	0	0	0	1	5	1	0	0	0	0
29	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0

	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
1	3	1	0	0	0	1	0	1	3	0	13	9	1	0	0	0	0	1	1	61
2	1	7	1	0	0	3	3	0	7	0	21	17	2	0	0	3	0	3	8	24
3	0	3	3	2	0	1	1	1	0	0	6	21	0	0	1	1	0	3	1	3
4	0	0	0	0	0	0	1	0	2	0	4	13	0	0	0	0	0	3	0	3
5	1	2	1	0	0	1	0	1	0	0	15	0	67	3	0	0	3	1	8	117
6	2	7	1	1	0	0	3	3	10	1	13	0	89	10	0	1	1	2	21	5
7	0	0	2	1	1	0	0	1	1	1	7	0	30	1	0	2	1	0	4	0
8	0	0	0	0	0	0	0	0	1	0	3	0	16	0	0	0	0	0	1	0
9	2	0	1	0	1	3	3	0	3	1	14	0	0	34	0	0	0	1	4	7
10	0	0	2	0	0	0	1	0	0	0	5	0	0	23	0	0	0	1	2	3
11	0	1	1	0	0	0	1	0	1	0	2	0	0	0	25	0	0	0	0	0
12	0	1	0	0	0	0	0	2	0	0	6	0	0	0	16	1	0	2	7	2

Table IV-7 (continued)

	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
13	3	1	0	0	1	1	1	0	0	0	9	0	0	1	2	16	0	1	0	75
14	1	3	0	0	0	2	7	0	4	1	21	0	1	6	1	68	2	2	13	3
15	0	0	0	0	0	0	0	0	1	0	3	0	0	0	1	37	0	1	4	0
16	0	0	0	0	0	0	0	0	2	0	4	0	0	0	0	39	0	0	1	0
17	0	0	2	0	0	1	0	0	2	2	4	0	0	1	1	0	3	0	3	1
18	0	0	0	0	0	0	0	1	2	0	9	0	0	0	0	0	32	3	4	2
19	0	0	0	0	0	0	1	0	1	0	3	0	0	0	0	0	53	0	3	1
20	247	7	6	0	2	1	1	1	2	3	23	0	2	0	1	1	1	54	8	144
21	6	309	7	1	0	2	4	0	3	2	12	0	2	2	1	4	0	83	13	15
22	1	4	142	2	0	1	0	0	1	0	7	0	0	0	0	0	1	60	2	1
23	0	1	0	109	0	0	0	1	5	0	9	0	1	0	2	0	0	58	0	0
24	4	0	1	0	51	1	1	1	2	0	9	0	1	0	0	0	0	1	60	106
25	4	9	0	0	7	162	5	0	6	2	26	0	4	1	1	2	0	3	279	15
26	1	0	2	0	0	3	134	2	7	0	19	1	1	0	0	3	1	2	237	9
27	0	0	0	0	0	0	0	68	2	0	12	0	0	0	0	0	0	2	64	1
28	3	5	0	1	0	4	1	0	241	1	25	0	2	0	0	5	0	6	2	6
29	0	0	0	0	1	0	0	0	0	53	16	0	0	0	0	2	0	1	3	3

	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58
1	12	1	0	124	0	0	0	2	0	2	1	2	1	14	7	1	7	2	1
2	119	1	0	326	42	3	2	3	0	0	2	3	2	18	89	5	12	105	1
3	1	11	2	187	67	4	0	0	0	0	0	1	2	1	46	43	1	72	0
4	4	0	32	0	44	0	0	1	0	0	0	1	2	1	2	10	0	0	0
5	8	2	0	5	0	1	4	0	0	3	2	0	2	99	6	2	79	8	0
6	121	3	2	4	1	211	1	0	0	8	4	1	2	72	112	8	45	116	0
7	3	16	0	2	0	132	0	3	0	0	2	3	5	6	35	27	0	60	0
8	1	1	13	0	1	0	0	0	0	0	0	1	1	0	1	17	2	1	0
9	45	6	0	2	0	3	0	1	1	0	1	0	0	50	39	0	0	6	1
10	3	14	1	3	0	0	0	3	0	0	1	0	3	17	43	1	0	5	0
11	11	0	0	1	1	0	0	0	0	1	0	0	0	0	0	0	0	5	0
12	2	15	0	3	0	0	2	1	0	0	0	0	0	1	6	3	1	38	0
13	10	0	0	5	0	0	43	1	0	0	0	0	0	51	4	0	2	6	2
14	102	6	0	11	1	1	75	186	2	2	5	1	5	73	96	3	4	92	9
15	2	12	0	1	0	0	0	101	21	1	4	0	2	3	24	13	0	22	0
16	2	1	10	0	0	0	0	0	30	0	0	0	2	0	7	10	0	2	0
17	4	10	2	5	0	1	0	1	0	0	4	1	1	5	17	2	1	28	0
18	32	5	0	3	0	1	1	0	0	0	4	0	1	6	6	0	2	25	1
19	6	15	2	2	0	0	0	3	0	0	5	0	2	6	2	0	1	33	0
20	7	0	0	4	2	1	0	0	0	155	4	0	0	118	12	0	9	6	2
21	91	3	1	6	1	0	0	7	0	167	185	3	2	90	95	3	1	8	0
22	1	21	1	1	1	0	0	1	0	2	120	60	2	7	43	4	0	1	2
23	0	2	18	0	1	0	0	0	1	0	1	63	2	1	4	3	0	2	0
24	10	0	0	3	2	0	0	2	0	1	2	0	0	82	4	2	57	3	0
25	130	3	2	4	1	4	5	5	0	3	2	0	2	137	116	5	74	121	4
26	12	28	3	6	1	3	1	6	0	2	3	3	110	18	109	59	13	150	0
27	3	1	28	2	1	0	0	0	0	0	0	1	90	6	11	17	1	8	0
28	7	30	2	1	1	0	6	6	0	4	5	2	4	12	10	0	4	11	59
29	1	4	0	1	0	0	0	0	0	0	0	0	0	4	9	0	1	0	28

Table IV-8

Results of Bayesian Assignment: Deterministic Assignment Based on Square Root of Major Diagonal, 10% Relaxation

Segment

Item	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	91	5	0	0	2	1	0	1	1	0	0	0	2	0	1	0	1	0	1
2	5	218	6	6	6	2	3	0	1	0	0	1	1	6	1	0	0	0	1
3	0	1	255	8	1	0	8	0	0	1	1	1	0	1	3	1	2	1	0
4	0	0	5	157	0	0	0	0	2	2	0	0	0	0	0	0	1	0	0
5	0	4	0	0	119	3	0	1	1	0	0	0	0	2	0	1	0	0	0
6	1	7	3	0	1	134	5	1	0	0	0	3	0	1	0	0	4	0	0
7	0	0	2	0	0	0	119	4	2	0	0	0	0	0	0	0	2	0	0
8	0	0	0	1	0	0	1	68	0	0	0	0	0	0	0	0	0	0	0
9	0	1	0	0	0	0	0	1	49	1	0	3	0	0	1	1	1	0	0
10	0	1	2	0	0	0	0	0	0	41	0	2	0	0	0	0	1	0	0
11	0	0	0	0	1	0	0	0	0	0	13	0	0	0	1	0	0	0	0
12	0	1	0	0	1	0	0	0	0	0	0	119	0	0	0	0	0	0	1
13	0	0	0	0	1	0	0	0	1	0	0	0	44	0	0	0	0	0	0
14	1	2	1	0	1	2	0	2	0	1	0	6	4	116	2	4	0	1	0
15	0	2	0	0	1	0	0	1	0	0	0	2	0	0	48	0	0	0	0
16	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	56	0	1	2
17	0	1	4	0	1	1	1	0	1	0	0	0	0	0	0	0	72	0	1
18	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	10	1
19	0	0	4	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	49
20	0	2	2	1	1	0	0	0	2	0	2	0	1	0	1	0	0	0	1
21	1	1	1	0	1	1	0	0	3	0	0	4	0	0	0	0	2	0	0
22	0	0	0	1	0	0	0	0	1	0	0	1	0	2	0	1	1	0	0
23	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0
24	0	2	0	0	2	0	0	0	0	0	0	1	0	0	0	0	0	0	0
25	0	3	0	1	0	0	1	0	1	0	0	4	0	1	0	1	3	0	0
26	0	1	2	0	1	1	4	0	3	0	1	0	0	2	1	1	0	0	0
27	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	2	0	0
28	0	0	4	0	3	0	0	0	4	0	0	0	1	5	1	0	0	0	0
29	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0

	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
1	3	1	0	0	0	1	0	1	3	0	13	9	1	0	0	0	0	1	1	61
2	1	7	1	0	0	3	3	0	7	0	21	17	2	0	0	3	0	3	8	24
3	0	3	3	2	0	1	1	1	0	0	6	21	0	0	1	1	0	3	1	3
4	0	0	0	0	0	0	1	0	2	0	4	13	0	0	0	0	0	3	0	3
5	1	2	1	0	0	1	0	1	0	0	15	0	67	3	0	0	3	1	8	117
6	2	7	1	1	0	0	3	3	10	1	13	0	89	10	0	1	1	2	21	5
7	0	0	2	1	1	0	0	1	1	1	7	0	30	1	0	2	1	0	4	0
8	0	0	0	0	0	0	0	0	1	0	3	0	16	0	0	0	0	0	1	0
9	2	0	1	0	1	3	3	0	3	1	14	0	0	34	0	0	0	1	4	7
10	0	0	2	0	0	0	1	0	0	0	5	0	0	23	0	0	0	1	2	3
11	0	1	1	0	0	0	1	0	1	0	2	0	0	0	25	0	0	0	0	0
12	0	1	0	0	0	0	0	2	0	0	6	0	0	0	16	1	0	2	7	2

Table IV-8 (continued)

	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
13	3	1	0	0	1	1	1	0	0	0	9	0	0	1	2	16	0	1	0	75
14	1	3	0	0	0	2	7	0	4	1	21	0	1	6	1	68	2	2	13	3
15	0	0	0	0	0	0	0	0	1	0	3	0	0	0	1	37	0	1	4	0
16	0	0	0	0	0	0	0	0	2	0	4	0	0	0	0	39	0	0	1	0
17	0	0	2	0	0	1	0	0	2	2	4	0	0	1	1	0	3	0	3	1
18	0	0	0	0	0	0	0	1	2	0	9	0	0	0	0	0	32	3	4	2
19	0	0	0	0	0	0	1	0	1	0	3	0	0	0	0	0	53	0	3	1
20	247	7	6	0	2	1	1	1	2	3	23	0	2	0	1	1	1	54	8	144
21	6	309	7	1	0	2	4	0	3	2	12	0	2	2	1	4	0	83	13	15
22	1	4	142	2	0	1	0	0	1	0	7	0	0	0	0	0	1	60	2	1
23	0	1	0	109	0	0	0	1	5	0	9	0	1	0	2	0	0	58	0	0
24	4	0	1	0	51	1	1	1	2	0	9	0	1	0	0	0	0	1	60	106
25	4	9	0	0	7	162	5	0	6	2	26	0	4	1	1	2	0	3	279	15
26	1	0	2	0	0	3	134	2	7	0	19	1	1	0	0	3	1	2	237	9
27	0	0	0	0	0	0	0	68	2	0	12	0	0	0	0	0	0	2	64	1
28	3	5	0	1	0	4	1	0	241	1	25	0	2	0	0	5	0	6	2	6
29	0	0	0	0	1	0	0	0	0	53	16	0	0	0	0	2	0	1	3	3

	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58
1	12	1	0	124	0	0	0	2	0	2	1	2	1	14	7	1	7	2	1
2	119	1	0	326	42	3	2	3	0	0	2	3	2	18	89	5	12	105	1
3	1	11	2	187	67	4	0	0	0	0	0	1	2	1	46	43	1	72	0
4	4	0	32	0	44	0	0	1	0	0	0	1	2	1	2	10	0	0	0
5	8	2	0	5	0	1	4	0	0	3	2	0	2	99	6	2	79	8	0
6	121	3	2	4	1	211	1	0	0	8	4	1	2	72	112	8	45	116	0
7	3	16	0	2	0	132	0	3	0	0	2	3	5	6	35	27	0	60	0
8	1	1	13	0	1	0	0	0	0	0	0	1	1	0	1	17	2	1	0
9	45	6	0	2	0	3	0	1	1	0	1	0	0	50	39	0	0	6	1
10	3	14	1	3	0	0	0	3	0	0	1	0	3	17	43	1	0	5	0
11	11	0	0	1	1	0	0	0	0	1	0	0	0	0	0	0	0	5	0
12	2	15	0	3	0	0	2	1	0	0	0	0	0	1	6	3	1	38	0
13	10	0	0	5	0	0	43	1	0	0	0	0	0	51	4	0	2	6	2
14	102	6	0	11	1	1	75	186	2	2	5	1	5	73	96	3	4	92	9
15	2	12	0	1	0	0	0	101	21	1	4	0	2	3	24	13	0	22	0
16	2	1	10	0	0	0	0	0	30	0	0	0	2	0	7	10	0	2	0
17	4	10	2	5	0	1	0	1	0	0	4	1	1	5	17	2	1	28	0
18	32	5	0	3	0	1	1	0	0	0	4	0	1	6	6	0	2	25	1
19	6	15	2	2	0	0	0	3	0	0	5	0	2	6	2	0	1	33	0
20	7	0	0	4	2	1	0	0	0	155	4	0	0	118	12	0	9	6	2
21	91	3	1	6	1	0	0	7	0	167	185	3	2	90	95	3	1	8	0
22	1	21	1	1	1	0	0	1	0	2	120	60	2	7	43	4	0	1	2
23	0	2	18	0	1	0	0	0	1	0	1	63	2	1	4	3	0	2	0
24	10	0	0	3	2	0	0	2	0	1	2	0	0	82	4	2	57	3	0
25	130	3	2	4	1	4	5	5	0	3	2	0	2	137	116	5	74	121	4
26	12	28	3	6	1	3	1	6	0	2	3	3	110	18	109	59	13	150	0
27	3	1	28	2	1	0	0	0	0	0	0	1	90	6	11	17	1	8	0
28	7	30	2	1	1	0	6	6	0	4	5	2	4	12	10	0	4	11	59
29	1	4	0	1	0	0	0	0	0	0	0	0	0	4	9	0	1	0	28

Table IV-9

Estimated Segment Sizes Using Various Procedures

Assignment	Probabilistic				Deterministic				WLS
Q	SRMS		SRMD		SRMS		SRMD		
Relaxation	.01	.10	.01	.10	.01	.10	.01	.10	
Partition									
1	92.3	107.1	90.2	99.1	93	108	93	99	93.6
2	145.7	209.8	159.9	231.6	165	237	177	253	271.2
3	173.2	251.9	179.3	270.0	199	288	199	292	215.8
4	133.1	163.4	122.8	159.8	134	163	123	178	115.7
5	89.2	143.3	86.7	134.1	92	149	87	144	122.2
6	88.0	125.9	108.5	146.7	108	154	127	145	158.4
7	89.7	156.3	73.6	139.6	105	182	82	142	96.0
8	69.5	92.6	63.6	80.4	64	84	64	79	44.6
9	41.6	68.6	31.0	63.0	46	78	31	72	41.2
10	37.2	46.4	35.7	44.5	46	56	32	46	33.8
11	20.9	50.5	13.4	29.6	13	32	13	17	22.8
12	87.4	131.5	96.7	136.7	110	170	110	147	99.8
13	41.5	59.4	27.1	50.4	39	55	27	54	56.5
14	58.9	82.1	74.9	109.0	103	144	103	136	125.3
15	42.1	62.1	25.5	49.7	49	71	24	61	29.2
16	50.2	63.1	44.0	59.0	50	62	50	68	48.3
17	78.1	90.3	65.9	83.2	75	85	75	93	50.7
18	10.4	27.0	7.3	16.0	11	30	9	13	24.8
19	32.9	39.8	33.1	49.0	43	51	43	57	56.7
20	165.2	251.5	173.4	262.6	174	267	174	279	225.1
21	138.1	235.5	176.9	297.9	181	310	185	361	197.5
22	96.2	154.9	70.8	130.1	127	207	88	172	70.6
23	90.7	110.5	85.9	103.2	99	113	95	117	78.9
24	40.8	71.2	34.3	56.8	43	76	40	64	69.7
25	122.8	158.0	127.4	176.3	145	184	145	187	228.8
26	73.2	128.9	82.7	151.4	102	182	102	168	179.9
27	52.6	93.5	45.1	71.6	56	100	49	83	73.0
28	184.2	232.5	202.9	267.9	222	281	222	309	145.6
29	60.5	91.5	52.3	67.9	67	104	52	67	38.1
30	2704.5	487.5	2445.7	389.3	2845	513	2512	320	215.4

Table IV-9 (continued)

Assignment	Probabilistic				Deterministic				WLS
Q	SRMS		SRMD		SRMS		SRMD		
Relaxation	.01	.10	.01	.10	.01	.10	.01	.10	
Partition									
31	227.5	245.0	188.4	200.5	61	66	61	61	13.2
32	257.4	250.4	214.3	232.9	236	229	200	219	98.8
33	63.0	113.2	45.3	85.8	39	71	45	82	45.5
34	58.8	63.8	53.1	55.7	40	43	40	52	21.7
35	240.8	237.9	214.5	211.9	191	189	179	185	73.1
36	120.9	139.8	113.2	112.1	110	125	110	98	37.4
37	358.7	342.3	355.6	342.4	314	301	344	294	237.5
38	576.9	708.1	569.3	680.4	571	700	568	753	360.9
39	505.4	732.7	471.9	636.3	491	711	456	607	399.2
40	435.9	594.4	507.2	714.2	489	664	589	750	300.0
41	230.5	292.6	210.5	226.7	204	256	211	211	-3.41
42	130.5	167.7	116.8	132.5	120	155	106	119	61.2
43	433.1	488.2	462.5	526.6	616	691	611	712	332.4
44	230.7	253.2	259.4	283.6	171	187	182	168	205.9
45	326.9	349.0	353.3	359.7	338	360	361	365	264.1
46	124.0	145.4	140.7	155.6	122	143	128	140	117.7
47	249.6	306.1	274.7	330.9	264	323	293	333	242.1
48	64.3	99.4	52.5	70.8	55	85	42	55	43.6
49	311.6	316.7	361.4	375.3	346	352	378	351	229.0
50	337.6	376.2	336.5	366.0	304	336	343	357	261.2
51	160.6	201.0	143.4	174.2	134	167	119	146	94.3
52	179.8	269.9	190.6	277.2	169	260	182	245	126.3
53	699.1	790.8	809.9	915.2	631	712	738	898	733.4
54	994.2	888.0	1194.5	1056.7	822	735	934	955	1031.1
55	218.8	234.0	184.2	187.3	201	215	173	238	26.1
56	197.1	332.3	216.5	348.1	148	251	178	317	309.0
57	1001.8	888.6	986.7	861.9	1095	973	1184	936	308.6
58	133.9	166.7	116.1	133.6	92	113	92	110	59.6

Table IV-10

Chi Squares/Correlations between Segment Size Estimates

Method	1	2	3	4	5	6	7	8	9
1	.	0.645	0.992	0.587	0.992	0.634	0.989	0.575	0.476
2	0.283	.	0.708	0.983	0.616	0.968	0.691	0.960	0.828
3	0.016	0.267	.	0.666	0.979	0.692	0.989	0.645	0.573
4	0.321	0.022	0.274	.	0.557	0.954	0.643	0.972	0.891
5	0.037	0.329	0.045	0.350	.	0.636	0.992	0.572	0.434
6	0.331	0.044	0.304	0.046	0.298	.	0.707	0.986	0.779
7	0.044	0.297	0.028	0.299	0.015	0.273	.	0.657	0.516
8	0.363	0.059	0.315	0.031	0.332	0.018	0.282	.	0.812
9	0.570	0.244	0.486	0.174	0.563	0.226	0.501	0.190	.

Methods

1 = Probabilistic, 1% relaxation, using q's computed from SRMS

2 = Probabilistic, 10% relaxation, using q's computed from SRMS

3 = Deterministic, 1% relaxation, using q's computed from SRMS

4 = Deterministic, 10% relaxation, using q's computed from SRMS

5 = Probabilistic, 1% relaxation, using q's computed from SRMD

6 = Probabilistic, 10% relaxation, using q's computed from SRMD

7 = Deterministic, 1% relaxation, using q's computed from SRMD

8 = Deterministic, 10% relaxation, using q's computed from SRMD

9 = Partition sizes as computed by weighted least squares procedure

Numbers above the diagonal are correlations between methods (n=58)

Numbers below diagonal are (Chi Square/58)

Table IV-11

Chi square/Correlations between Recovered Switching Matrices

Method	1	2	3	4	5	6	7	8	9	10	11
1	.	.983	.989	.987	.988	.983	.989	.986	.985	.965	.828
2	.138	.	.999	.999	.999	.999	.998	.999	.998	.974	.859
3	.109	.012	.	.999	.999	.998	.999	.999	.999	.977	.855
4	.124	.004	.013	.	.999	.999	.999	.999	.999	.976	.852
5	.133	.013	.006	.013	.	.999	.998	.998	.999	.977	.855
6	.140	.001	.014	.005	.011	.	.998	.999	.999	.973	.859
7	.111	.010	.002	.011	.006	.011	.	.999	.999	.976	.854
8	.129	.004	.015	.002	.012	.004	.012	.	.999	.975	.853
9	.138	.011	.003	.009	.004	.012	.004	.014	.	.973	.856
10	.229	.119	.112	.120	.123	.121	.113	.123	.116	.	.841
11	.705	.635	.645	.654	.661	.633	.643	.648	.651	.765	.

Methods

1 = Actual Switching Matrix

2 = Probabilistic assignment, 1% relaxation, SRMS

3 = Probabilistic assignment, 10% relaxation, SRMS

4 = Deterministic assignment, 1% relaxation, SRMS

5 = Deterministic assignment, 10% relaxation, SRMS

6 = Probabilistic assignment, 1% relaxation, SRMD

7 = Probabilistic assignment, 10% relaxation, SRMD

8 = Deterministic assignment, 1% relaxation, SRMD

9 = Deterministic assignment, 1% relaxation, SRMD

10 = Sticky clustering procedure

11 = Partition sizes as computed by weighted least squares procedure

Numbers above the diagonal are correlations between methods (n=841)
Numbers below diagonal are (Chi Square/841)

particular. If, on the other hand, within-partition probabilities had been completely relaxed (i.e., such that $q_{ix} = q_i$ for all partitions), then each partition would have equal posterior probabilities, regardless of the household's observed purchases. Thus it is expected that increased relaxation would tend to "flatten" the estimated partition sizes. This indeed seems to have been the case.

The chi-square and correlation results underscore the differences between the relaxation levels. For example, with probabilistic assignment, there is only a .645 correlation between the segment size estimates from the 10% relaxation-square root of market share method and the 1% relaxation-square root of market share method (chi-square/N = .283). For the square root of the major diagonal method, the correlation is .666 (chi-square/N = .274).

A second effect apparent from Tables IV-1 through IV-8 is that when within-partition probabilities are relaxed, there is a greater number of non-partition purchases assigned to each partition. For example, in Table IV-1, approximately 13 non-partition purchases were assigned to the 12-ounce Ivory loyal partition (partition 1), when a 10% relaxation is used. In Table IV-2, only 5 non-partition purchases were assigned to the 12-ounce Ivory loyal partition.

The method of computing the initial q_{ix}'s does not seem to greatly affect the sizes of partitions as estimated by the Bayesian procedure. Estimates of partition sizes using the square root of the market share were nearly identical to those computed using the square root of the major diagonal, as long as the same level of relaxation is used. For example, from Table IV-8, the correlations between the square root method and the major diagonal method are .992 (1% relaxation) and

.983 (10% relaxation). Chi-square/N are .037 (1% relaxation) and .022 (10% relaxation). There does appear to be a slight tendency for the square root of the major diagonal to flatten the estimated partition sizes compared to the square root of the market shares.

The WLS regression procedure produced segment size estimates that differed significantly from all of the different Bayesian approaches. Referring to Table IV-10, the correlations between the WLS estimates and the Bayesian estimates were uniformly low (ranging between .434 and .891) and chi-square/N was uniformly high (ranging between .244 and .570). One might argue that the WLS estimates are actually better estimates of segment size, and that the poor association between the WLS estimates and the Bayesian estimates is due to the Bayesian estimates being inaccurate. However, if one of the methods is correct, it should be able to recover the observed switching data. Table IV-11 demonstrates that the WLS method was substantially inferior to the other methods with regard to recovering the item-switching matrix. This result implies that the proportionality assumption of the WLS approach may be violated in the LDD category, and that substantial evidence may be lost through aggregating households.

Table IV-9 also shows that the structure obtained from the sticky clustering procedure does slightly worse than the Bayesian estimates in fitting the observed switching matrix. A priori, one would expect the sticky clustering structure to fit the observed switching matrix as well as or better than any method using the hybrid structure, since it contains more segments (81 versus 58). The slightly lower fit of the sticky structure may be due in part to the low purchase rate in the category. For example, two households belonging to the same segment

may appear to have different purchase probabilities if only a few purchases are observed. The sticky clustering procedure would be more likely to assign the two households to different segments than the Bayesian approach. Thus, the sticky clustering method may be biased toward "over-identifying" partitions when the purchase rate is low.

Among the various Bayesian assignment methods, partition sizes estimated using the deterministic assignment rule were very close to those estimated using proportional assignment with 1% relaxation. However, as stated earlier, when within-partition q_{ix}'s were relaxed more liberally (10%), the resultant partition size estimates were substantially different, especially with regard to the size of the "all-brands" partition (30). Deterministic assignment and probabilistic assignment using 1% relaxation yield almost identical results. This being the case, it might be conclude that there is little uncertainty about the segment membership of each household, and all the choices made by the households can be grouped without resorting to the more complex purchase assignment procedures.

Besides requiring less additional analysis, the deterministic assignment method using 1% relaxation has the advantage of being more compatible with the underlying assumptions of the proposed model. Conceptually, by relaxing within-partition probabilities, we also are relaxing the assumptions of the discrete market partition framework. Under a strict interpretation of the model, households assigned to a partition should only buy brands within that partition, and would never buy a brand or item not contained in that partition. The 1% relaxation is a but a slight divergence from this assumption: here, the assumed probability of within-partition purchase is dropped from 100% to 99%.

probability of within- partition purchase is dropped from 100% to 99%. In the more liberal 10% relaxation, this assumed probability drops to 90%. Under a strict interpretation of the partitions as representing evoked sets or consideration sets, such a relaxation might be viewed as overly liberal. Further, the 10% relaxation results in a greater number of non-partition item purchases assigned to each partition.

Thus, for reasons of consistency and ease of estimation, the deterministic assignment procedure was chosen as the best segmentation method. Each household in the panel (as well as their purchases records) was assigned to one and only one of the 58 hybrid partitions.

CHAPTER V

MODEL ESTIMATION

The Bayesian analysis described in the previous chapter assigned each household in the panel to a unique partition. The items contained within the partition are interpreted as representing the consideration or choice set of the households assigned to the partition. Thus, the Bayesian assignment acted as a method for disaggregating the total set of households into unique segments. Ideally, the households were assigned in such a way that the segments are homogeneous with respect to choice behavior. The estimated response sensitivities of each segment to marketing mix variables are of great interest to the multi-brand firm, as they can provide an index both of the loci and severity of competition among the brands within the portfolio.

This chapter reports the steps involved in the final phase of calibrating the proposed segmented logit model, i.e, parameter estimation. Parameter estimation itself was divided into two phases, corresponding to the estimation of the purchase probability component (N_{xt}) and the primary demand component (P_{ixt}). The approach is similar to the estimation of a nested logit model (Currim 1982), except that the higher node choice (primary demand) and the lower node choice (purchase probability) were modeled in two distinct steps rather than simultaneously. This approach was adopted in part due to the enormous

number of choice occasions for the higher node. Details of the parameter estimation and a discussion are presented below.

Purchase Probability

To develop estimates for the purchase probability component (P_{ixt}) of the proposed segmented logit model, information on segment membership was used to divide the total set of all observed purchase occasions into 58 subsamples, corresponding to each of the identified partitions/segments. Secondly, the week and the store at which each recorded purchase took place was noted. Next, the audited shelf price for each item in the purchase store/week was obtained from the store scanner data set. The values of independent variables were then computed with the following equation:

Z_{it} = (observed shelf price per ounce of item i at time t in the store where the purchase occurred)
- (observed mean price per ounce of item i over 52 weeks in the store where the purchase occurred)

The price variables were normalized in this fashion because, in general, prices levels can vary significantly between stores and because the items have different sizes. Converting the raw item price into price per ounce facilitates comparisons between items of different sizes. Further normalization by subtracting mean price per ounce compensates for differences between store price levels and forces base price effects into the constant parameter for each item in the logit analysis. The variables Z_{it} then represent the observed depth of price discount for each item at the store in which the purchase was made. As a final data

treatment, purchases records of items outside the inferred consideration set were deleted from each subsample.

Next, we specified a unique polytomous multinomial logit model for each of the switching segments (30 through 58), where the independent variables are the computed Z_{it}'s for only those items belonging to the segment's consideration set. By definition, P_{iit} for each item-loyal segment is 1; therefore, no share-model estimates are necessary for item-loyal segments. Within each segment-level choice model, one item was chosen as a baseline and a unique intercept parameter was estimated for each of the remaining considered brands. In addition, each segment included one parameter for the price discount variable Z_{it}.

$$P_{ixt} = \frac{\exp\{\ \beta_{0ix} + \beta_x Z_{it}\ \}}{\sum_{j \in A_x} \exp\{\ \beta_{0jx} + \beta_x Z_{jt}\ \}} \tag{5.1}$$

While the proposed segmented logit model is very general as to the specification of independent variables, only price was used as an independent variable in the estimated model. The reason for examining price alone were threefold. First, the remaining potential independent variables (feature advertising and display) were so collinear with price as to make separate parameter estimates unstable or impossible for many segments. Secondly, feature advertising and display were measured on scales which, at best, could be considered dichotomous, and thus did not reflect intensity of promotion. For example, the data records if a

special display had been set up for item i in store s at time t, but does not measure the size and prominence of the display. Price, on the other hand, provides a convenient ratio level independent variable of both the presence and intensity of promotional activity. Third, obtaining analytical optimal values for feature advertising and special display is very difficult, both because of the lack of appropriate ratio scale properties and because it is difficult to ascertain the cost associated with these types of promotion. Theoretically, optimal levels of advertising and display could be obtained with integer programming methods assuming that costs were known.

It should be noted that the observed collinearity between price and other forms of promotion suggests that we are not truly modeling the effect of price alone, but the effect of price plus the effect of the other unspecified independent variables with which price is associated. The reported parameter estimates and optimal values for price should be interpreted in this light. We will leave it to future research to separate these effects empirically within the structure of the SLM.

Maximum likelihood estimates of the parameters of P_{ixt} were obtained using the PROC MLOGIT procedure of SAS, and are presented in Table V-1.

Despite the small number of choices made by some segments, 26 of the 29 segment-level models contained at least one significant parameter estimate. Of the 29 estimated price discount sensitivity parameters, 21 were significant and all had the desired negative sign. Further, there were significant differences among the estimated price sensitivities for each segment. These results are somewhat encouraging

TABLE V-1

Parameter Estimates for the Purchase Probability Component

(Values significant at .05 underlined)

	Segment							
	30	31	32	33	34	35	36	37
Price	-.740	-.663	-.118	-.046	-6.37	-.890	-.763	-.612
Item Specific Intercepts								
Ivory 12	.834	-.396	-	-	-	-	-	-
Ivory 22	1.520	.245	-	-	-	-	-	-
Ivory 32	.383	.381	-	-	-	-	-	-
Ivory 48	-.124	0.000	-	-	-	-	-	-
Joy 12	1.301	-	1.097	-	-	-	-	-
Joy 22	1.359	-	1.328	-	-	-	-	-
Joy 32	.347	-	.326	-	-	-	-	-
Joy 48	-.626	-	0.000	-	-	-	-	-
Ajax 22	.799	-	-	-.578	-	-	-	-
Ajax 32	.280	-	-	0.000	-	-	-	-
Derm. 22	-1.760	-	-	-	.125	-	-	-
Derm. 32	.078	-	-	-	0.000	-	-	-
Palm. 12	.450	-	-	-	-	-.541	-	-
Palm. 22	1.185	-	-	-	-	.264	-	-
Palm. 32	.053	-	-	-	-	-.182	-	-
Palm. 48	-.318	-	-	-	-	0.000	-	-
Dove all	.085	-	-	-	-	-	-	-
Lux 22	.252	-	-	-	-	-	.167	-
Lux 32	-.204	-	-	-	-	-	0.000	-
Dawn 12	1.228	-	-	-	-	-	-	.518
Dawn 22	1.449	-	-	-	-	-	-	.486
Dawn 32	.463	-	-	-	-	-	-	.483
Dawn 48	.044	-	-	-	-	-	-	0.000
Sunl. 12	.719	-	-	-	-	-	-	-
Sunl. 22	1.440	-	-	-	-	-	-	-
Sunl. 32	1.192	-	-	-	-	-	-	-
Sunl. 48	.483	-	-	-	-	-	-	-
Store all	1.265	-	-	-	-	-	-	-
Gen. all	0.000	-	-	-	-	-	-	-
n choices	2512	60	192	36	30	144	55	269
log (L)	-7826	-79.2	-241.6	-23.5	-17.5	-183.4	-37.4	-364.5
chi square	1265	8.04	49.18	2.83	6.56	32.39	1.49	16.81

TABLE V-1 (Continued)

(Values significant at .05 underlined)

	Segment							
	38	39	40	41	42	43	44	45
Price	-.814	-.118	-.473	-.711	-.810	-.314	-.661	-.570
			Item Specific Intercepts					
Ivory 12	-	-1.02	-	-	-	-.547	-	-
Ivory 22	-	-	.060	-	-	.496	-.444	-
Ivory 32	-	-	-	1.529	-	.000	.051	-
Ivory 48	-	-	-	-	.600	-	.000	-
Joy 12	-	-.189	-	-	-	-	-	-
Joy 22	-	-	-.216	-	-	-	-	.349
Joy 32	-	-	-	.535	-	-	-	.000
Joy 48	-	-	-	-	-.349	-	-	-
Ajax 22	-	-	-.955	-	-	-	-	-
Ajax 32	-	-	-	-.121	-	-	-	-
Derm. 22	-	-	-2.33	-	-	-	-	-
Derm. 32	-	-	-	-.157	-	-	-	-
Palm. 12	-	-.330	-	-	-	-	-	-
Palm. 22	-	-	-.208	-	-	-	-	-
Palm. 32	-	-	-	-.088	-	-	-	-
Palm. 48	-	-	-	-	-.356	-	-	-
Dove all	-	-	-	.456	-	-	-	-
Lux 22	-	-	-1.34	-	-	-	-	-
Lux 32	-	-	-	.200	-	-	-	-
Dawn 12	-	.489	-	-	-	-	-	-
Dawn 22	-	-	-.123	-	-	-	-	-
Dawn 32	-	-	-	1.028	-	-	-	-
Dawn 48	-	-	-	-	.144	-	-	-
Sunl. 12	.101	0.000	-	-	-	-	-	-
Sunl. 22	1.105	-	-	-	-	-	-	-
Sunl. 32	1.103	-	-	1.248	-	-	-	-
Sunl. 48	0.000	-	-	-	0.000	-	-	-
Store all	-	-	-	.993	-	-	-	-
Gen. all	-	-	-	0.000	-	-	-	-
n choices	423	378	558	198	85	570	176	344
log (L)	-484.9	-568.5	-1057	-434.2	-126.9	-576.3	-185.3	-229.7
chi square	202.92	79.71	206.8	81.20	19.76	99.74	16.14	17.43

TABLE V-1 (Continued)

(Values significant at .05 underlined)

	Segment							
	46	47	48	49	50	51	52	53
Price	-.744	-.524	-.867	-.212	-.398	-.788	-1.016	-.599
Item Specific Intercepts								
Ivory 12	-	-	-	-	-	-	-	-
Ivory 22	-	-	-	-	-	-	-	-
Ivory 32	-	-	-	-	-	-	-	-
Ivory 48	-	-	-	-	-	-	-	-
Joy 12	-	-	-	-	-	-	-	-.325
Joy 22	-	-	-	-	-	-	-	-.566
Joy 32	-	-	-	-	-	-	-	-
Joy 48	-	-	-	-	-	-	-	-
Ajax 22	-	-	-	-	-	-	-	-1.36
Ajax 32	-	-	-	-	-	-	-	-
Derm. 22	-	-	-	-	-	-	-	-
Derm. 32	-	-	-	-	-	-	-	-
Palm. 12	-.207	-	-	-	-	-	-	-.989
Palm. 22	0.000	.556	-	-	-	-	-	-.897
Palm. 32	-	0.000	-.438	-	-	-	-	-
Palm. 48	-	-	0.000	-	-	-	-	-
Dove all	-	-	-	-	-	-	-	-
Lux 22	-	-	-	-	-	-	-	-
Lux 32	-	-	-	-	-	-	-	-
Dawn 12	-	-	-	-.017	-	-	-	.009
Dawn 22	-	-	-	0.000	.380	-	-	-.513
Dawn 32	-	-	-	-	0.000	-.370	-	-
Dawn 48	-	-	-	-	-	0.000	-	-
Sunl. 12	-	-	-	-	-	-	-	-.362
Sunl. 22	-	-	-	-	-	-	-	0.000
Sunl. 32	-	-	-	-	-	-	-.398	-
Sunl. 48	-	-	-	-	-	-	0.000	-
Store all	-	-	-	-	-	-	-	-
Gen. all	-	-	-	-	-	-	-	-
n choices	110	264	39	342	323	110	136	690
log (L)	-72.1	-168.6	-25.1	-236.9	-216.1		-71.5	-1439
chi square	8.33	28.78	3.63	0.28	15.52		45.56	154.03

TABLE V-1 (Continued)

(Values significant at .05 underlined)

SEGMENT

		54	55	56	57	58
Price		-.593	-.798	-.919	-.675	-3.067
Item Specific Intercepts						
Ivory	12	-	-	-	-	-
Ivory	22	-.428	-	-	-.354	-
Ivory	32	-1.23	.224	-	-.574	-
Ivory	48	-	-1.12	-	-	-
Joy	12	-	-	.416	-	-
Joy	22	-4.79	-	-.492	-.319	-
Joy	32	-.039	-.406	-	-.887	-
Joy	48	-	-.984	-	-	-
Ajax	22	-.797	-	-	-	-
Ajax	32	-1.06	-	-	-	-
Derm.	22	-	-	-	-3.09	-
Derm.	32	-	-	-	-1.29	-
Palm.	12	-	-	-	-	-
Palm.	22	-.240	-	-	-.521	-
Palm.	32	-1.33	-.585	-	-2.06	-
Palm.	48	-	-1.49	-	-	-
Dove	all	-1.71	-	-	-1.82	-
Lux	22	-	-	-	-2.66	-
Lux	32	-	-	-	-2.01	-
Dawn	12	-	-	-	-	-
Dawn	22	-.074	-	-	-	-
Dawn	32	-1.03	-	-	-	-
Dawn	48	-	-	-	-	-
Sunl.	12	-	-	.072	-	-
Sunl.	22	-.078	-	0.000	-.418	-
Sunl.	32	0.000	-.007	-	0.000	-
Sunl.	48	-	0.000	-	-	-
Store	all	-	-	-	-	.911
Gen.	all	-	-	-	-	0.000
n choices		874	166	162	1143	78
log (L)		-2020	-313.4	-205.3	-2557	-43.4
chi square		443.7	63.52	38.47	750.1	21.30

as the Bayesian assignment seems to have identified groups of households that differ in terms of sensitivity to price as well as choice sets.

Further, some of the discrepancies in price sensitivity between segments have particular significance for marketing decision makers. Note that segment 39 (the 12-ounce segment) is much less sensitive to price differences than segment 40 (the 22-ounce segment). The effect of item size on price sensitivity appeared to be quite systematic, as segments loyal to smaller sizes were generally less price sensitive than segments loyal to larger sizes; note the monotonicity of β_p for segments 39 through 42. The pattern of sensitivity also held quite strongly for the "chained" segments, with the segments loyal to the smaller sizes of the same brand generally having less price sensitivity. While intuitively appealing, this finding is of importance to a marketer or retailer deciding which item to discount for a particular week, and to whom these discounts should be offered. We can also see that such differences exist between the various brand loyal segments: for example, the Joy-loyal segments (32 and 45) seem much less sensitive to price discounts than the Palmolive-loyal segments (35, 46, 47 and 48). As an index of cannibalization, the segment specific price sensitivities imply that size-to-size cannibalization is more intensive for Palmolive. These and related issues are pursued in greater depth in Chapter VII.

Primary Demand

To obtain estimates of the parameters of the primary demand component (N_{xt}) of the segmented logit model, a more complex data handling procedure was required. Since the primary demand component models the decision between purchase and non-purchase, information on

the independent variables during non-purchase occasions is necessary. In order to do this, an estimation data set consisting of 52 observations (one for each week) for each panel household was generated. If the household made a purchase in week t, then the t-th observation for the household consisted of a vector of observed price discounts (Z_{it}) for the store/week in which the purchase was made. For the remaining (non-purchase) weeks, the observed store loyalties of each household to were used to generate observations. If the household was observed to be completely loyal to store A (100% of item purchases occurred at store A), then the non-purchase week observations were vectors containing observed price discounts (Z_{it}) for the items in store A. If the household was observed to switch between stores, the proportion of purchases that household made at each of the stores was computed. Then, for each of the non-purchase weeks, one of the stores was selected with probability proportional to the household's store loyalty vector. The observed price discounts (Z_{it}) for the selected stores were used as the observations for the non-purchase weeks. For example, if 80% of the household's purchases occurred at store s, then the store had an 80% probability of being selected for each of the household's non-purchase weeks.

Thus, the estimation data set consisted of observed price discounts for 120,224 (52 weeks, 2312 households) choice occasions, where the choice was between purchase and non-purchase.

To estimate primary demand (N_{xt}), a binomial logit model was specified for each segment (including single brand loyal segments), where the alternatives were (buy, not buy) and the independent variables were the observed weekly price discounts (Z_{it}) for the set of items

contained in the segment's consideration set. For each segment, the model specified one intercept parameter and a separate slope parameter for the price discount of each of item:

$$N_{xt} = \frac{N_x}{(1 + \exp - (\gamma_{0x} + \sum_{j \in A_x} \gamma_{jrx} Z_{jrt}))} \tag{2.2}$$

Maximum likelihood estimates of the parameters were obtained with the PROC CATMOD procedure of SAS, and are presented in Table V-2.

A noteworthy result of the primary demand estimation is that the intercept terms varied widely between segments. Since the intercept scales category purchase probability, this result implies that the different segments had widely different purchase rates. For example, the primary demand intercept for segment 1 (Ivory 12-ounce loyal) implies a category purchase probability of .085, or a purchase approximately every 12 weeks when no prices were being discounted. The intercept for segment 30 (all items) implies a category purchase probability of .143, or a purchase every 7 weeks. In general, the single-item loyal segments had larger intercept terms, implying lower purchase rates. This result is not surprising, as households with lower purchase rates were more likely to be assigned to the single-item partitions by the Bayesian method described in Chapter IV.

A second noteworthy result of the primary demand estimation is that only a handful of the slope parameters were significantly different from zero (at alpha=.05), while every intercept parameter was significant. For the single-item loyal segments, only 4 of 29 price discount parameters exhibited significance at .05. In the multi-item segments, only 16 of 155 total price parameters were significant. Of the

Table V-2

Primary Demand Parameters for Single-Item Loyal Segments

(values significant at .05 underlined)

Segment	1	2	3	4	5	6	7	8
Intercept	2.38	2.51	2.52	2.52	2.84	2.56	2.41	2.66
Brand effect	-.872	-.154	.251	.074	-.118	.029	.292	.035
Likelihood Ratio								
df	9	78	65	47	9	91	36	26
x2	74.7	119.5	91.6	51.4	16.4	84.3	34.1	15.5
prob(x2)	.000	.002	.017	.306	.060	.678	.562	.947

Segment	9	10	11	12	13	14	15
Intercept	2.73	2.36	1.67	2.73	2.49	2.88	3.39
Brand effect	.351	.387	1.772	-1.04	-.465	-.184	.168
Likelihood Ratio							
df	50	52	3	16	10	87	37
x2	36.6	55.3	12.3	13.7	11.6	82.8	31.6
prob(x2)	.922	.351	.007	.624	.312	.608	.722

Segment	16	17	18	19	20	21	22
Intercept	2.54	2.59	3.75	2.85	2.82	2.92	2.65
Brand effect	.114	.290	-.372	-.482	-.580	.057	.621
Likelihood Ratio							
df	59	30	6	24	6	63	31
x2	72.6	31.2	2.4	14.1	4.3	67.7	32.0
prob(x2)	.110	.404	.879	.944	.637	.320	.416

Segment	23	24	25	26	27	28	29
Intercept	2.82	2.45	2.55	3.03	2.41	2.43	2.58
Brand effect	.174	.506	.093	-.060	.305	.904	.151
Likelihood Ratio							
df	52	48	65	92	77	158	19
x2	54.4	62.0	66.2	69.0	114.7	156.6	33.9
prob(x2)	.383	.085	.360	.965	.003	.517	.019

Table V-2 (continued)

Primary Demand Parameter Estimates for Switching Segments

(values significant at .05 underlined)

Segment		30	31	32	33	34	35	36	37
Intercept		1.79	2.09	2.10	1.92	2.37	1.92	1.69	1.88
Brand									
Ivory	12	1.83	.100	-	-	-	-	-	-
Ivory	22	.003	.456	-	-	-	-	-	-
Ivory	32	.036	-.163	-	-	-	-	-	-
Ivory	48	.063	.391	-	-	-	-	-	-
Joy	12	-2.00	-	-1.96	-	-	-	-	-
Joy	22	.006	-	.439	-	-	-	-	-
Joy	32	.079	-	-.019	-	-	-	-	-
Joy	48	-.089	-	-.096	-	-	-	-	-
Ajax	22	.093	-	-	-.067	-	-	-	-
Ajax	32	.012	-	-	-.244	-	-	-	-
Derm.	22	.516	-	-	-	4.88	-	-	-
Derm.	32	-.025	-	-	-	-1.93	-	-	-
Palm.	12	.091	-	-	-	-	-.348	-	-
Palm.	22	.054	-	-	-	-	-.057	-	-
Palm.	32	-.094	-	-	-	-	-.029	-	-
Palm.	48	-.082	-	-	-	-	-.001	-	-
Dove	all	.068	-	-	-	-	-	-	-
Lux	22	.027	-	-	-	-	-	-.274	-
Lux	32	.170	-	-	-	-	-	.158	-
Dawn	12	.329	-	-	-	-	-	-	2.53
Dawn	22	.144	-	-	-	-	-	-	-.047
Dawn	32	.070	-	-	-	-	-	-	.241
Dawn	48	.058	-	-	-	-	-	-	-.136
Sunl.	12	.035	-	-	-	-	-	-	-
Sunl.	22	.020	-	-	-	-	-	-	-
Sunl.	32	-.026	-	-	-	-	-	-	-
Sunl.	48	.039	-	-	-	-	-	-	-
Store	all	-.042	-	-	-	-	-	-	-
Gen.	all	-.460	-	-	-	-	-	-	-
	df	30	5	5	3	3	5	3	5

Likelihood Ratio

	30	31	32	33	34	35	36	37
df	665	102	152	81	10	250	42	149
X2	927.6	82.2	179.5	79.7	10.3	225.0	56.4	184.8
prob(x2)	.0001	.926	.064	.519	.415	.871	.068	.025

TABLE V-2 (Continued)

(values significant at .05 underlined)

Segment		38	39	40	41	42	43	44
Intercept		1.84	1.92	2.08	2.04	1.90	1.78	2.06
Brand								
Ivory	12	-	-4.47	-	-	-	-.223	-
Ivory	22	-	-	.201	-	-	.012	.170
Ivory	32	-	-	-	-.498	-	.229	.111
Ivory	48	-	-	-	-	.312	-	.350
Joy	12	-	4.42	-	-	-	-	-
Joy	22	-	-	.158	-	-	-	-
Joy	32	-	-	-	-.501	-	-	-
Joy	48	-	-	-	-	-.570	-	-
Ajax	22	-	-	.073	-	-	-	-
Ajax	32	-	-	-	.169	-	-	-
Derm.	22	-	-	-.568	-	-	-	-
Derm.	32	-	-	-	-.463	-	-	-
Palm.	12	-	.745	-	-	-	-	-
Palm.	22	-	-	.038	-	-	-	-
Palm.	32	-	-	-	.022	-	-	-
Palm.	48	-	-	-	-	.384	-	-
Dove	all	-	-	-	.172	-	-	-
Lux	22	-	-	-.005	-	-	-	-
Lux	32	-	-	-	-.102	-	-	-
Dawn	12	-	-.064	-	-	-	-	-
Dawn	22	-	-	.118	-	-	-	-
Dawn	32	-	-	-	-.143	-	-	-
Dawn	48	-	-	-	-	.330	-	-
Sunl.	12	.047	-.033	-	-	-	-	-
Sunl.	22	.022	-	.075	-	-	-	-
Sunl.	32	.059	-	-	.086	-	-	-
Sunl.	48	-.001	-	-	-	.334	-	-
Store	all	-	-	-	-.071	-	-	-
Gen.	all	-	-	-	-.290	-	-	-
	df	5	6	9	12	6	4	4

Likelihood Ratio

	38	39	40	41	42	43	44
df	399	129	531	467	247	163	206
x2	460.4	150.1	613.5	473.3	249.7	196.6	225.5
prob(x2)	.018	.099	.008	.410	.440	.038	.168

TABLE V-2 (Continued)

(values significant at .05 underlined)

Segment	45	46	47	48	49	50	51
Intercept	1.97	2.12	1.91	1.97	1.85	1.98	1.69
Brand							
Ivory 12	-	-	-	-	-	-	-
Ivory 22	-	-	-	-	-	-	-
Ivory 32	-	-	-	-	-	-	-
Ivory 48	-	-	-	-	-	-	-
Joy 12	-	-	-	-	-	-	-
Joy 22	.323	-	-	-	-	-	-
Joy 32	-.136	-	-	-	-	-	-
Joy 48	-	-	-	-	-	-	-
Ajax 22	-	-	-	-	-	-	-
Ajax 32	-	-	-	-	-	-	-
Derm. 22	-	-	-	-	-	-	-
Derm. 32	-	-	-	-	-	-	-
Palm. 12	-	.323	-	-	-	-	-
Palm. 22	-	-.136	-.021	-	-	-	-
Palm. 32	-	-	-.083	.559	-	-	-
Palm. 48	-	-	-	.253	-	-	-
Dove all	-	-	-	-	-	-	-
Lux 22	-	-	-	-	-	-	-
Lux 32	-	-	-	-	-	-	-
Dawn 12	-	-	-	-	.482	-	-
Dawn 22	-	-	-	-	.105	.048	-
Dawn 32	-	-	-	-	-	.071	-.168
Dawn 48	-	-	-	-	-	-	.049
Sunl. 12	-	-	-	-	-	-	-
Sunl. 22	-	-	-	-	-	-	-
Sunl. 32	-	-	-	-	-	-	-
Sunl. 48	-	-	-	-	-	-	-
Store all	-	-	-	-	-	-	-
Gen. all	-	-	-	-	-	-	-
df	3	3	3	3	3	3	3
Likelihood Ratio							
df	120	97	189	106	66	98	51
x2	120.0	84.8	208.1	109.4	77.2	98.2	66.9
prob(x2)	.482	.808	.163	.392	.163	.475	.067

TABLE V-2 (Continued)

(values significant at .05 underlined)

Segment		52	53	54	55	56	57	58
Intercept		1.92	1.92	1.92	2.11	2.05	1.88	2.37
Brand								
Ivory	12	-	-	-	-	-	-	-
Ivory	22	-	-	.111	-	-	-.038	-
Ivory	32	-	-	-.188	-.16	-	.063	-
Ivory	48	-	-	-	-.15	-	-	-
Joy	12	-	-.274	-	-	-.236	-	-
Joy	22	-	-.030	-.085	-	.291	.093	-
Joy	32	-	-	.001	.274	-	.192	-
Joy	48	-	-	-	.294	-	-	-
Ajax	22	-	.016	.092	-	-	-	-
Ajax	32	-	-	.122	-	-	-	-
Derm.	22	-	-	-	-	-	1.036	-
Derm.	32	-	-	-	-	-	.086	-
Palm.	12	-	.330	-	-	-	-	-
Palm.	22	-	-.012	-.034	-	-	-.031	-
Palm.	32	-	-	-.037	-.04	-	-.005	-
Palm.	48	-	-	-	.160	-	-	-
Dove	all	-	-	.260	-	-	.163	-
Lux	22	-	-	-	-	-	-.120	-
Lux	32	-	-	-	-	-	.409	-
Dawn	12	-	-.327	-	-	-	-	-
Dawn	22	-	.140	.201	-	-	-	-
Dawn	32	-	-	.250	-	-	-	-
Dawn	48	-	-	-	-	-	-	-
Sunl.	12	-	.053	-	-	.229	-	-
Sunl.	22	-	.080	.082	-	.070	.005	-
Sunl.	32	.074	-	-.117	.000	-	.004	-
Sunl.	48	.000	-	-	.052	-	-	-
Store	all	-	-	-	-	-	-	-.628
Gen.	all	-	-	-	-	-	-	-1.51
	df	3	10	14	9	5	14	3

Likelihood Ratio

	52	53	54	55	56	57	58
df	89	529	641	313	256	586	121
x2	128.9	623.2	774.0	344.3	259.9	677.4	97.8
prob(x2)	.004	.002	.000	.108	.420	.005	.940

16 significant parameters, 5 were negative, implying that price discounts for these items actually reduced the primary demand of the segment. One way of interpreting these results is that primary demand is small and constant. Conceptually, this outcome implies that (at least for the LDD category) consumers buy at approximately a constant rate, and are not particularly influenced by price discounts in deciding whether or not to make a purchase in the category. However, the significance of the price sensitivity parameters of the purchase probability model imply that once the decision to purchase is made, price information plays a strong role in determining which alternative is chosen.

An alternative explanation of these results is that the dependent variables (price discounts for each alternative) exhibit multicollinearity, which will tend to increase the standard errors of the estimates and therefore reduce significance. Though this might be the case, the former explanation is preferred for two reasons. First, retailers in grocery goods seldom, if ever, discount only one category, and fluctuations of primary demand might well be due to the store traffic built by discounts in another category. Even if the parameters of the primary demand model had turned out to be significant, we might still have questioned whether the observed increase was due to discounts in the LDD category alone, or to discounts in the other promoted categories. Secondly, the empirical results tend to agree with common sense regarding the LDD category. Conversations with industry personnel indicated that the LDDs are not considered to be "loss leaders", or items that are effective in building store traffic when heavily discounted. A convenience sample of LDD consumers at an Austin, Texas

food retailer tended to corroborate this view, as nearly all indicated that they bought in response to being out of stock at home rather than in response to advertised price discounts of particular brands. These self-reports of behavior are consistent with the empirical results of the primary demand model.

Although the primary demand for many product categories might be influenced by marketing activity, it does not seem to be affected significantly in the LDD category. For this reason, the individual item price parameters for the primary demand model were all set to zero, thereby reducing N_{xt} to a constant for each segment:

$$N_{xt} = N_x/(1 + \exp\{\gamma_{0x}\}) \tag{5.3}$$

The estimation of the share functions and the primary demand functions completes the empirical estimation of the SLM. In Chapter VI, the validity of the SLM is tested with holdout data via model comparison. In Chapter VII, the segments are compared in terms of membership characteristics, preferred items, and item elasticities. In Chapter VIII, the estimated SLM is used to obtain optimal prices levels.

CHAPTER VI

MODEL VALIDATION AND COMPARISON

Chi-square and likelihood ratio statistics indicated that the segmented logit model provided a uniformly good fit to the panel data from which it was calibrated. This provides some tentatively encouraging evidence of the model's validity, but in and of itself does not constitute a strong test of the model. The good fit which was evidenced on the calibration data might be due in large part to the large number of model parameters. A stronger test of the model's validity is in its ability to predict data outside the calibration data set, and how these predictions fare against the predictions of other reasonable model forms.

In this chapter, the predictive validity of the segmented logit model is compared and contrasted with a number of consumer and store sales models. The data set used to assess predictive validity consisted of the observed store-level sales and pricing data for the year subsequent to the calibration data set (i.e., April 1986 through April 1987). Comparison models and methods for generating predictions are described below.

The predictive validity of the structured logit model was compared against five other models. All the comparison models, like the SLM, were calibrated on the 1986-1987 UPC data. Two of the comparison models were calibrated using the panel (consumer) data, and the

remaining three comparison models were calibrated using the store sales data. Note that since the predictive validation is based on fit to store data, the store sales models have a built-in advantage in terms of data similarity.

Comparison Panel Models

Unsegmented Logit Model

A model that serves as a natural baseline comparison for the segmented logit model is the unsegmented logit model (ULM). In the ULM, sales quantity were modelled as

$$Q_{it} = N_t P_{it} \tag{6.1}$$

where the terms are defined in the same way as for the SLM, except that notation for segment membership is suppressed. The unsegmented logit model assumes that the population as an aggregate is homogeneous with respect to the choice process, and that each consumer (household) has a fully enumerated choice set: i.e., each household chooses from among all available alternatives.

The ULM was estimated on all observed choice occasions from the panel data set, using the same independent variables specified in the SLM. The program used to obtain maximum likelihood parameter estimates (TSP) is limited to approximately 3000 choice observations, a number significantly smaller than the number of choice occasions present in the panel data set. To obtain parameter estimates for the entire panel data set, a bootstrapping method was employed. First, 18 separate random

bootstrap samples of approximately 3000 choice occasions were selected from the total set of all purchase occasions. The parameter estimates were averaged across all 18 bootstrap samples to obtain estimates for the total sample. The individual bootstrap sample estimates as well as the mean estimates are presented in Table VI-1. While there was some variance for the item specific intercept estimates between samples, the price parameter estimate exhibited little variability.

Compared to the SLM, the ULM has the advantage of being much more parsimonious: the ULM required estimation of only 29 logit parameters while the SLM require estimation of 156 logit parameters. Of course, the SLM has the ability to represent structure due to underlying consumer evoked sets.

Random Structure Model

An observed improvement in fit of the SLM over the ULM might be attributed to the larger number of parameters contained in the SLM. To investigate whether or not the number of parameters alone could account for differences in fit, a random structure model (RSM) was calibrated on the panel data.

The first step in calibrating the RSM was generating a new "random" discrete structure matrix. The first 30 partitions in the random structure matrix were equivalent to the SLM structure matrix described in Chapter III (29 single-item loyal partitions and one all-items partition). The remaining partitions in the random structure matrix were generated by randomly assigning items to 29 multi-item partitions, subject to the constraint that each item was contained in exactly as many multi-item partitions as in the SLM structure. For

Table VI-1

Bootstrap Logit Model Parameter Estimates

Bootstrap Sample

Parameter	1	2	3	4	5	6	7	8	9	10
Price	-0.52	-0.58	-0.58	-0.60	-0.53	-0.61	-0.59	-0.62	-0.61	-0.58
Ivory 22	1.01	1.13	0.96	1.06	0.95	1.00	0.90	1.03	0.93	0.93
Ivory 32	0.60	0.71	0.76	0.60	0.64	0.59	0.46	0.67	0.64	0.72
Ivory 48	-0.17	-0.31	-0.27	-0.53	-0.69	-0.25	-0.68	-0.38	-0.41	-0.24
Joy 12	0.24	0.56	0.38	0.34	0.34	0.34	0.32	0.43	0.25	0.28
Joy 22	0.93	1.09	0.97	0.81	0.91	0.94	0.66	0.96	0.80	0.99
Joy 32	0.29	0.35	0.04	0.03	0.07	0.22	0.04	0.13	0.03	0.20
Joy 48	-1.20	-1.21	-1.12	-1.04	-1.41	-1.11	-1.49	-0.93	-1.25	-0.97
Ajax 22	-0.29	-0.34	-0.30	-0.50	-0.59	-0.48	-0.41	-0.13	-0.34	0.00
Ajax 32	-0.81	-0.53	-0.90	-0.95	-0.67	-0.88	-0.80	-0.98	-0.98	-1.11
Derm. 22	-2.06	-1.91	-1.38	-1.90	-2.30	-1.66	-2.14	-2.00	-1.77	-1.76
Derm. 32	-0.64	-0.30	-0.77	-0.86	-0.95	-0.35	-0.83	-0.47	-0.84	-0.65
Palm. 12	-0.25	-0.27	-0.39	-0.55	-0.56	-0.42	-0.44	-0.33	-0.46	-0.31
Palm. 22	0.74	0.95	0.96	0.70	0.73	0.82	0.62	0.87	0.77	0.92
Palm. 32	-0.36	-0.12	-0.05	-0.43	-0.33	-0.14	-0.57	-0.38	-0.43	-0.33
Palm. 48	-0.81	-0.72	-0.67	-1.03	-0.92	-0.86	-1.05	-0.74	-0.95	-0.81
Dove	-0.89	-0.51	-0.58	-0.79	-0.68	-0.81	-0.95	-0.67	-0.90	-0.72
Lux 22	-0.90	-0.65	-0.94	-1.17	-0.90	-1.22	-1.11	-1.00	-1.00	-0.85
Lux 32	-0.70	-0.58	-0.62	-0.93	-0.60	-0.67	-1.08	-0.69	-0.75	-0.62
Dawn 12	0.72	0.93	0.81	0.58	0.77	0.76	0.65	0.87	0.81	0.78
Dawn 22	1.04	1.12	1.10	0.99	0.95	1.09	0.85	0.99	0.90	1.16
Dawn 32	0.24	0.36	0.35	-0.04	0.23	0.34	0.04	0.24	0.25	0.31
Dawn 48	-0.45	-0.39	-0.20	-0.62	-0.41	-0.24	-0.72	-0.34	-0.42	-0.39
Sunl. 12	0.12	0.12	0.08	-0.30	0.01	0.11	-0.15	0.14	-0.19	0.19
Sunl. 22	0.99	1.06	1.04	0.90	0.92	0.88	0.71	0.97	0.88	1.02
Sunl. 32	0.92	0.97	0.96	0.77	0.85	0.84	0.66	0.84	0.66	0.89
Sunl. 48	-0.30	-0.13	-0.06	-0.22	-0.40	-0.11	-0.36	-0.21	-0.35	-0.18
Store	0.17	0.39	0.23	0.14	0.21	0.27	0.03	0.14	0.06	0.30
Generic	-1.24	-0.87	-1.24	-1.32	-1.37	-1.07	-1.02	-1.18	-1.11	-1.10

Table VI-1 (continued)

Bootstrap Logit Model Parameter Estimates

Bootstrap Sample

Parameter	11	12	13	14	15	16	17	18	mean	s.d.
Price	-0.50	-0.61	-0.60	-0.59	-0.61	-0.60	-0.53	-0.51	-0.57	0.04
Ivory 22	1.05	1.04	1.05	1.03	1.19	1.14	1.12	0.96	1.03	0.08
Ivory 32	0.71	0.73	0.61	0.56	0.82	0.84	0.87	0.49	0.67	0.11
Ivory 48	-0.28	-0.22	-0.33	-0.09	-0.11	-0.14	-0.18	-0.69	-0.33	0.19
Joy 12	0.35	0.50	0.37	0.34	0.58	0.54	0.49	0.56	0.40	0.11
Joy 22	0.88	1.09	1.05	0.92	1.04	1.29	1.00	0.90	0.96	0.13
Joy 32	0.13	0.38	0.31	0.35	0.34	0.42	0.11	0.21	0.20	0.13
Joy 48	-1.16	-1.03	-1.42	-0.84	-1.12	-1.13	-0.89	-1.10	-1.13	0.17
Ajax 22	-0.30	-0.43	-0.34	-0.57	-0.18	-0.01	-0.20	-0.35	-0.32	0.16
Ajax 32	-1.19	-0.82	-0.89	-0.91	-0.63	-0.42	-0.73	-1.03	-0.85	0.19
Derm. 22	-1.75	-1.65	-1.65	-1.71	-1.21	-1.83	-1.72	-1.95	-1.80	0.25
Derm. 32	-0.70	-0.27	-0.94	-0.48	-0.55	-0.34	-0.63	-0.40	-0.61	0.22
Palm. 12	-0.35	-0.02	-0.50	-0.46	-0.22	-0.12	-0.21	-0.38	-0.35	0.14
Palm. 22	0.92	0.97	0.80	0.94	1.05	0.96	1.11	0.78	0.87	0.13
Palm. 32	-0.48	-0.05	-0.19	-0.17	-0.06	0.16	0.05	-0.16	-0.22	0.19
Palm. 48	-0.81	-0.59	-0.98	-0.76	-0.74	-0.51	-1.05	-0.79	-0.82	0.15
Dove	-0.70	-0.74	-0.52	-0.80	-0.77	-0.36	-0.94	-0.73	-0.72	0.15
Lux 22	-0.85	-0.81	-0.93	-0.87	-0.86	-0.73	-0.90	-1.26	-0.94	0.16
Lux 32	-0.85	-0.68	-0.50	-0.48	-0.60	-0.41	-0.53	-0.91	-0.68	0.17
Dawn 12	0.77	0.86	0.90	0.61	0.94	1.06	0.84	0.60	0.79	0.12
Dawn 22	0.97	1.26	1.11	1.11	1.25	1.16	1.11	0.93	1.06	0.11
Dawn 32	0.25	0.41	0.52	0.33	0.32	0.41	0.30	-0.01	0.27	0.14
Dawn 48	-0.26	-0.41	-0.46	-0.56	-0.17	-0.15	-0.11	-0.49	-0.38	0.16
Sunl. 12	-0.12	0.09	0.00	0.15	0.45	0.24	0.04	-0.07	0.05	0.17
Sunl. 22	0.93	1.10	1.00	1.00	1.18	1.24	1.23	0.84	0.99	0.13
Sunl. 32	0.78	0.92	0.89	0.94	1.09	1.18	0.89	0.81	0.88	0.13
Sunl. 48	-0.29	-0.12	-0.12	-0.34	-0.01	-0.06	-0.15	-0.30	-0.21	0.11
Store	0.17	0.21	0.36	0.13	0.45	0.37	0.13	0.24	0.22	0.11
Generic	-0.96	-1.07	-1.23	-1.43	-0.73	-1.00	-1.10	-0.97	-1.11	0.17

example, 12-ounce Ivory liquid was contained in 4 multi-item partitions in the original structure (partitions 30, 31, 39 and 43). It was also contained in 4 multi-item partitions in the random structure (partitions 38, 41, 45, and 51). Thus, the random structure matrix differed from the SLM matrix only with regard to the multi-item partitions. Two of the 29 random partitions were not assigned any items and were eliminated. The final discrete random structure matrix consisted of 27 multi-item partitions and is presented in Table VI-2.

Next, the purchase records were grouped by randomly assigning each record to one of the partitions containing the purchased brand. For example, if the purchase was 12-ounce Ivory liquid, there was a 1/6 chance of assigning the purchase to each of the 6 random structure partitions containing 12-ounce Ivory liquid (partitions 1, 30, 38, 41, 45, and 51). The number of purchase records assigned to each random partition are presented in Table VI-3. Note that the all-item partition (30) is very large, while the single-item partitions are relatively small. This is due to the fact that the assignment method used here results in partitions sizes that are roughly proportional to the number of brands contained within the partition.

Finally, the grouped purchase records were then interpreted as if they were generated by evoked-set segments represented by the respective partitions. Separate logit models were estimated on each random segment using the same method described in Chapter V. Parameter estimates for the RSM are presented in Table VI-4.

Because the number of parameters needed for each segment-level logit model is equal to the number of brands contained the partition, the RSM has exactly the same total number of logit parameters as the SLM

Table VI-2

Random Structure Multi-Item Partitions

Partitions

Item																										
Ivory 12	0	0	0	0	0	0	0	1	0	1	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0
Ivory 22	1	0	1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0
Ivory 32	0	0	1	1	1	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	1	0
Ivory 48	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	1	0
Joy 12	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	1	0
Joy 22	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0
Joy 32	0	0	0	0	1	0	0	0	0	0	1	0	1	0	1	0	0	0	0	0	0	0	1	0	0	1
Joy 48	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
Ajax 22	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	1	0	0	0	1	0	0	0	0	0
Ajax 32	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	1	0	0
Derm. 22	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0
Derm. 32	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
Palm. 12	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0
Palm. 22	0	1	0	1	0	0	1	0	0	0	0	0	0	1	1	0	0	0	0	1	0	0	0	0	1	0
Palm. 32	0	1	0	0	1	0	0	0	1	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0
Palm. 48	1	0	1	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Dove	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0
Lux 22	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Lux 32	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
Dawn 12	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0
Dawn 22	1	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1	0	0	0	0	0	1	0	0
Dawn 32	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	1	1	0	1
Dawn 48	0	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Sunl. 12	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1
Sunl. 22	0	1	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0	1	0	0	1	0	0	0
Sunl. 32	0	1	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0
Sunl. 48	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0
Store	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
Generic	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0

Table VI-3

Random Structure Model Segment Sizes

Segment	N	Segment	N	Segment	N	Segment	N
1	64	15	47	29	32	43	287
2	161	16	25	30	2031	44	145
3	92	17	34	31	391	45	253
4	53	18	26	32	852	46	310
5	96	19	40	33	307	47	388
6	120	20	134	34	357	48	436
7	59	21	134	35	396	49	105
8	24	22	62	36	384	50	438
9	45	23	52	37	258	51	340
10	38	24	77	38	207	52	137
11	15	25	167	39	428	53	788
12	51	26	122	40	515	54	388
13	52	27	59	41	445	55	508
14	119	28	140	42	247	56	197

Table VI-4

Random Structure Model Parameter Estimates

Segment	30	31	32	33	34	35	36
Price	-0.544	-0.434	-0.678	-0.594	-0.578	-0.602	-0.742
Ivory 12	0.000	-	-	-	-	-	-
Ivory 22	0.704	0.000	-	0.000	-	-	-
Ivory 32	0.165	-	-	-0.506	0.000	0.000	0.000
Ivory 48	-0.452	-	-	-	-	-	-0.455
Joy 12	0.451	-	-	-	-	0.142	-
Joy 22	0.484	-	-	-	0.288	-	-
Joy 32	-0.271	-	-	-	-	-0.481	-
Joy 48	-0.678	-	-	-	-	-1.203	-
Ajax 22	-0.342	-	-	-	-	-	-
Ajax 32	-0.542	-	-	-	-	-	-
Derm. 22	-1.50	-	-	-2.246	-	-	-
Derm. 32	-0.322	-	-	-	-	-	-0.400
Palm. 12	-0.121	-	-	-	-	-	-
Palm. 22	0.517	-	0.000	-	-0.016	-	-
Palm. 32	-0.738	-	-1.168	-	-	-1.036	-
Palm. 48	-0.981	-1.555	-	-1.791	-	-0.932	-
Dove	-0.546	-	-	-	-	-	-
Lux 22	-0.509	-	-1.420	-	-1.035	-	-
Lux 32	-0.487	-	-	-	-	-	-
Dawn 12	0.921	-	0.322	-	-	-	-
Dawn 22	0.809	0.016	0.366	-	-	-	-
Dawn 32	-0.013	-	-	-	-	-	-
Dawn 48	-0.091	-	-0.580	-0.859	-	-0.301	-
Sunl. 12	0.145	-0.811	-	-	-	-	-
Sunl. 22	0.818	-	0.215	-	-	-	-
Sunl. 32	0.595	-	0.154	-	-	-	0.304
Sunl. 48	-0.352	-	-	-	-	-	-0.420
Store	0.558	-	0.266	-	-	-	-
Generic	-0.618	-	-	-	-	-	-
Log (L)	-6456	-479	-1726	-411	-456	-719	-575

Table VI-4 (continued)

Segment	37	38	39	40	41	42	43
Price	-0.271	-1.018	-0.552	-0.730	-0.788	-0.384	-0.537
Ivory 12	-	0.000	-	0.000	-	-	-
Ivory 22	-	-	-	0.702	0.000	-	-
Ivory 32	-	-	-	-	-	-	0.000
Ivory 48	-	-	-	-	-	0.000	-
Joy 12	-	-	-	-	-	-	0.173
Joy 22	-	-	0.000	-	-	-	-
Joy 32	-	-	-	-	-0.818	-	-0.138
Joy 48	-	-	-	-	-1.906	-	-
Ajax 22	-	-	-1.106	-	-	0.125	-
Ajax 32	-	-	-	-	-	-	-0.637
Derm. 22	-	-	-	-	-	-	-
Derm. 32	-	-	-	-	-1.273	-	-
Palm. 12	-	-	-	-0.247	-	-	-
Palm. 22	0.000	-	-	-	-	-	-
Palm. 32	-	-	-1.553	-	-	-	-
Palm. 48	-	-	-1.344	-	-	-	-
Dove	-	-	-	-0.505	-	-	-
Lux 22	-	-	-	-	-1.515	-	-
Lux 32	-	-	-	-0.481	-1.329	-	-
Dawn 12	0.232	-	-	-	-	-	-
Dawn 22	-	-	-	0.626	-	-	-
Dawn 32	-	-	-	-	-0.609	-	-
Dawn 48	-	-	-	-	-	-	-
Sunl. 12	-	-	-0.649	-	-	-	-
Sunl. 22	-	-	0.097	-	-	0.893	-
Sunl. 32	-	0.714	-	-	-	-	-
Sunl. 48	-	-	-	-0.230	-	-	-
Store	-	-	-	-	-	-	-
Generic	-	-	-	-	-1.410	-	-
Log (L)	-177	-120	-663	-932	-830	-241	-384

Table VI-4 (continued)

Segment	44	45	46	47	48	49	50
Price	-0.418	-0.763	-0.569	-0.474	-0.413	3.069	-0.691
Ivory 12	-	0.000	-	-	-	0.000	-
Ivory 22	-	-	-	-	-	-	-
Ivory 32	-	-	-	-	-	-	-
Ivory 48	-	-	0.000	-	-	-	-
Joy 12	-	-	-	-	0.000	-	-
Joy 22	-	-	-	0.000	-	-	-
Joy 32	-	-0.030	-	-	-	-	-
Joy 48	-	-	-	-	-	-	-
Ajax 22	-	-	-	-0.752	-	-	-
Ajax 32	-	-	-	-	-	-	-
Derm. 22	0.000	-	-	-	-2.367	-	-
Derm. 32	-	-	-	-	-	-	-
Palm. 12	-	-	-	-1.117	-0.866	-0.318	-
Palm. 22	1.968	0.385	-	-	-	-	0.000
Palm. 32	1.036	-0.645	-	-	-	-	-
Palm. 48	-	-	-	-	-	-	-
Dove	-	-	-	-	-	-	-
Lux 22	-	-	-	-	-	-	-
Lux 32	-	-	-	-	-	-	-
Dawn 12	-	-	-	-	-	-	0.277
Dawn 22	-	-	-	0.266	0.563	-	-
Dawn 32	-	-	-	-	-	-	-0.387
Dawn 48	-	-	-	-	-	-	-
Sunl. 12	-	-	-	-	-	-	-
Sunl. 22	-	-	0.948	-	-	-	0.161
Sunl. 32	-	-	-	-	0.187	-	-
Sunl. 48	-	-	-	-0.773	-	-	-
Store	-	-	0.763	-	-	-	-
Generic	-	-	-	-	-	-	-
Log (L)	-120	-323	-308	-568	-587	-68	-575

Table VI-4 (continued)

Segment	51	52	53	54	55	56
Price	-0.411	-0.851	-0.562	-0.694	-0.676	-0.474
Ivory 12	-	-	-	-	-	-
Ivory 22	0.000	-	0.000	-	-	-
Ivory 32	-	-	-0.325	-	0.000	-
Ivory 48	-	-	-	-	-0.726	-
Joy 12	-	-	-	-	0.001	-
Joy 22	-	-	-0.156	-	-	-
Joy 32	-	-	-0.632	-	-	0.000
Joy 48	-	-	-	-	-1.179	-
Ajax 22	-0.894	-	-	-	-	-
Ajax 32	-1.240	-	-	0.000	-	-
Derm. 22	-	-	-	-	-	-
Derm. 32	-	-	-	0.525	-	-
Palm. 12	-	-	-	-	-	-
Palm. 22	-	-	-	-	0.102	-
Palm. 32	-1.177	0.000	-	-	-	-
Palm. 48	-	-	-	-	-	-
Dove	-1.258	-	-	-	-0.697	-
Lux 22	-	-	-	-	-	-
Lux 32	-	-	-	0.442	-	-
Dawn 12	-	-	0.169	-	-	-
Dawn 22	-	-	-	1.891	-	-
Dawn 32	-	-	-0.648	1.423	-	0.161
Dawn 48	-	-	-	-	-	-
Sunl. 12	-0.474	-	-	-	-	-0.016
Sunl. 22	-	-	0.087	-	-	-
Sunl. 32	-	1.075	-	-	0.205	-
Sunl. 48	-	-	-1.125	-	-	-
Store	-	-	-	-	-	-
Generic	-	-	-1.450	-	-	-
Log (L)	-560	-73	-1626	-461	-915	-213

(156) and as such provides a reasonable baseline for determining the predictive effect adding additional parameters.

Comparison Store Models

Three additional models were estimated on the store data from the calibration period. These models offer a number of advantages over the three panel models (SLM, ULM, and RSM) described above. First, these models have the substantial advantage of being estimable on widely available simple store market share or sales data. The three panel models all require individual choice records for input. Second, the store-level data on which these models are calibrated can be seen as a population from which the panel data is drawn. Thus, the panel models are prone to sources of sampling error not present in the store models. The sole advantage of the panel models is in their ability to convey heterogeneous individual or segment-level information not available in the aggregate store data.

Simple Power Function Model

In the simple power function model (SPF), observed store-level item sales were modelled with the following equation (error terms omitted):

$$Q_{ist} = \exp(\beta_{0is})(PPO_{ist})^{\beta_{is}} \tag{6.2}$$

where PPO_{ist} = price per ounce of item i in store s at t.

After taking the natural logarithm of both sides, the SPF has a linear form:

$$\log(Q_{ist}) = \beta_{0is} + \beta_{is}\log(PPO_{ist}) \tag{6.3}$$

The SPF has the property that the slope β parameter of equation (6.3) is directly interpretable as a constant self-price sales elasticity for the item. Within each store, the SPF specifies separate log-linear models for each item, using own price as the sole independent variable. Unlike the panel models, the independent variable (price/ounce) is not normalized to account for interstore price variability because equations are store specific, and because logarithms are undefined for non-positive values. Unfortunately, the SPF can not be used to find optimal values of the independent variable. This becomes apparent when specifying the revenue equation:

$$R_{ist} = Q_{ist}PPO_{ist} \tag{6.4}$$

The first-order condition for the revenue equation,

$$\frac{\partial R_{ist}}{\partial(PPO_{ist})} = (\beta_{ist} + 1)PPO_{ist}{}^{\beta_{ist}} = 0 \tag{6.5}$$

only has a solution at (price per ounce) = 0. Thus, while the SPF may potentially provide a good description of the data, its usefulness in obtaining optimal values of price is limited.

To estimate the SPF, 379 separate OLS regression models were specified, one for each stocked item within each store. If an item's price never varied over the estimation period, β_{is} was set to 0, and the intercept was scaled such that $\exp\{\beta_{0is}\}$ reflected the mean sales level

for the item in the store. In total, the estimated SPF contained 713 parameters. The parameter estimates for the simple power function models are reported in Table VI-5. In the interest of parsimony, fit statistics and standard errors are omitted.

Cross-Elasticity Power Function Model

The cross-elasticity power function model (CPF) is similar to the SPF, except that sales quantities of each item are a function of the prices of all available items.

$$Q_{ist} = \exp(\beta_{0is})\prod_{j} (PPO_{jst})^{\beta_{ijs}} \tag{6.6}$$

Like the simple power function model, the cross elasticity power function model can be linearized via a log transformation:

$$\log(Q_{ist}) = \beta_{0is} + \sum_{j} \beta_{is}\log(PPO_{ist}) \tag{6.7}$$

the β_{ijs} here again are directly interpretable as constant price elasticities: if $i=j$, then the parameter represents self-price elasticities for item i, and if $i \neq j$, then the parameter represents cross elasticity of item i sales with respect to item j price. The CPF is identical to the models specified by Urban (1969) and implemented by Reibstein and Gatignon (1984).

The CPF has several advantages over the SPF. First, it incorporates competitive cross-effects. The cross-elasticity parameters can provide an index of the strength of substitutability or

Table VI-5

Simple Power Function Parameter Estimates

	Store							
	1		2		3		4	
Brand	β_1	β_0	β_1	β_0	β_1	β_0	β_1	β_0
Ivory 12	0.00	3.50	0.00	3.59	0.00	3.26	0.00	3.20
Ivory 22	-4.29	11.88	-4.99	13.10	-5.24	13.18	-3.25	9.64
Ivory 32	-2.81	8.48	-3.60	9.96	-3.23	8.97	-2.26	7.33
Ivory 48	-0.40	3.29	6.90	-9.23	-1.36	4.69	-1.37	4.99
Joy 12	0.00	3.52	0.00	3.77	0.00	3.30	0.00	3.26
Joy 22	-4.42	11.72	-3.92	10.91	-4.25	11.09	-3.20	9.31
Joy 32	-3.80	9.93	-2.41	7.61	-2.30	7.03	-2.62	7.62
Joy 48	-3.01	7.37	-3.86	9.17	-3.28	7.70	-5.25	11.16
Ajax 22	-2.42	6.41	-2.52	7.10	-1.96	5.39	-2.29	5.82
Ajax 32	-4.35	9.67	-4.67	10.25	-4.36	9.34	-3.75	8.32
Derm. 22	*	*	*	*	*	*	*	*
Derm. 32	0.00	2.45	0.00	2.70	0.00	2.29	0.00	2.45
Palm. 12	-1.64	6.19	-1.13	5.43	-1.62	5.95	-1.58	6.08
Palm. 22	-3.53	9.87	-3.55	9.97	-2.66	8.04	-2.36	7.68
Palm. 32	-4.40	10.14	-3.48	8.71	-3.16	7.62	-1.19	4.35
Palm. 48	-3.24	7.43	-4.01	9.03	-3.92	8.42	-2.20	5.46
Dove	-6.22	12.52	-6.30	12.54	-5.66	11.21	-4.28	9.11
Lux 22	0.00	3.34	0.00	3.44	0.00	2.22	0.00	2.56
Lux 32	-3.44	7.95	-6.94	13.43	-7.65	14.13	-4.99	9.98
Dawn 12	5.50	-6.69	15.76	-26.17	10.14	-16.03	6.33	-8.31
Dawn 22	-5.18	13.33	-1.98	7.98	-4.61	12.05	-5.10	13.12
Dawn 32	0.00	3.25	0.00	3.78	0.00	3.00	0.00	3.14
Dawn 48	-2.86	7.51	-1.71	5.98	-3.02	7.56	-3.33	8.12
Sunl. 12	-0.68	4.25	-4.76	12.37	-2.05	6.56	-3.08	8.85
Sunl. 22	-3.76	10.06	-3.66	9.85	-4.09	10.30	-3.07	8.83
Sunl. 32	-5.32	12.12	-5.60	12.66	-4.63	10.87	-4.23	10.14
Sunl. 48	-3.85	8.41	-4.07	8.76	-3.09	7.04	-2.19	5.66
Store	-5.69	10.41	-8.96	14.60	-5.27	9.70	-1.96	5.83
Generic	-2.02	3.51	-0.81	3.63	-1.79	3.50	-0.37	2.37

Table VI-5 (continued)

	Store							
	5		6		7		8	
Brand	β_1	β_0	β_1	β_0	β_1	β_0	β_1	β_0
Ivory 12	0.00	3.63	0.00	3.03	0.00	2.87	0.00	3.23
Ivory 22	-4.39	12.26	-6.73	15.75	-4.67	11.88	-6.75	15.99
Ivory 32	-2.27	7.72	-3.29	8.84	-2.94	8.36	-4.13	10.54
Ivory 48	0.33	2.39	-1.53	5.07	-0.55	3.24	-10.31	20.30
Joy 12	0.00	3.84	0.00	3.02	0.00	3.10	0.00	3.36
Joy 22	-3.08	9.71	-4.02	10.56	-2.93	8.52	-2.98	8.90
Joy 32	-2.89	8.71	-3.60	9.18	-2.24	6.77	-1.84	6.21
Joy 48	-4.29	10.08	-6.28	12.86	-1.13	3.63	-5.42	11.21
Ajax 22	-2.40	6.72	-2.40	5.86	-2.45	6.06	52.59	-94.68
Ajax 32	-4.24	9.58	-3.52	7.83	-2.93	6.98	-3.24	7.75
Derm. 22	*	*	*	*	*	*	*	*
Derm. 32	0.00	2.86	0.00	2.10	0.00.	1.51	0.00	2.35
Palm. 12	-0.28	3.97	-2.36	6.96	-2.22	6.68	-2.02	6.79
Palm. 22	-1.88	7.19	-3.36	8.98	-2.64	7.73	-3.14	8.99
Palm. 32	-3.05	8.23	-3.40	7.76	-3.60	8.30	-4.25	9.61
Palm. 48	-3.05	7.56	-4.60	9.38	-3.44	7.47	-4.93	9.94
Dove	-1.39	4.37	-7.89	14.54	-3.47	7.42	-3.45	7.79
Lux 22	0.00	3.08	0.00	2.65	0.00	2.94	0.00	2.45
Lux 32	-5.16	10.80	-7.66	13.85	-3.67	7.79	-4.86	10.01
Dawn 12	-13.8	31.41	0.60	2.27	-5.12	13.32	0.29	3.38
Dawn 22	-2.64	9.11	-4.77	12.02	-4.98	12.32	-2.50	8.46
Dawn 32	0.00	3.71	0.00	2.60	0.00	2.58	0.00	3.25
Dawn 48	-1.03	4.76	-3.48	7.58	-2.36	5.90	-2.66	7.12
Sunl. 12	-1.98	7.10	-1.95	6.31	-4.45	11.09	-2.93	8.57
Sunl. 22	-3.47	9.82	-4.41	10.65	-3.27	8.71	-4.00	10.30
Sunl. 32	-4.94	11.91	-6.31	13.23	-4.15	9.88	-3.85	9.35
Sunl. 48	-2.86	7.31	-4.09	8.60	-2.48	5.75	-1.91	4.60
Store	-12.78	19.72	-0.47	3.51	-1.89	5.40	-5.56	10.13
Generic	-2.99	4.74	-3.40	3.99	-2.07	2.64	-0.18	2.33

Table VI-5 (continued)

	Store							
	9		10		11		12	
Brand	β_1	β_0	β_1	β_0	β_1	β_0	β_1	β_0
Ivory 12	4.98	-6.50	0.71	1.79	1.46	0.72	-1.84	6.46
Ivory 22	0.31	2.70	-0.98	5.44	-0.05	4.12	-0.54	4.46
Ivory 32	-1.50	5.48	-1.47	5.71	-1.78	6.57	-0.75	4.41
Ivory 48	-2.19	5.60	-1.83	5.21	-2.51	6.69	0.53	1.52
Joy 12	4.16	-4.77	1.35	0.93	2.36	-0.68	-7.66	18.23
Joy 22	0.71	2.05	0.52	2.71	0.33	3.28	-2.87	8.46
Joy 32	0.31	1.86	-2.12	6.61	-2.74	8.13	-4.06	10.13
Joy 48	-4.10	8.73	-1.31	4.25	9.06	-14.01	1.01	0.31
Ajax 22	-6.00	12.48	-6.75	14.63	-4.09	10.12	0.90	0.17
Ajax 32	-2.11	4.73	-5.59	11.23	-3.95	8.63	3.81	-4.12
Derm. 22	*	*	*	*	*	*	-7.02	12.89
Derm. 32	-1.85	4.12	*	*	-1.40	4.61	-0.81	3.16
Palm. 12	*	*	*	*	*	*	0.19	2.41
Palm. 22	-3.38	8.76	-3.85	10.08	-3.71	10.17	-1.74	5.95
Palm. 32	-3.03	7.09	-4.32	9.97	-1.98	6.08	-1.38	4.41
Palm. 48	-1.26	3.48	-2.16	5.37	-1.92	5.09	-1.30	3.43
Dove	-0.10	1.76	-0.98	3.60	-1.48	4.65	11.29	-14.82
Lux 22	*	*	*	*	*	*	-7.53	13.68
Lux 32	*	*	*	*	*	*	0.00	1.13
Dawn 12	*	*	*	*	*	*	0.00	3.72
Dawn 22	-0.19	3.70	-1.02	5.52	-1.20	6.52	-1.79	7.06
Dawn 32	-1.13	4.38	-2.29	6.94	-1.28	5.69	-0.32	3.65
Dawn 48	-2.96	6.67	-3.77	8.75	-4.29	10.00	-2.04	5.96
Sunl. 12	*	*	*	*	*	*	-3.75	9.56
Sunl. 22	-4.65	11.10	-4.98	11.86	-3.37	9.29	-1.38	5.42
Sunl. 32	-5.54	11.87	-3.35	8.24	-4.27	10.13	-0.02	2.80
Sunl. 48	-4.38	8.97	-2.23	4.92	-5.05	10.04	-2.39	5.02
Store	-0.68	4.07	-0.83	4.68	-0.99	5.04	-3.50	6.67
Generic	-7.50	8.67	-3.14	5.50	-5.94	7.95	-0.87	3.08

Table VI-5 (continued)

	Store			
	13		14	
Brand	β_1	β_0	β_1	β_0
Ivory 12	-0.13	3.26	-26.02	53.71
Ivory 22	-0.20	3.67	-1.90	7.52
Ivory 32	-0.89	4.94	-1.15	6.41
Ivory 48	-1.54	5.33	-2.01	6.78
Joy 12	3.82	-4.38	0.00	2.65
Joy 22	0.17	2.85	-0.54	5.43
Joy 32	-1.96	6.63	-2.17	8.00
Joy 48	19.70	-32.57	2.06	-0.92
Ajax 22	1.22	-0.30	-1.64	5.89
Ajax 32	-0.85	3.19	-3.98	8.25
Derm. 22	-2.72	6.25	-4.75	10.64
Derm. 32	-0.85	2.58	-16.31	28.12
Palm. 12	-4.92	12.25	-1.52	5.09
Palm. 22	3.32	-2.44	-1.76	6.92
Palm. 32	2.01	-0.24	-4.68	11.26
Palm. 48	0.34	1.53	-4.26	9.97
Dove	-62.38	99.05	-13.61	24.84
Lux 22	0.00	2.00	-5.94	11.65
Lux 32	0.56	1.09	-23.34	39.97
Dawn 12	0.00	4.10	-49.44	99.72
Dawn 22	-0.73	4.90	-0.45	5.39
Dawn 32	-0.86	5.17	-4.33	11.94
Dawn 48	-1.87	6.21	-2.10	7.30
Sunl. 12	-3.24	8.98	-13.04	27.73
Sunl. 22	-0.66	4.28	-2.31	8.24
Sunl. 32	-84.58	126.63	-4.23	10.74
Sunl. 48	-3.00	6.18	-2.30	4.97
Store	-3.60	7.03	-2.25	6.79
Generic	-1.13	3.46	-0.57	3.88

complementarity between pairs of items. Secondly, the CPF implies optimal price levels. Thus, the model provides information useful for managerial pricing decisions. Finally, because the model has more independent variables than the simple power function model, it should provide a better fit (in terms of R^2) to the estimation data.

The CPF does have some disadvantages vis-a-vis the SPF. First, it is potentially prone to multicollinearity problems. Since the SPF has only one explanatory variable (self-price), multicollinearity will not be a problem. If, however, the items within a store are priced over time according to some pattern, the parameter estimates from the CPF will exhibit instability. While the multicollinearity problem will not affect model fit, it will potentially limit the model's ability to predict a holdout sample. Second, the CPF will be computationally more expensive to estimate.

With each store, a separate CPF model was calibrated for each item that was stocked for more than 20 weeks, resulting in 367 total OLS regressions. The dependent variables in each regression model were the largest possible set of non-redundant item prices. First, for some stores, particular items were priced at a constant level throughout the estimation periods. These item prices were eliminated as regressors. Second, some items (such as 12-ounce Ivory and 12-ounce Joy) had perfectly correlated pricing patterns over the estimation periods within each store. If this was the case, then the price of the smaller market share item was eliminated as a regressor. Thus, for some items, the CPF did not have self-price as a regressor variable. After elimination of constant and/or redundant price variables, each regression model contained between 15 and 22 dependent price terms. In total, the

estimated CPF contained 6877 parameters, which are reported in Table VI-6. In the interest of parsimony, significance levels and standard errors of the estimates are suppressed.

Logistic Market Share Model

The third model calibrated on the store scanner data was a multinomial logit model of the store-level item market shares. This model had the form:

$$MS_{ist} = \frac{\exp(\beta_{0is} + \beta_s(PPO_{ist}))}{\Sigma \exp(\beta_{0js} + \beta_s(PPO_{jst}))} \quad (6.8)$$

where MS_{ist} = market share of i in store s at t.

Henceforth, this model will be referred to as the logistic market share (LMS) model. In form, this model more closely resembles the models calibrated on the panel data than the SPF or the CPF. However, like the SPF and the CPF, the LMS model requires only observed store data in order to be estimated. Further, the parameters of the LMS model are store specific, allowing it to capture interstore response heterogeneity, a feature lacking in all three panel models. Essentially, the data is partitioned by store into 14 segments instead of by consideration set into 58 segments. Finally, the LMS model has the same set of independent variables and model form as the ULM. Since the LMS is not affected by sampling error and includes interstore variability, it should consistently provide better predictions than the unstructured logit.

The LMS model also has some important advantages vis-a-vis the SPF and CPF. First, unlike the SPF, the LMS model captures competitive

Table VI-6

Cross-elasticity Power Function Parameter Estimates (β_{ijs})

Store 1

Brand j

Brand i	Iv 22	Iv 32	Iv 48	Jo 22	Jo 32	Jo 48	Aj 22	Aj 32
Ivory 12	-0.72	-0.73	8.62	-0.24	1.20	1.08	0.35	1.09
Ivory 22	-5.14	2.13	22.38	0.42	-1.46	-1.03	-0.15	-0.57
Ivory 32	-0.91	-1.81	12.47	0.48	1.40	0.54	-0.49	0.16
Ivory 48	-3.42	-0.26	-1.39	-1.06	0.71	0.62	-0.89	1.09
Joy 12	0.46	-0.08	6.35	0.66	-0.16	-0.15	-0.67	0.73
Joy 22	-0.45	4.00	18.70	-3.88	1.63	-0.71	0.36	-1.14
Joy 32	0.30	0.52	-8.29	1.46	-2.27	-1.60	-0.03	0.69
Joy 48	3.29	0.65	24.13	0.10	-2.14	-1.10	-0.78	-0.75
Ajax 22	2.10	3.76	19.70	2.81	1.45	-0.48	-4.80	0.28
Ajax 32	-4.88	0.08	-15.08	-2.05	-2.36	1.44	1.07	-5.04
Derm. 32	2.14	-0.37	42.66	0.92	1.74	-0.27	-0.50	-1.00
Palm. 12	-1.15	3.07	10.30	0.15	1.81	0.09	0.46	-0.75
Palm. 22	0.39	-0.40	-1.23	-0.05	-1.23	1.17	-0.79	0.41
Palm. 32	-0.51	-2.82	0.52	-0.33	-0.96	-0.28	0.78	-0.12
Palm. 48	-7.90	1.12	-26.93	-2.78	-1.21	0.48	1.13	0.08
Dove	-3.59	1.28	-5.25	-0.62	-2.50	-3.23	-0.54	-0.11
Lux 32	-5.10	-2.41	13.88	0.53	1.25	-0.20	-0.05	-0.28
Dawn 12	0.41	0.30	10.38	0.02	-0.25	-0.17	0.22	-0.11
Dawn 22	-0.98	2.05	14.50	-0.09	-0.45	0.44	0.46	-0.19
Dawn 32	0.78	0.72	2.98	-0.43	-0.23	0.38	0.07	0.08
Dawn 48	4.71	-1.02	12.30	-0.51	3.52	0.54	0.11	1.17
Sunl. 12	2.52	-0.79	1.14	0.95	-1.68	-1.55	0.40	0.51
Sunl. 22	2.71	1.44	3.07	-2.10	2.51	-2.40	-0.40	-0.88
Sunl. 32	-0.97	-1.82	10.25	0.38	-1.10	6.46	-0.22	-0.44
Sunl. 48	6.75	-11.36	45.01	8.00	0.95	-7.18	-2.41	0.61
Store	0.46	2.87	28.85	-0.37	1.19	-0.87	1.19	-0.85
Generic	-3.39	3.61	29.82	-0.93	1.82	1.08	-0.34	0.40

Brand j

Brand i	Pa 12	Pa 22	Pa 32	Pa 48	Dove	Da 12	Da 22
Ivory 12	-1.62	-0.29	-0.39	0.13	0.39	2.81	0.75
Ivory 22	3.29	-0.51	-1.14	0.69	3.00	-30.47	-0.52
Ivory 32	-0.84	-0.05	1.73	0.90	-0.95	-31.00	1.29
Ivory 48	-3.88	0.23	-0.22	0.80	3.46	-36.20	1.59
Joy 12	-1.98	-0.41	0.06	0.43	-0.62	-3.58	0.16
Joy 22	4.25	-0.64	-0.30	0.80	-1.56	9.71	-1.20
Joy 32	-0.77	-0.31	-0.43	0.93	-0.82	17.64	0.51
Joy 48	-0.19	0.20	0.82	1.15	1.95	-31.45	-4.37
Ajax 22	-0.48	1.22	3.02	-0.20	-3.85	1.18	-4.86
Ajax 32	0.50	0.62	-1.13	2.67	1.98	-41.30	11.86
Derm. 32	0.63	0.00	1.13	0.68	-0.66	-90.41	-4.79
Palm. 12	-1.01	0.14	-0.16	0.31	-0.34	-1.88	3.34
Palm. 22	-1.84	-3.40	1.07	0.58	-1.81	0.10	-1.49
Palm. 32	1.45	-0.08	-3.63	0.13	-0.73	18.52	-0.07

Table VI-6 (continued)

	Brand j						
Brand i	Pa 12	Pa 22	Pa 32	Pa 48	Dove	Da 12	Da 22
Palm. 48	-4.02	2.21	-2.43	-0.70	6.47	-52.58	10.62
Dove	0.85	1.79	-0.24	1.79	-4.59	-14.92	1.67
Lux 32	1.91	-1.12	0.20	1.16	1.09	-35.01	4.46
Dawn 12	0.44	-0.12	-0.82	0.05	-0.38	-27.58	-0.41
Dawn 22	0.25	0.20	0.32	1.42	1.77	-37.34	-3.89
Dawn 32	2.56	-0.81	-0.54	0.15	0.06	54.10	-2.76
Dawn 48	-3.93	-0.30	0.31	0.65	-0.05	-18.67	0.48
Sunl. 12	-3.19	1.06	1.31	0.03	-3.21	-27.33	2.90
Sunl. 22	0.87	0.05	2.61	2.15	-1.39	-42.77	4.08
Sunl. 32	6.27	0.53	-0.64	-0.90	1.90	-19.92	1.63
Sunl. 48	0.82	-0.35	4.49	4.18	0.91	-263.66	-8.35
Store	-0.87	0.32	0.50	0.75	-0.49	-24.18	2.51
Generic	-4.39	-0.80	-1.04	1.22	-3.79	-68.61	0.33

	Brand j						
Brand i	Da 48	Su 12	Su 22	Su 32	Store	Gnrc.	β_{0is}
Ivory 12	0.35	-0.09	0.10	-0.38	0.91	-1.01	-18.94
Ivory 22	2.29	-0.05	-0.18	-0.05	-1.37	4.27	22.27
Ivory 32	-0.25	0.14	0.52	0.27	-7.01	1.36	46.47
Ivory 48	2.52	-0.48	-0.11	-1.21	-24.15	7.51	103.12
Joy 12	0.22	-0.20	0.52	0.04	5.44	-2.31	-3.83
Joy 22	1.94	-0.26	0.20	-0.34	7.66	-2.82	-61.08
Joy 32	-0.82	0.07	0.30	-0.10	-8.94	-1.61	-0.83
Joy 48	3.38	-1.67	0.45	-2.38	27.99	-0.09	-9.71
Ajax 22	-2.68	1.79	0.87	0.64	14.85	-12.14	-47.27
Ajax 32	0.59	-0.49	0.56	-0.20	-61.39	10.06	169.08
Derm. 32	4.31	1.13	-0.52	0.46	17.04	1.16	74.08
Palm. 12	0.49	-0.76	0.14	-0.21	-11.19	2.98	-8.99
Palm. 22	0.40	-0.18	0.56	0.07	5.44	-0.28	10.95
Palm. 32	-2.84	-1.50	0.49	0.78	-6.26	-1.76	-8.09
Palm. 48	3.81	-3.41	0.52	-1.67	-79.66	20.11	235.24
Dove	0.81	1.23	-0.14	0.23	-33.67	6.37	88.05
Lux 32	0.29	-0.24	-0.33	0.10	-16.86	5.33	62.09
Dawn 12	1.78	0.70	-0.15	0.19	-1.78	3.22	36.93
Dawn 22	1.94	-1.17	0.12	-0.22	-11.23	6.70	58.77
Dawn 32	-0.06	-0.11	0.04	-0.64	5.96	-3.72	-111.81
Dawn 48	-1.74	1.08	-0.41	0.65	7.65	-2.67	0.76
Sunl. 12	-2.43	-0.05	2.08	1.42	-7.15	-3.00	64.40
Sunl. 22	2.39	0.22	-3.77	0.36	-29.75	7.61	98.36
Sunl. 32	-6.00	0.48	0.43	-4.05	-17.08	-0.49	40.57
Sunl. 48	5.76	1.61	0.79	1.34	64.44	0.00	348.56
Store	2.55	-0.20	-0.01	0.64	-9.82	1.38	-4.66
Generic	3.01	0.04	0.90	0.36	-20.61	6.99	109.47

Table VI-6 (continued)

Store 2

Brand j

Brand i	Iv 22	Iv 32	Jo 22	Jo 32	Jo 48	Aj 22	Aj 32
Ivory 12	-3.40	-0.99	0.19	-0.22	2.44	0.24	0.26
Ivory 22	-3.78	2.99	-1.06	0.19	0.83	1.42	-0.66
Ivory 32	-0.23	-3.58	1.77	2.11	0.72	0.14	0.67
Ivory 48	3.13	1.32	-1.39	0.79	-0.05	0.44	-0.00
Joy 12	0.47	-0.77	1.11	0.60	1.57	0.24	-0.22
Joy 22	0.09	2.60	-5.16	-1.51	1.76	2.24	-0.27
Joy 32	3.96	0.23	0.22	-0.26	-0.82	1.91	-0.16
Joy 48	4.60	1.88	0.86	-0.19	-3.67	2.22	0.64
Ajax 22	2.20	0.77	1.80	3.52	0.09	-0.68	3.15
Ajax 32	0.92	-4.45	1.23	6.27	1.85	-2.88	-3.52
Derm. 32	4.16	1.85	0.94	3.13	-0.17	2.01	2.20
Palm. 12	-1.38	-0.75	0.04	0.42	2.50	-0.27	-0.45
Palm. 22	2.25	0.10	1.94	-4.40	0.82	0.56	-0.29
Palm. 32	3.82	-0.91	-0.16	1.09	-3.67	2.39	0.13
Palm. 48	-1.97	-4.21	-1.05	-2.71	1.88	0.16	0.12
Dove	4.74	0.59	-0.84	5.55	1.47	2.18	1.88
Lux 32	1.05	-0.04	-1.07	1.80	-3.15	0.55	-0.98
Dawn 12	4.00	0.57	0.82	3.15	0.05	1.46	0.37
Dawn 22	1.40	-0.20	0.01	-0.32	0.88	0.84	0.29
Dawn 32	2.36	-1.57	1.24	2.00	0.29	0.52	0.67
Dawn 48	0.73	-2.02	0.47	1.54	-0.10	0.67	0.09
Sunl. 12	3.03	0.27	0.89	0.93	1.47	-0.21	0.03
Sunl. 22	5.77	-4.13	-0.05	4.93	-1.73	-4.02	2.36
Sunl. 32	-0.37	-5.20	2.45	1.43	-0.04	1.64	0.89
Sunl. 48	-1.23	-2.17	2.12	0.12	-3.58	0.68	1.14
Store	3.24	-1.73	1.09	1.56	0.97	0.62	0.62
Generic	2.83	-2.22	-0.27	1.25	-0.52	0.96	0.11

Brand j

Brand i	Pa 22	Pa 32	Pa 48	Dove	Lu 32	Da 22	Su 12
Ivory 12	0.07	-0.19	-0.32	1.57	1.69	-0.01	0.11
Ivory 22	0.48	-1.41	-0.63	-1.01	-0.32	2.91	1.02
Ivory 32	-0.26	-0.22	-0.51	0.96	0.93	1.56	1.58
Ivory 48	0.07	-0.78	0.25	-1.33	-1.13	1.02	1.00
Joy 12	-0.24	0.02	-0.05	1.19	1.23	2.01	0.91
Joy 22	1.39	-0.08	0.17	-0.49	-0.02	-0.35	0.06
Joy 32	-0.41	-0.79	0.35	-1.60	-0.41	0.36	1.68
Joy 48	0.32	-0.80	0.19	0.85	-1.07	-3.91	0.80
Ajax 22	0.62	-2.39	0.68	1.38	1.85	0.16	3.45
Ajax 32	-0.92	1.57	-0.66	1.94	-1.85	8.15	2.64
Derm. 32	-0.11	-1.59	0.60	-0.47	-0.24	2.59	2.89
Palm. 12	0.48	-0.01	-0.27	1.94	-0.30	2.50	1.35
Palm. 22	-2.61	1.26	0.87	1.56	-1.00	1.89	0.25
Palm. 32	-0.41	-4.16	0.08	-1.13	-1.61	0.95	0.40
Palm. 48	0.58	-0.06	-4.89	1.62	-0.99	0.94	-0.25
Dove	-0.02	1.27	-0.37	-7.87	1.95	-0.03	-0.82
Lux 32	-0.42	-1.78	0.74	2.92	-8.15	-1.88	1.79

Table VI-6 (continued)

Brand i		Pa 22	Pa 32	Pa 48	Dove	Lu 32	Da 22	Su 12
Dawn	12	-0.18	-1.72	0.21	-1.64	-0.05	-0.84	4.13
Dawn	22	0.50	-0.72	-0.14	-0.03	-0.24	-1.99	0.97
Dawn	32	-0.15	-0.48	-0.17	0.29	0.04	1.10	1.76
Dawn	48	-0.31	0.29	0.27	-0.91	-0.31	0.40	1.02
Sunl.	12	0.22	1.06	-1.48	-2.08	0.59	3.44	-3.20
Sunl.	22	-1.20	3.89	2.36	-2.31	0.99	0.34	0.87
Sunl.	32	-0.45	-0.07	-0.38	1.96	0.01	0.51	0.41
Sunl.	48	-0.16	1.41	1.35	4.04	2.63	4.64	-0.18
Store		0.20	0.16	-0.25	-0.02	0.72	2.22	2.50
Generic		0.19	0.33	0.51	0.62	-0.53	1.86	1.79

Brand i		Su 22	Su 32	Su 48	Store	Gnrc.	β_{0is}
Ivory	12	0.31	-0.54	-0.22	0.79	-0.13	1.56
Ivory	22	1.10	1.04	-0.54	-16.58	0.82	18.99
Ivory	32	0.14	0.06	-0.58	3.45	-1.98	-8.89
Ivory	48	0.45	0.53	-0.51	-6.06	1.37	1.92
Joy	12	-0.07	-0.28	0.23	-3.90	-2.44	-3.90
Joy	22	1.75	-0.20	-0.17	-3.77	-1.48	6.22
Joy	32	0.46	0.31	-0.26	-2.15	-1.42	-2.55
Joy	48	0.66	-1.36	-1.14	1.24	-2.80	-0.01
Ajax	22	0.41	-0.28	-0.98	0.73	-2.74	-24.77
Ajax	32	-1.68	-1.23	-1.77	-4.50	-0.43	-3.59
Derm.	32	0.94	0.33	-0.87	-10.80	-1.41	-16.40
Palm.	12	0.06	0.01	0.10	-1.99	-0.43	-4.55
Palm.	22	1.00	0.13	0.73	-0.83	-5.42	-1.04
Palm.	32	0.51	0.10	0.15	-0.12	-0.35	6.27
Palm.	48	0.05	0.36	0.78	8.36	-3.10	10.75
Dove		1.00	-1.26	-1.26	8.27	-1.13	-22.74
Lux	32	-0.07	-1.09	-0.29	0.03	-0.63	18.90
Dawn	12	0.46	0.27	-0.96	-2.08	-0.27	-11.84
Dawn	22	0.11	0.24	-0.34	-2.83	1.14	4.74
Dawn	32	0.12	0.27	-0.01	-2.81	-2.52	-6.36
Dawn	48	0.10	0.60	-0.41	-4.42	1.49	3.34
Sunl.	12	1.90	0.99	-1.27	-0.80	-1.74	-6.42
Sunl.	22	-3.91	-0.27	-0.34	2.08	-1.55	-4.76
Sunl.	32	-0.29	-4.18	1.15	-5.07	-5.16	13.07
Sunl.	48	0.44	0.52	-3.76	5.45	-0.08	-17.77
Store		0.29	0.10	-0.50	-5.00	-4.41	-9.16
Generic		-0.01	-0.23	-0.19	-6.42	-2.80	0.59

Table VI-6 (continued)

Store 3

Brand j

Brand i	Iv 22	Jo 22	Jo 32	Jo 48	Aj 22	Aj 32
Ivory 12	-1.11	0.01	-0.97	-3.10	0.69	2.40
Ivory 22	-9.87	1.92	-2.59	-11.57	2.14	-3.66
Ivory 32	0.38	1.99	5.48	-2.71	0.98	-0.42
Ivory 48	-0.27	0.98	3.53	-1.04	1.08	2.29
Joy 12	0.36	0.89	2.65	0.56	2.51	0.56
Joy 22	-0.78	-2.64	-0.37	-5.47	1.57	0.08
Joy 32	-0.68	0.74	1.82	-0.87	1.47	-0.47
Joy 48	-2.88	0.14	-0.86	6.90	0.83	-0.51
Ajax 22	9.39	3.07	7.36	-5.24	-2.07	-2.25
Ajax 32	-2.07	1.31	-2.38	-7.59	-0.14	-4.67
Derm. 32	0.71	1.18	2.91	-1.13	-0.17	1.03
Palm. 12	0.79	-0.58	-0.29	-3.77	1.42	-0.31
Palm. 22	-1.79	2.76	-4.31	-3.61	0.49	-1.13
Palm. 32	4.47	-0.81	2.69	-17.06	13.93	-0.75
Palm. 48	5.41	-2.56	0.99	5.04	-4.18	-0.34
Dove	-1.46	0.36	1.24	4.67	1.05	5.79
Lux 32	-4.02	1.46	4.39	0.65	-1.39	-0.88
Dawn 12	0.75	-0.02	0.21	1.60	0.78	0.66
Dawn 22	0.70	0.14	-0.01	-0.33	1.46	1.07
Dawn 32	3.41	0.51	-0.57	-0.39	1.20	0.62
Dawn 48	1.58	-0.98	-1.83	-9.29	4.33	-3.05
Sunl. 12	-1.06	-2.12	-0.96	6.61	2.58	-0.80
Sunl. 22	6.86	-0.53	7.22	3.71	-1.20	0.21
Sunl. 32	7.77	-1.64	-0.86	-0.04	0.42	-2.47
Sunl. 48	-4.76	-1.60	-5.15	3.74	-1.07	1.84
Store	1.15	1.22	2.13	0.99	-1.12	-0.13
Generic	3.51	-0.92	5.57	6.82	-2.00	-2.06

Brand j

Brand i	Pa 22	Pa 32	Pa 48	Dove	Lu 32	Da 22
Ivory 12	0.51	1.99	1.41	2.90	-0.73	1.46
Ivory 22	-1.61	-0.82	10.36	5.44	-0.19	-0.47
Ivory 32	-1.66	-0.48	3.66	-2.80	1.00	0.30
Ivory 48	-0.41	-2.17	-0.66	-1.84	-1.48	-5.78
Joy 12	-0.41	-0.59	1.64	-1.18	2.66	2.94
Joy 22	-0.05	-0.97	2.62	0.74	-0.87	-1.77
Joy 32	-2.01	-0.94	5.05	-3.88	2.64	-0.96
Joy 48	-0.82	-5.31	-8.73	-4.26	-2.17	-9.84
Ajax 22	-1.48	0.14	6.06	-5.86	-1.97	-5.21
Ajax 32	0.58	1.96	3.05	2.52	-1.55	4.31
Derm. 32	-1.28	-1.83	-0.03	-1.16	1.52	-3.39
Palm. 12	0.45	-1.03	1.41	-0.01	-0.43	0.74
Palm. 22	-3.53	0.47	8.25	1.37	-0.62	0.32
Palm. 32	0.33	-4.42	15.78	-1.42	-1.58	-0.66
Palm. 48	0.43	4.22	-5.14	1.20	-0.77	0.92
Dove	0.47	-0.89	-3.32	-5.70	1.48	0.64

Table VI-6 (continued)

Brand i		Brand j					
		Pa 22	Pa 32	Pa 48	Dove	Lu 32	Da 22
Lux	32	-0.42	0.25	0.60	-0.43	-5.25	5.16
Dawn	12	0.61	-1.12	-0.07	-0.05	-0.54	-1.67
Dawn	22	0.31	0.15	-1.61	0.11	-0.53	-3.15
Dawn	32	0.19	0.15	-1.54	1.94	-1.97	-2.51
Dawn	48	0.19	1.14	9.84	0.14	-2.16	-4.66
Sunl.	12	0.52	-0.94	0.05	-4.84	2.09	1.66
Sunl.	22	-0.65	2.15	0.17	-1.80	2.32	2.50
Sunl.	32	1.93	2.06	2.30	-2.38	-2.81	1.05
Sunl.	48	1.38	-2.72	-6.74	2.05	8.01	2.21
Store		-0.20	0.84	-2.62	-0.89	0.03	0.16
Generic		-0.79	2.66	0.25	-4.11	1.09	2.44

Brand i		Brand j					
		Su 12	Su 22	Su 32	Su 48	Store	β_{0is}
Ivory	12	-2.80	0.81	-0.53	0.18	-2.20	1.23
Ivory	22	1.51	-0.87	0.76	-0.22	4.09	16.57
Ivory	32	1.03	-0.71	0.26	0.01	-3.25	-4.59
Ivory	48	-1.94	-1.27	-1.97	1.28	3.18	15.09
Joy	12	0.48	0.29	-0.59	0.41	-5.43	-13.14
Joy	22	-0.75	0.49	-1.32	0.46	0.46	19.03
Joy	32	-1.06	-0.76	0.23	0.99	1.21	-0.71
Joy	48	-1.90	-1.66	-2.70	2.88	0.64	55.00
Ajax	22	4.48	-1.10	0.44	-0.90	-4.39	-3.78
Ajax	32	-1.00	-0.78	-0.12	-0.52	-0.84	15.11
Derm.	32	0.48	-1.14	-0.75	-0.15	10.43	-5.00
Palm.	12	-0.15	0.38	0.08	0.22	-6.97	13.12
Palm.	22	-0.93	-0.45	-0.18	0.46	-4.09	13.01
Palm.	32	-0.97	1.29	1.04	-0.51	3.60	-23.83
Palm.	48	-0.99	2.75	0.08	-1.91	-10.03	4.95
Dove		-4.30	-0.60	-1.80	0.85	-4.73	10.52
Lux	32	2.75	-0.05	-1.43	0.92	-6.71	5.00
Dawn	12	0.91	0.65	-0.76	0.30	-4.43	5.00
Dawn	22	-1.96	-0.40	-0.08	0.52	1.23	8.77
Dawn	32	0.81	-0.13	-0.30	0.25	1.73	-2.40
Dawn	48	-2.46	-0.03	0.01	-0.45	0.69	14.56
Sunl.	12	-4.33	1.42	0.19	1.48	-0.59	1.25
Sunl.	22	3.06	-3.66	1.41	-1.95	-0.90	-31.38
Sunl.	32	1.79	2.79	-2.17	-2.81	-10.53	5.26
Sunl.	48	0.65	1.09	-2.66	-0.41	-23.86	41.05
Store		0.32	-0.11	0.13	-0.50	-6.09	7.89
Generic		1.38	0.62	0.01	-1.05	1.65	-24.04

Table VI-6 (continued)

Store 4

Brand j

Brand i	Iv 22	Iv 32	Jo 22	Jo 32	Jo 48	Aj 22	Aj 32
Ivory 12	4.66	-0.66	-0.32	3.40	-2.89	0.94	1.09
Ivory 22	-1.09	1.29	-0.16	0.52	-0.31	0.43	-0.16
Ivory 32	0.86	-2.38	-0.18	-0.60	0.26	-0.01	0.20
Ivory 48	2.19	-0.55	-1.02	-2.75	-1.77	-0.51	1.25
Joy 12	2.97	0.74	-0.30	-0.85	-1.76	-0.31	0.30
Joy 22	3.74	2.21	-1.96	1.23	-3.50	0.35	-1.13
Joy 32	4.00	0.07	0.83	-2.14	-1.61	0.67	0.74
Joy 48	11.87	4.16	-0.69	-1.35	-7.34	3.90	-0.49
Ajax 22	-0.89	-1.29	3.13	0.53	-3.32	-3.02	0.98
Ajax 32	-1.56	-1.39	-1.80	-1.92	1.88	1.49	-3.50
Derm. 32	3.92	-1.52	1.03	2.82	-1.80	0.82	0.27
Palm. 12	3.15	-0.48	0.05	1.39	-0.52	0.37	0.06
Palm. 22	4.54	-1.54	0.39	-2.80	-0.58	0.56	-0.32
Palm. 32	4.80	17.31	-0.01	1.40	0.30	3.79	-2.27
Palm. 48	6.40	-4.82	-0.03	9.38	-2.88	2.13	1.72
Dove	0.46	-1.16	-0.75	-0.71	-0.35	0.13	1.46
Lux 32	5.69	-0.02	0.79	1.63	-4.45	-0.31	-1.05
Dawn 12	1.80	-0.51	-0.02	0.46	-0.72	0.27	0.65
Dawn 22	3.95	-1.92	0.23	0.24	-0.71	0.53	0.92
Dawn 32	1.91	-0.87	-0.65	-0.83	-0.84	0.28	1.05
Dawn 48	4.83	7.06	0.18	-2.14	4.28	-0.04	-0.98
Sunl. 12	2.72	0.40	-0.58	-0.50	-1.58	2.30	0.15
Sunl. 22	3.31	-2.24	-0.76	-0.84	-1.62	2.14	1.39
Sunl. 32	7.55	-2.27	1.87	0.96	0.52	-1.84	1.87
Sunl. 48	0.30	0.91	3.03	0.67	-7.70	3.70	1.79
Store	2.48	0.46	-0.04	0.31	-1.02	0.48	-0.10
Generic	6.92	1.13	0.67	5.14	-2.56	2.75	1.37

Brand j

Brand i	Pa 22	Pa 48	Dove	Lu 32	Da 12	Da 22
Ivory 12	-0.86	0.49	-3.10	-1.83	13.45	-0.22
Ivory 22	-0.28	-0.44	-1.92	-2.19	-14.72	-3.03
Ivory 32	0.12	-1.04	-0.08	-1.46	-1.34	-1.33
Ivory 48	-0.27	-0.15	0.91	-4.37	-2.81	-5.15
Joy 12	-0.50	0.29	-1.03	-0.90	10.80	0.10
Joy 22	0.26	-0.47	-2.52	-1.92	-10.85	-3.65
Joy 32	0.00	0.25	-1.38	-1.13	-26.71	-3.96
Joy 48	0.08	-0.03	-2.43	-3.61	-38.84	-3.39
Ajax 22	1.61	-1.30	3.35	0.49	-25.99	4.18
Ajax 32	0.89	-1.38	3.11	5.04	19.05	4.62
Derm. 32	-0.87	0.79	-1.34	-1.08	-17.33	-3.79
Palm. 12	-0.16	-0.18	-0.39	-1.30	-1.83	1.05
Palm. 22	-2.14	-0.72	-1.36	-3.60	-26.11	-0.32
Palm. 32	1.03	0.69	7.84	-2.23	-62.74	2.12
Palm. 48	-1.85	0.24	-2.80	-0.05	-40.20	-2.09
Dove	-0.03	0.07	-5.30	-0.21	3.50	-4.87
Lux 32	-0.73	0.33	-2.63	-6.47	-14.90	3.14

Table VI-6 (continued)

Brand i	Pa 22	Pa 48	Dove	Lu 32	Da 12	Da 22
			Brand j			
Dawn 12	-0.65	0.29	-0.50	-0.80	9.34	-1.19
Dawn 22	-0.25	-1.40	-0.21	-1.75	1.73	-3.96
Dawn 32	-0.29	0.81	-1.38	-1.47	15.85	-3.23
Dawn 48	0.20	-1.38	0.77	-3.90	-7.89	-4.63
Sunl. 12	-0.23	1.19	-2.25	-0.34	-4.37	0.86
Sunl. 22	-0.10	1.06	0.25	-1.76	10.48	-2.67
Sunl. 32	1.31	-2.00	-1.80	-4.09	-53.18	-3.20
Sunl. 48	-0.71	3.94	-0.61	1.27	-16.20	-3.05
Store	-0.04	0.40	-1.58	-0.69	4.11	0.39
Generic	-1.51	-0.37	-0.31	-3.42	17.48	-1.26

Brand i	Su 12	Su 22	Su 32	Su 48	Store	β_{0is}
			Brand j			
Ivory 12	-1.25	0.23	-0.07	0.01	2.98	-27.06
Ivory 22	0.12	0.82	0.10	0.06	5.50	36.02
Ivory 32	-0.96	0.50	0.25	0.37	1.62	13.57
Ivory 48	-0.68	0.75	-0.47	-0.50	3.88	25.64
Joy 12	-0.39	0.74	-0.33	0.01	-0.85	-15.02
Joy 22	0.32	0.91	-0.41	0.01	6.71	27.16
Joy 32	0.88	0.73	0.59	0.16	2.53	53.77
Joy 48	-0.11	1.97	0.48	0.72	5.47	62.95
Ajax 22	-1.18	-0.55	0.70	0.93	-9.55	57.62
Ajax 32	0.80	-0.35	0.02	-0.90	3.94	-47.86
Derm. 32	-0.09	0.58	0.23	0.21	6.89	27.02
Palm. 12	0.37	0.82	0.61	0.25	3.40	-6.93
Palm. 22	-0.16	0.40	0.15	0.29	1.07	64.66
Palm. 32	0.69	2.06	0.07	-1.84	-15.02	80.26
Palm. 48	0.40	0.81	0.64	0.58	3.92	60.78
Dove	-1.13	0.40	1.22	0.23	4.61	7.97
Lux 32	0.05	-0.58	0.50	0.68	-1.05	36.59
Dawn 12	-0.36	0.66	-0.37	-0.23	-0.31	-11.98
Dawn 22	-0.06	0.15	0.68	-0.21	3.40	2.56
Dawn 32	-1.40	0.59	-0.34	0.27	0.34	-17.11
Dawn 48	-0.23	0.51	-1.19	-0.97	6.92	3.94
Sunl. 12	-2.87	1.56	0.93	-0.26	-0.78	9.67
Sunl. 22	-2.60	-2.66	0.45	-0.13	5.80	-12.19
Sunl. 32	3.27	1.02	-3.53	-0.12	6.48	97.66
Sunl. 48	0.22	0.83	0.74	-1.60	4.86	21.25
Store	-0.67	0.29	-0.00	0.00	-2.62	-2.99
Generic	1.72	1.28	-1.25	-1.38	0.21	-49.88

Table VI-6 (continued)

Store 5

Brand j

Brand i	Iv 22	Iv 32	Jo 22	Jo 32	Jo 48	Aj 22
Ivory 12	0.42	-1.82	-0.74	-0.70	2.41	0.27
Ivory 22	-4.49	1.63	0.40	0.38	1.95	-0.49
Ivory 32	-0.51	-3.40	-0.77	1.43	0.43	0.18
Ivory 48	-1.63	1.52	0.36	-2.07	1.39	-0.72
Joy 12	-0.49	-1.35	0.29	-0.31	0.40	0.26
Joy 22	0.38	1.09	-2.65	-0.00	-0.16	0.53
Joy 32	-3.32	-1.38	0.81	-2.22	0.19	-0.10
Joy 48	1.94	-0.93	-0.83	-1.35	-3.04	-0.97
Ajax 22	0.93	-3.30	-0.06	0.08	-3.18	-2.36
Ajax 32	-0.72	-4.66	-1.78	-1.46	2.22	0.55
Derm. 32	1.58	2.69	0.71	-0.70	1.04	-0.88
Palm. 12	0.91	-3.08	0.35	1.34	-0.51	0.22
Palm. 22	-2.42	1.63	2.06	-6.39	1.24	-0.07
Palm. 32	0.98	-2.89	-0.86	-0.98	2.22	3.27
Palm. 48	0.69	0.97	-0.08	1.20	1.86	1.15
Dove	2.45	0.00	-1.23	-0.86	-0.12	2.33
Lux 22	0.00	0.00	9.15	0.00	0.00	-2.25
Lux 32	-2.00	3.49	3.11	-2.65	1.28	-0.73
Dawn 12	-2.38	-1.22	0.43	0.13	1.67	-0.15
Dawn 22	-0.34	0.12	-0.43	0.18	0.56	-0.58
Dawn 32	-1.42	-2.51	0.82	-0.78	1.35	0.09
Dawn 48	2.63	-1.96	0.69	0.30	0.35	0.85
Sunl. 12	-0.88	-2.02	0.33	-1.82	3.74	-0.43
Sunl. 22	4.31	-5.04	-0.09	4.77	-2.02	-0.62
Sunl. 32	-3.62	-5.77	-0.89	-1.59	5.83	0.31
Sunl. 48	-1.63	3.24	1.44	0.84	-4.62	-0.45
Store Generic	-0.31 -0.05	-1.69 -1.30	0.35 0.06	1.09 -0.41	1.49 1.95	0.22 0.20

Brand j

Brand i	Aj 32	Pa 22	Pa 32	Pa 48	Lu 32	Da 22
Ivory 12	0.44	0.06	0.73	-0.39	0.16	2.35
Ivory 22	-0.04	-0.20	-1.07	-0.75	0.66	1.42
Ivory 32	1.51	0.01	0.71	-0.01	0.93	1.21
Ivory 48	0.04	0.54	1.32	0.64	0.04	-1.54
Joy 12	0.14	-0.20	0.37	0.39	0.09	0.27
Joy 22	0.25	-0.05	-0.85	-0.09	-0.17	0.20
Joy 32	-0.26	-0.42	0.94	0.31	1.62	1.03
Joy 48	-0.25	0.19	1.37	-1.03	0.63	0.29
Ajax 22	2.14	0.33	0.02	1.97	1.43	2.42
Ajax 32	-3.85	1.45	1.63	-0.60	-1.10	2.39
Derm. 32	0.10	-0.18	1.93	-0.34	-0.24	-0.02
Palm. 12	0.32	-0.74	0.63	1.35	0.33	4.66
Palm. 22	-0.51	-1.21	0.77	-1.00	-0.99	-0.39
Palm. 32	-0.44	0.11	-2.88	0.67	-0.01	2.45
Palm. 48	0.11	-0.67	-1.19	-2.31	0.93	-1.91

Table VI-6 (continued)

Brand j

Brand i	Aj 32	Pa 22	Pa 32	Pa 48	Lu 32	Da 22
Dove	0.84	-0.55	-1.17	0.00	-1.43	-1.51
Lux 22	-6.65	8.68	0.06	-2.17	12.93	-3.40
Lux 32	-2.23	0.05	1.58	-0.73	-4.96	3.18
Dawn 12	0.59	-0.29	-0.00	-0.07	0.87	-0.13
Dawn 22	0.62	-0.07	0.47	-0.19	0.96	-1.28
Dawn 32	0.07	-0.54	1.12	0.73	1.48	-0.46
Dawn 48	1.76	-0.06	-1.47	-0.37	-2.88	-1.44
Sunl. 12	-0.42	0.14	1.91	-0.51	0.78	4.80
Sunl. 22	1.79	-0.99	2.47	1.72	1.39	1.09
Sunl. 32	1.24	0.37	-0.29	-1.18	1.91	-1.73
Sunl. 48	0.32	0.13	1.42	1.33	1.63	7.13
Store	0.95	-0.43	0.33	0.37	1.71	1.26
Generic	0.88	-0.48	-0.21	0.24	0.82	2.14

Brand j

Brand i	Su 12	Su 22	Su 32	Su 48	β_{0is}
Ivory 12	-1.73	0.31	-0.96	-0.15	2.57
Ivory 22	0.90	0.31	-0.21	-0.87	5.11
Ivory 32	-0.74	0.15	-0.45	-0.27	3.22
Ivory 48	-1.53	0.61	-0.63	-0.10	6.47
Joy 12	-0.56	0.18	-0.90	-0.21	6.69
Joy 22	-0.37	0.40	-0.89	-0.62	9.29
Joy 32	-0.57	-0.15	-0.19	-0.16	10.91
Joy 48	-1.35	-0.14	0.17	-0.62	13.11
Ajax 22	0.35	-0.81	-0.48	-0.87	4.92
Ajax 32	-0.02	-1.03	-0.76	-0.61	16.23
Derm. 32	-0.88	0.82	0.25	-1.05	-5.80
Palm. 12	-0.30	-0.15	0.15	0.09	-6.42
Palm. 22	-2.28	0.93	-1.02	-0.14	21.25
Palm. 32	-1.74	0.40	-0.52	0.02	3.15
Palm. 48	1.16	0.71	-0.23	-1.45	0.16
Dove	-1.85	0.15	0.91	0.39	4.80
Lux 22	0.00	2.93	0.00	-3.63	-21.43
Lux 32	-0.93	0.17	-0.23	0.35	4.15
Dawn 12	-0.36	-0.14	-0.88	-0.35	8.57
Dawn 22	-0.45	-0.44	-0.40	0.01	6.90
Dawn 32	-1.14	0.11	-0.88	-0.00	7.64
Dawn 48	1.29	-0.11	-0.81	-1.44	6.19
Sunl. 12	-2.81	0.65	-0.17	-0.80	-0.71
Sunl. 22	1.11	-4.02	0.30	-0.73	-5.99
Sunl. 32	-2.47	0.46	-4.65	0.15	25.25
Sunl. 48	1.20	0.67	-0.55	-2.93	-13.77
Store	0.34	0.05	-0.58	-0.71	-3.60
Generic	0.49	0.24	-0.77	-0.53	-2.76

Table VI-6 (continued)

Store 6

Brand j

Brand i	Iv 22	Iv 32	Jo 22	Jo 32	Jo 48	Aj 22	Aj 32
Ivory 12	-1.54	-0.23	-0.94	-1.82	2.59	0.76	0.43
Ivory 22	-7.59	3.49	-0.40	-3.03	0.71	-0.17	-1.05
Ivory 32	-0.33	-4.31	-0.29	-0.09	2.69	2.67	1.63
Ivory 48	-0.90	1.69	-0.89	0.10	3.93	0.65	0.21
Joy 12	-0.61	-1.96	1.02	0.10	1.59	0.60	0.67
Joy 22	-1.65	0.99	-3.78	-0.67	1.56	0.34	0.36
Joy 32	-0.42	-1.11	1.97	-2.43	1.22	-0.02	1.28
Joy 48	-3.18	-1.43	0.28	0.87	2.19	-0.48	0.23
Ajax 22	-0.55	-2.50	5.29	2.70	0.73	-5.53	4.65
Ajax 32	3.85	-3.48	-1.19	0.47	2.55	-1.42	-4.07
Derm. 32	0.59	-2.01	2.20	-0.06	3.29	1.39	0.13
Palm. 12	-1.23	1.23	-2.03	-2.15	-0.14	1.85	-0.37
Palm. 22	-1.91	2.41	1.41	-3.45	4.81	1.42	-0.66
Palm. 32	4.39	-2.02	-1.47	-2.80	-5.43	5.38	0.26
Palm. 48	-5.89	-5.03	-1.82	1.37	-1.72	-0.08	-1.26
Dove	-2.43	1.39	-0.26	-4.89	-0.23	0.19	0.01
Lux 22	0.00	0.00	53.68	0.00	0.00	-1.11	0.96
Lux 32	-3.05	-3.53	1.32	-2.52	1.24	-2.67	1.34
Dawn 12	-1.28	0.32	-0.04	-1.90	0.40	1.08	0.19
Dawn 22	-2.36	0.38	-1.43	-2.13	1.42	1.45	0.54
Dawn 32	-2.27	2.27	0.37	0.26	0.34	1.77	1.12
Dawn 48	2.22	2.18	1.54	3.02	2.21	1.75	1.23
Sunl. 12	-2.29	-0.88	0.65	-0.81	3.79	0.83	1.52
Sunl. 22	0.24	1.36	0.20	-1.52	2.70	0.03	0.06
Sunl. 32	0.60	-1.95	-2.88	-1.91	4.01	2.41	2.09
Sunl. 48	-8.48	4.20	3.25	-6.22	-3.19	-0.67	-0.04
Store	-2.10	-1.05	0.64	-0.99	0.21	-0.12	0.62
Generic	-1.98	-2.15	-2.08	-7.51	1.45	-1.05	-0.05

Brand j

Brand i	Pa 22	Pa 32	Pa 48	Dove	Lu 32	Da 22
Ivory 12	0.81	1.36	0.55	-1.28	1.42	2.43
Ivory 22	0.86	-0.49	-0.35	1.80	0.04	-0.62
Ivory 32	0.27	-0.07	-0.08	-3.71	0.40	0.47
Ivory 48	0.69	0.14	-0.69	-0.50	0.73	2.46
Joy 12	-0.37	0.05	0.95	-3.06	1.00	0.58
Joy 22	-0.03	0.13	0.65	-3.60	1.11	-0.65
Joy 32	-0.03	-0.27	-0.35	-2.62	-0.24	1.06
Joy 48	-1.81	-0.25	-1.38	-5.53	5.16	1.93
Ajax 22	-0.05	0.40	1.22	-0.81	1.84	1.99
Ajax 32	-0.52	4.23	0.40	-2.05	-3.22	4.54
Derm. 32	0.37	-0.78	-1.24	-4.21	0.23	1.25
Palm. 12	1.22	-0.45	0.52	1.11	0.26	4.10
Palm. 22	-2.63	-0.02	0.50	-5.12	0.69	-0.70
Palm. 32	2.08	-3.94	2.35	2.89	-2.27	1.22
Palm. 48	-0.85	3.77	-4.20	5.78	1.27	4.85
Dove	1.19	1.68	1.08	-9.07	-1.76	1.07

Table VI-6 (continued)

Brand i	Pa 22	Pa 32	Pa 48	Dove	Lu 32	Da 22
Lux 22	4.30	-8.87	-9.23	0.00	-26.79	-7.84
Lux 32	-0.47	-0.53	2.10	-2.00	-4.45	-2.80
Dawn 12	0.20	-0.93	0.79	0.11	0.32	-1.78
Dawn 22	0.91	0.59	0.94	1.77	1.72	-6.52
Dawn 32	0.44	-0.87	0.09	2.60	3.38	0.07
Dawn 48	-0.37	-1.29	-0.19	-4.09	-1.12	-0.87
Sunl. 12	-0.70	1.37	-1.14	-2.76	2.51	5.11
Sunl. 22	0.40	2.89	2.70	-2.16	0.25	0.60
Sunl. 32	-0.16	-1.69	-0.44	3.03	0.22	0.80
Sunl. 48	1.23	-1.08	0.29	-0.89	2.28	-6.17
Store	-0.07	0.49	0.45	-0.46	0.17	0.34
Generic	1.76	2.85	-0.38	0.06	-0.07	1.97

Brand j

Brand i	Su 12	Su 22	Su 32	Su 48	Store	β_{0is}
Ivory 12	-1.11	0.49	-0.49	0.06	1.87	-5.05
Ivory 22	-0.04	0.71	-0.77	-0.38	-1.42	18.75
Ivory 32	0.34	0.31	-0.59	-0.63	0.02	4.48
Ivory 48	0.10	0.09	0.11	-0.33	3.65	-15.50
Joy 12	0.68	-0.01	-0.87	-0.39	-1.55	4.47
Joy 22	0.04	0.22	-0.44	-0.04	-0.99	14.15
Joy 32	-0.10	-0.24	-1.03	-0.57	-1.00	10.13
Joy 48	1.81	0.02	-0.88	-0.55	-5.47	13.64
Ajax 22	3.11	-0.15	-0.94	-2.33	-2.20	-12.23
Ajax 32	-0.35	0.14	-1.06	-0.01	-2.89	6.02
Derm. 32	2.01	0.49	0.26	-1.09	-3.13	-0.26
Palm. 12	0.51	0.13	0.80	0.57	2.41	-10.75
Palm. 22	0.28	0.43	-0.57	0.03	1.82	5.69
Palm. 32	0.33	0.84	1.41	0.35	-2.72	-1.27
Palm. 48	-1.98	-0.74	-0.93	0.37	0.42	14.62
Dove	-1.62	0.41	0.46	1.22	-0.09	21.28
Lux 22	8.33	3.69	0.00	-5.63	-17.30	-6.21
Lux 32	3.49	-0.33	-0.82	-0.32	0.13	25.13
Dawn 12	0.82	0.32	-0.59	-0.07	-0.11	7.18
Dawn 22	-1.20	0.86	-0.78	0.31	0.79	9.94
Dawn 32	0.60	0.54	-0.98	-0.24	1.94	-15.52
Dawn 48	0.31	-0.68	-1.44	0.24	0.55	-8.68
Sunl. 12	-2.65	0.77	-0.29	-1.19	2.93	-7.66
Sunl. 22	-1.00	-3.37	0.23	-0.54	2.36	-5.43
Sunl. 32	0.73	1.24	-5.82	-0.67	4.17	-2.05
Sunl. 48	-3.32	1.07	-1.59	-2.87	-0.72	43.02
Store	-0.26	-0.14	-0.67	-0.27	-1.41	10.23
Generic	-1.72	0.62	0.40	0.24	-1.13	17.60

Table VI-6 (continued)

Store 7

Brand j

Brand i	Iv 22	Iv 32	Jo 22	Jo 32	Jo 48	Aj 22	Aj 32
Ivory 12	0.61	1.48	0.60	2.16	1.33	0.43	0.14
Ivory 22	-4.26	0.45	2.91	1.38	-0.60	-0.22	0.26
Ivory 32	0.77	-2.83	-0.45	1.40	-0.30	0.81	1.13
Ivory 48	-1.49	-1.29	1.37	-2.23	-1.07	-0.90	0.85
Joy 12	0.07	2.44	-0.20	-0.71	0.94	1.63	-1.54
Joy 22	1.52	-0.22	-2.06	-0.02	-3.80	0.75	1.53
Joy 32	0.21	-0.70	1.45	-0.02	-1.27	1.57	0.98
Joy 48	1.54	-1.76	0.63	3.66	-3.12	0.36	0.90
Ajax 22	1.04	-3.21	1.76	-0.68	-5.90	-1.43	1.46
Ajax 32	1.92	0.61	-1.97	-0.23	2.74	-0.96	-3.63
Derm. 32	0.00	3.48	-3.31	0.00	0.00	-2.49	-2.35
Palm. 12	2.02	2.60	-1.01	-1.08	3.70	2.42	-1.13
Palm. 22	-0.25	2.15	2.34	-1.89	-0.71	0.17	-1.15
Palm. 32	4.18	-1.46	5.68	5.05	-7.70	3.10	0.30
Palm. 48	-1.55	-1.55	-1.12	0.46	-0.28	0.91	1.17
Dove	-2.60	0.39	-1.87	1.45	5.21	0.00	-0.21
Lux 22	0.00	0.00	22.21	0.00	0.00	-2.06	1.50
Lux 32	-1.76	-2.57	0.45	2.01	-0.75	-2.64	-1.39
Dawn 12	0.84	0.33	-0.34	0.01	0.03	1.19	-0.25
Dawn 22	1.13	0.60	-0.33	1.43	2.61	-1.00	-0.08
Dawn 32	8.03	-1.62	0.19	2.95	3.61	-0.50	0.61
Dawn 48	0.10	-6.37	0.65	1.14	-4.55	1.05	1.99
Sunl. 12	-0.44	2.95	2.50	-1.53	6.78	-2.59	-0.83
Sunl. 22	5.67	-1.82	0.78	5.54	-0.26	-1.40	1.71
Sunl. 32	1.19	-3.96	-1.56	-2.05	-1.15	6.54	1.31
Sunl. 48	0.36	1.27	-0.05	2.78	-1.94	0.97	1.21
Store Generic	0.17 -6.03	0.15 3.52	-1.48 -3.54	-0.14 -4.22	1.29 2.77	-0.20 -3.75	-0.38 -2.43

Brand j

Brand i	Pa 22	Pa 32	Pa 48	Dove	Lu 32	Da 22	Da 48
Ivory 12	-0.29	-0.13	-1.03	-1.30	0.01	-1.94	1.16
Ivory 22	-0.47	-0.76	-1.06	0.37	-1.04	-4.76	0.46
Ivory 32	0.59	0.57	-0.50	-1.40	1.08	-1.77	-0.47
Ivory 48	-0.08	-1.33	2.25	2.88	-2.24	-2.75	4.50
Joy 12	0.22	-0.76	-0.50	-1.02	0.63	-1.76	0.88
Joy 22	0.61	-1.04	0.94	2.20	-1.34	-2.81	4.55
Joy 32	-0.56	0.17	-0.25	-1.14	1.52	-4.73	-1.15
Joy 48	-0.35	1.02	3.31	-2.62	1.40	-3.92	-1.88
Ajax 22	-0.05	0.91	3.05	-0.15	3.63	-5.03	-0.20
Ajax 32	0.25	1.93	-1.93	-4.74	-1.14	-1.05	-1.32
Derm. 32	-0.37	2.63	2.27	0.00	4.40	12.33	0.64
Palm. 12	2.28	-0.37	-2.11	-0.61	0.42	0.08	-0.33
Palm. 22	-2.42	1.00	1.74	-2.14	1.95	0.47	-0.03
Palm. 32	-1.61	-3.92	-0.26	-2.17	-0.93	4.31	-1.40
Palm. 48	-0.82	-1.30	-2.68	1.94	-1.85	-1.97	2.45
Dove	0.45	1.24	0.99	-6.47	2.19	8.77	-0.20

Table VI-6 (continued)

Brand i		Brand j						
		Pa 22	Pa 32	Pa 48	Dove	Lu 32	Da 22	Da 48
Lux	22	-5.19	-4.87	-0.39	0.00	-10.00	10.68	1.36
Lux	32	-1.54	1.57	-1.45	-2.03	-0.89	4.84	-3.98
Dawn	12	-0.10	0.09	0.91	-0.89	0.25	-5.82	-0.55
Dawn	22	0.21	0.94	-1.45	0.81	-0.85	1.17	0.21
Dawn	32	-1.15	0.78	-3.17	-1.80	-2.63	14.16	0.13
Dawn	48	-1.69	-1.11	1.31	2.02	-0.98	2.24	-2.27
Sunl.	12	0.61	0.79	-2.10	0.76	0.29	0.73	1.32
Sunl.	22	-1.89	4.15	1.75	-1.12	0.34	-1.03	-0.40
Sunl.	32	0.66	-2.54	2.36	0.04	0.54	-3.73	-4.43
Sunl.	48	-0.36	0.65	-4.59	-2.68	2.47	2.30	-2.73
Store		0.98	0.74	0.09	-2.88	0.68	-3.26	-1.25
Generic		1.58	2.84	1.97	-2.02	2.91	-9.62	-1.79

Brand i		Brand j					
		Su 12	Su 22	Su 32	Su 48	Store	β_{0is}
Ivory	12	-1.11	0.05	-0.48	-0.26	-1.22	1.69
Ivory	22	0.90	0.16	-0.21	-0.33	-0.58	15.99
Ivory	32	1.21	0.07	0.72	-0.14	-2.17	4.95
Ivory	48	-1.27	0.29	-2.20	0.04	5.23	4.28
Joy	12	0.48	0.57	0.17	-0.35	-0.86	1.91
Joy	22	-0.20	0.01	-0.41	-0.16	-0.17	3.57
Joy	32	0.86	0.24	0.02	0.07	0.53	7.11
Joy	48	0.54	1.53	1.99	0.23	1.98	-6.37
Ajax	22	-2.33	-0.12	1.09	1.35	3.82	7.45
Ajax	32	-0.05	1.02	-0.00	-1.17	-0.06	17.71
Derm.	32	-0.20	0.00	0.00	0.00	0.00	-27.28
Palm.	12	2.03	0.26	-0.08	-0.59	-1.18	-11.72
Palm.	22	-0.32	0.74	1.13	0.55	0.48	-2.83
Palm.	32	-0.10	-0.59	0.02	0.33	-1.77	-2.07
Palm.	48	-1.77	0.28	-1.07	0.63	-1.35	17.68
Dove		-6.28	0.70	1.57	0.78	1.92	-10.48
Lux	22	-6.31	0.00	0.00	0.00	0.00	-12.96
Lux	32	1.40	-1.29	0.64	-0.34	-3.83	23.04
Dawn	12	0.80	0.82	-0.00	-0.09	1.90	5.92
Dawn	22	-0.19	-0.27	-0.47	-0.77	-2.61	-0.33
Dawn	32	-0.14	-0.40	0.05	-1.10	-4.06	-26.35
Dawn	48	-2.54	-1.18	-1.22	-0.03	-2.38	24.72
Sunl.	12	-1.90	1.21	-0.52	-2.44	-2.99	-3.54
Sunl.	22	4.54	-3.37	0.35	-1.52	0.94	-20.56
Sunl.	32	0.16	2.36	-2.60	1.16	2.55	10.05
Sunl.	48	1.20	-1.61	0.88	-2.68	-6.12	12.88
Store		-0.19	1.04	0.46	-0.36	-1.31	12.69
Generic		0.61	2.03	0.16	-0.03	1.66	27.57

Table VI-6 (continued)

Store 8

Brand j

Brand i	Iv 22	Iv 32	Jo 22	Jo 32	Jo 48
Ivory 12	-0.23	1.89	0.56	0.76	2.41
Ivory 22	-6.55	2.09	-0.55	-1.51	0.43
Ivory 32	0.77	-3.73	-0.31	-0.63	0.25
Ivory 48	-2.01	-0.44	0.32	-0.51	-0.45
Joy 12	-0.69	0.31	0.81	1.23	0.70
Joy 22	-2.11	1.83	-3.89	-3.69	0.62
Joy 32	0.51	0.01	1.18	0.64	-0.40
Joy 48	0.14	-0.83	0.28	0.86	-5.19
Ajax 22	0.00	-0.11	97.18	0.00	0.00
Ajax 32	-2.92	-1.67	0.10	1.10	-0.88
Derm. 32	-5.13	-2.67	0.64	-1.12	-0.32
Palm. 12	2.07	-0.53	0.37	2.60	-1.16
Palm. 22	-2.29	2.79	0.54	-4.98	-2.07
Palm. 32	6.65	-2.75	-0.79	0.27	-1.51
Palm. 48	-3.29	0.78	-2.78	-0.89	0.89
Dove	-0.71	2.40	-0.76	-1.10	-3.13
Lux 22	0.00	0.00	19.24	0.00	0.00
Lux 32	0.25	1.33	-1.85	-1.46	-2.54
Dawn 12	0.02	-2.32	0.49	1.10	1.03
Dawn 22	0.87	-0.19	-0.16	-0.90	0.33
Dawn 32	1.48	-0.07	0.27	-0.90	0.12
Dawn 48	2.92	1.99	1.54	3.11	-0.69
Sunl. 12	-1.00	-1.86	0.35	-3.19	-2.99
Sunl. 22	1.16	1.92	1.28	2.11	-0.15
Sunl. 32	4.31	-4.70	-1.12	-3.41	0.74
Sunl. 48	-8.82	-2.52	5.52	-5.77	-4.43
Store	-0.79	-1.11	-0.13	0.19	1.62
Generic	2.23	-3.17	0.09	-0.69	-0.99

Brand j

Brand i	Pa 22	Pa 32	Pa 48	Dove	Lu 32
Ivory 12	0.35	-0.98	-0.06	-0.44	1.14
Ivory 22	0.74	-0.86	-0.44	0.17	-0.89
Ivory 32	0.41	0.34	-0.09	0.15	0.01
Ivory 48	0.09	1.80	1.22	0.84	1.11
Joy 12	-0.69	-0.17	0.12	-0.95	1.47
Joy 22	1.04	0.30	0.80	0.82	0.98
Joy 32	-0.32	-0.00	0.11	-1.54	2.79
Joy 48	-1.26	-0.74	0.66	-1.89	0.05
Ajax 22	0.86	-3.00	-2.00	0.00	8.30
Ajax 32	0.71	0.56	1.34	0.48	0.74
Derm. 32	0.13	0.37	-0.51	-1.52	2.61
Palm. 12	0.05	-0.83	0.47	-0.66	-0.37
Palm. 22	-2.32	0.78	2.20	-1.72	-1.32
Palm. 32	0.16	-2.90	1.46	0.24	-3.53
Palm. 48	0.88	0.34	-4.69	4.33	1.62
Dove	1.19	-0.72	-0.35	-3.46	-0.14

Table VI-6 (continued)

Brand i		Brand j Pa 22	Pa 32	Pa 48	Dove	Lu 32
Lux	22	6.94	-6.57	1.80	0.00	-3.04
Lux	32	-0.37	0.15	2.25	-2.63	-4.62
Dawn	12	-0.37	0.64	-0.23	0.41	0.67
Dawn	22	0.74	-0.01	-1.42	0.69	0.01
Dawn	32	0.16	0.70	1.40	-1.53	-0.26
Dawn	48	-1.18	-0.33	1.02	-0.79	-1.57
Sunl.	12	1.45	2.33	2.06	0.76	1.08
Sunl.	22	-0.62	1.48	1.06	0.53	3.62
Sunl.	32	0.13	-2.81	-2.12	-1.95	-5.34
Sunl.	48	3.35	5.07	2.90	1.33	2.23
Store		0.06	-0.05	-0.40	-0.50	1.56
Generic		0.14	1.04	-0.60	-0.54	1.44

Brand i		Brand j Da 22	Su 12	Su 22	Su 32	β_{0is}
Ivory	12	-0.51	0.13	-0.01	-0.30	-4.86
Ivory	22	-1.80	0.46	0.46	-0.23	18.86
Ivory	32	2.36	-0.60	0.35	0.47	3.82
Ivory	48	-0.80	0.25	-0.21	-0.74	1.97
Joy	12	0.43	0.48	-0.32	-0.51	-0.48
Joy	22	-0.40	-2.34	1.31	-0.72	14.19
Joy	32	1.83	0.80	-0.74	0.10	-5.59
Joy	48	2.75	-0.68	0.10	-0.71	12.60
Ajax	22	1.08	4.26	-3.73	-2.35	-178.06
Ajax	32	3.00	1.10	0.31	-2.02	-1.26
Derm.	32	-1.12	-0.19	-0.34	0.17	18.76
Palm.	12	2.03	1.09	-0.41	0.91	-7.44
Palm.	22	-0.72	-0.89	0.67	-0.08	19.91
Palm.	32	-1.22	-1.30	0.05	-0.56	11.47
Palm.	48	0.28	-0.56	0.54	-0.85	8.57
Dove		-2.26	-1.71	-0.07	0.15	20.87
Lux	22	-12.75	-3.08	2.47	-0.30	-6.57
Lux	32	4.67	0.15	0.35	-0.02	8.67
Dawn	12	4.01	-1.11	-0.32	-0.26	-2.48
Dawn	22	-1.43	0.27	-0.02	0.60	5.22
Dawn	32	1.48	-1.35	0.80	-0.34	-0.28
Dawn	48	-1.21	1.54	-0.52	-0.73	-7.48
Sunl.	12	2.06	-4.84	1.35	0.31	8.12
Sunl.	22	3.11	0.64	-4.07	-0.82	-15.92
Sunl.	32	-3.31	-0.10	-0.40	-3.29	42.04
Sunl.	48	-1.22	-6.16	1.34	-1.22	18.82
Store		0.84	-0.12	0.13	0.22	0.97
Generic		2.86	-0.21	-0.33	0.61	-0.90

Table VI-6 (continued)

Store 9

Brand j

Brand i	Iv 22	Iv 32	Jo 22	Jo 32	Jo 48	Aj 22
Ivory 12	-0.44	-0.33	1.68	1.37	-0.11	0.68
Ivory 22	-3.45	-1.34	2.34	-4.25	1.59	-1.01
Ivory 32	0.91	-3.35	-3.33	12.63	2.69	1.94
Ivory 48	-0.45	-0.23	5.03	-1.75	1.42	1.79
Joy 12	-0.93	-0.09	1.97	-1.27	-2.40	0.02
Joy 22	0.53	-1.00	-2.48	8.26	0.55	0.66
Joy 32	-1.98	-0.71	1.72	-8.01	1.05	1.87
Joy 48	3.78	-3.01	-0.19	0.44	-1.32	4.31
Ajax 22	-0.46	-2.91	4.67	8.88	-0.01	-0.06
Ajax 32	0.60	-1.09	-0.31	-0.43	1.61	-1.35
Derm. 32	-1.82	-4.62	-1.84	8.15	-125.26	-5.70
Palm. 22	-1.47	0.28	0.19	-0.95	1.93	1.75
Palm. 32	-1.08	-0.50	2.19	-4.99	-2.17	0.15
Palm. 48	2.39	0.54	-1.52	-1.90	-4.28	-1.17
Dove	-0.89	-0.43	1.74	2.61	-1.61	2.40
Dawn 22	-0.96	0.50	0.83	0.05	-2.42	-1.02
Dawn 32	-1.76	-3.82	-1.70	4.48	3.21	-1.01
Dawn 48	1.06	-2.76	1.18	-4.70	3.58	5.09
Sunl. 22	1.34	0.42	-1.59	1.78	4.54	-1.22
Sunl. 32	-2.84	0.42	3.85	-7.72	0.01	-0.41
Store	-1.31	0.61	0.98	-1.10	-1.84	-0.70
Generic	0.75	-1.65	2.36	-3.28	1.85	-0.60

Brand i	Aj 32	Pa 22	Pa 32	Pa 48	Dove	Da 22
Ivory 12	0.24	-0.01	-0.12	0.32	0.52	0.05
Ivory 22	1.57	-0.35	1.40	0.33	-0.18	2.57
Ivory 32	-1.11	-3.70	-2.23	1.10	0.35	-3.26
Ivory 48	0.60	-0.12	0.70	0.76	-4.31	-0.10
Joy 12	0.25	-0.81	-0.25	0.60	2.55	0.43
Joy 22	-0.69	-1.80	-1.05	1.15	-1.07	0.07
Joy 32	1.27	-0.69	-1.28	0.13	4.38	2.21
Joy 48	1.25	-2.24	-2.05	-0.99	0.99	-2.40
Ajax 22	-1.04	-1.13	-2.80	1.03	2.52	-4.34
Ajax 32	-2.16	0.58	0.82	-2.00	6.84	-1.17
Derm. 32	-0.04	-4.16	3.19	2.59	-0.82	7.59
Palm. 22	1.00	-5.27	-0.63	-0.04	1.16	1.36
Palm. 32	-0.33	-0.34	-2.19	2.01	-0.86	1.63
Palm. 48	1.82	0.07	0.49	-3.44	4.96	-1.22
Dove	-0.76	-1.67	-2.00	-0.11	2.64	-0.22
Dawn 22	0.26	1.01	0.58	-0.81	2.89	-0.83
Dawn 32	0.44	-0.99	-0.91	-0.42	-1.30	3.09
Dawn 48	-1.16	-4.06	-1.81	-0.32	2.56	-1.08
Sunl. 22	1.00	-0.67	-1.21	0.36	-1.20	0.63
Sunl. 32	0.06	0.02	1.31	-0.61	4.67	3.86
Store	1.10	0.42	0.22	0.50	-0.98	2.23
Generic	-0.45	-1.01	0.50	0.91	2.96	1.47

Table VI-6 (continued)

Brand i	Da 32	Da 48	Su 22	Su 32	Store	β_{0is}
			Brand j			
Ivory 12	0.24	-0.43	-0.52	-1.59	-0.10	0.50
Ivory 22	-0.16	0.05	1.99	1.06	2.15	-2.96
Ivory 32	-2.38	-2.26	1.39	0.62	1.60	0.49
Ivory 48	0.82	1.42	-0.05	-0.77	5.14	-13.34
Joy 12	0.14	-0.73	0.69	-0.23	-1.59	5.85
Joy 22	-1.03	-1.09	0.15	-0.64	-1.42	3.45
Joy 32	-0.52	-0.48	-0.21	-0.38	-2.56	9.49
Joy 48	-0.83	-2.19	0.98	-3.27	-1.34	14.50
Ajax 22	-2.98	-4.70	-1.02	-2.49	0.91	12.57
Ajax 32	1.36	-1.20	-0.50	-0.82	-3.91	5.72
Derm. 32	4.87	0.00	5.50	5.24	-10.59	205.09
Palm. 22	0.10	-1.36	0.07	-0.48	0.82	6.38
Palm. 32	3.94	1.78	-1.06	-2.62	-0.24	10.05
Palm. 48	-0.41	-0.76	1.76	-0.53	-4.74	13.75
Dove	0.41	0.35	1.30	-0.31	-3.10	-0.24
Dawn 22	0.58	-0.14	0.40	-0.44	-1.46	4.97
Dawn 32	-2.95	-2.76	1.26	-0.16	0.92	9.62
Dawn 48	-1.14	-3.16	2.89	1.39	1.79	4.11
Sunl. 22	0.55	-1.48	-5.05	-0.80	-1.19	8.45
Sunl. 32	-1.57	0.52	1.30	-2.89	0.17	3.55
Store	0.12	0.01	-1.03	0.52	-0.49	4.24
Generic	1.27	-0.96	-0.09	-0.75	-2.24	-0.03

Table VI-6 (continued)

Store 10

Brand i	Iv 22	Iv 32	Jo 22	Jo 32	Jo 48	Aj 22
	Brand j					
Ivory 12	-0.24	-0.00	-0.21	-1.46	-1.83	-0.08
Ivory 22	-0.49	-1.38	0.32	-0.57	-0.99	2.56
Ivory 32	-0.81	-2.23	0.09	2.23	5.32	-0.43
Ivory 48	-0.68	-1.07	0.21	-1.78	-1.68	3.28
Joy 12	-0.09	-0.43	0.86	-0.07	-1.39	-0.12
Joy 22	0.56	1.69	0.53	2.03	-1.73	-1.29
Joy 32	-0.33	-0.46	0.88	-3.69	-3.02	1.23
Joy 48	-1.36	-1.50	-1.44	-0.47	0.19	0.62
Ajax 22	-0.71	1.25	-0.77	-8.14	8.13	-4.20
Ajax 32	-4.92	-0.83	4.72	-6.17	-5.62	3.46
Palm. 22	1.55	0.14	0.52	-0.41	-4.36	-0.11
Palm. 32	-0.57	-1.45	1.96	1.73	7.25	3.21
Palm. 48	3.45	0.72	3.95	1.25	0.71	-3.16
Dove	0.07	-0.40	1.28	0.20	2.78	2.44
Dawn 22	0.08	0.20	-1.06	0.94	0.92	-0.85
Dawn 32	0.47	-1.80	0.53	3.55	-4.33	-1.34
Dawn 48	-0.75	1.42	-0.61	1.19	0.03	0.13
Sunl. 22	2.14	1.00	-0.13	-1.31	4.32	-1.52
Sunl. 32	-2.43	0.04	1.04	-2.52	7.42	4.24
Store	-0.41	-0.28	1.22	-0.10	0.05	0.58
Generic	0.15	0.40	0.26	1.71	3.12	-1.63

Brand i	Aj 32	Pa 22	Pa 32	Pa 48	Dove	Da 22
	Brand j					
Ivory 12	0.89	0.17	-0.28	0.78	-1.81	1.06
Ivory 22	1.68	0.52	-0.30	-0.07	0.06	0.29
Ivory 32	-0.38	0.51	-1.18	1.09	2.28	-1.94
Ivory 48	2.53	-0.74	-1.30	1.56	1.89	0.27
Joy 12	0.02	0.68	0.51	0.62	-1.53	0.22
Joy 22	-0.79	-0.19	2.10	0.12	-1.78	0.13
Joy 32	2.17	0.26	-2.01	0.14	-0.46	-0.16
Joy 48	2.22	-2.02	1.57	1.94	-0.61	-0.45
Ajax 22	2.68	1.74	0.01	1.32	9.20	-2.76
Ajax 32	-4.41	0.78	-1.50	0.72	3.18	2.29
Palm. 22	0.45	-4.04	1.69	-0.56	0.91	0.62
Palm. 32	-0.21	1.00	-4.25	2.22	0.72	-2.54
Palm. 48	0.21	2.78	1.96	-6.54	-1.11	-0.18
Dove	0.05	-0.00	-2.03	1.26	-2.92	-0.24
Dawn 22	1.11	1.17	-1.04	-1.27	3.23	-2.10
Dawn 32	0.60	0.12	2.89	1.19	-6.63	0.08
Dawn 48	-1.23	-0.75	-3.75	-0.16	-1.27	0.01
Sunl. 22	-0.16	1.06	2.86	-0.47	-2.72	1.71
Sunl. 32	0.05	-0.40	-0.14	0.72	1.88	1.86
Store	-0.53	0.25	-0.86	0.33	-0.35	0.71
Generic	-0.16	-0.13	-0.74	-0.18	1.38	-0.25

Table VI-6 (continued)

Brand i	Da 32	Da 48	Su 22	Su 32	Store	β_{0is}
			Brand j			
Ivory 12	0.85	-0.75	-0.05	-0.32	0.51	7.94
Ivory 22	1.77	0.35	1.32	-0.88	0.16	-4.09
Ivory 32	-1.40	0.07	0.15	0.46	0.17	-3.34
Ivory 48	2.47	-2.44	-0.35	2.38	-1.62	-3.53
Joy 12	-0.39	-1.47	-0.42	0.02	0.83	7.34
Joy 22	-0.80	0.00	0.77	0.42	0.95	-0.86
Joy 32	0.89	-1.33	1.21	0.74	-0.15	9.84
Joy 48	-1.18	0.05	-1.75	0.32	1.40	7.24
Ajax 22	-0.87	-2.51	-1.10	1.88	-5.54	2.61
Ajax 32	-0.26	-2.20	-1.47	1.35	-2.86	24.67
Palm. 22	0.86	-0.05	1.77	-0.27	0.23	5.54
Palm. 32	0.45	-1.06	2.52	-0.49	-1.85	-14.28
Palm. 48	-0.22	-5.60	1.13	3.18	-0.08	-3.39
Dove	1.80	0.29	-0.64	-0.91	1.02	-5.26
Dawn 22	-0.24	-0.00	1.16	-0.75	-0.01	1.73
Dawn 32	-0.72	0.95	2.44	0.65	2.98	0.10
Dawn 48	-0.75	-1.60	0.23	-1.45	5.07	12.36
Sunl. 22	1.62	-0.55	-5.19	0.99	-0.72	-3.13
Sunl. 32	-2.88	-0.17	-2.22	-3.32	-0.17	-2.71
Store	0.23	-1.00	-0.02	-0.04	-1.50	5.70
Generic	-0.14	-2.01	0.56	0.07	-1.41	0.50

Table VI-6 (continued)

Store 11

Brand j

Brand i	Iv 22	Iv 32	Jo 22	Jo 32	Aj 32	De 32
Ivory 12	0.87	-0.26	-0.02	0.70	0.73	2.66
Ivory 22	-1.14	0.73	-0.08	-2.44	0.99	3.69
Ivory 32	-0.28	-1.70	0.76	2.60	-0.92	-0.92
Ivory 48	2.57	0.37	-1.10	0.10	0.52	3.07
Joy 12	0.02	-0.48	0.51	2.22	0.38	1.33
Joy 22	0.15	-0.79	-1.08	1.70	0.26	-0.61
Joy 32	0.74	0.45	0.46	-2.42	0.60	0.85
Joy 48	-1.56	1.08	0.22	0.28	1.51	-0.54
Ajax 22	0.10	-1.44	0.26	-0.58	2.17	4.41
Ajax 32	3.51	-0.88	2.22	-1.17	-5.25	-13.10
Derm. 32	-0.72	-0.40	0.04	2.99	-0.24	-5.00
Palm. 22	0.96	-0.42	-0.10	1.55	0.22	0.99
Palm. 32	0.47	1.27	0.25	1.56	0.53	2.70
Palm. 48	2.88	-2.55	-2.64	4.44	-0.32	-4.65
Dove	-0.74	-0.17	-0.04	0.46	1.20	5.25
Dawn 22	-0.53	0.34	0.18	1.85	0.49	2.53
Dawn 32	-0.70	-1.22	1.04	1.32	0.41	4.18
Dawn 48	-0.13	-0.68	-1.63	1.38	0.47	-3.29
Sunl. 22	0.01	0.44	-0.76	-0.08	0.24	0.11
Sunl. 32	-1.51	0.26	-1.29	-0.64	0.52	4.78
Store	0.19	0.19	0.76	0.46	0.22	1.35
Generic	-0.06	-0.27	2.23	1.61	0.18	-2.32

Brand	Pa 22	Pa 32	Pa 48	Dove	Da 22	Da 32
Ivory 12	-0.05	0.26	0.43	-0.98	-0.58	-0.04
Ivory 22	-0.18	1.50	0.29	-0.33	1.72	0.84
Ivory 32	0.38	0.06	-0.86	-3.45	0.22	-0.28
Ivory 48	0.75	-1.00	2.71	0.35	-0.69	-0.03
Joy 12	0.05	-0.84	-0.16	-1.73	-0.46	1.27
Joy 22	-0.06	-0.55	0.54	-0.21	0.38	1.11
Joy 32	0.02	-0.42	-0.45	-0.90	-0.66	1.60
Joy 48	-1.21	1.95	-0.39	1.06	1.26	-2.06
Ajax 22	-0.91	-1.96	0.08	6.65	-4.94	2.13
Ajax 32	1.22	3.27	-1.89	12.58	-0.68	1.05
Derm. 32	-0.31	-0.18	-0.96	-0.55	0.63	-0.60
Palm. 22	-4.24	1.64	-0.98	-2.87	0.05	1.88
Palm. 32	1.59	-2.22	0.06	-3.60	1.76	4.60
Palm. 48	-0.35	-2.81	-1.50	4.54	-3.06	-1.83
Dove	0.31	-0.89	0.53	-2.97	-0.40	-0.79
Dawn 22	-0.31	-0.18	-0.39	-1.68	-1.79	1.35
Dawn 32	0.63	0.05	-0.58	0.58	-0.40	-3.00
Dawn 48	-0.23	-1.23	-0.79	2.78	-1.22	0.59
Sunl. 22	0.17	-0.41	-0.28	1.29	1.37	0.12
Sunl. 32	-0.08	-0.57	0.75	1.49	0.85	-3.67
Store	-0.15	-0.23	0.32	0.13	-0.21	-0.41
Generic	0.34	-0.11	-0.78	-1.77	1.20	2.01

Table VI-6 (continued)

	Brand j				
Brand i	Da 48	Su 22	Su 32	Store	β_{0is}
Ivory 12	-0.73	0.72	0.45	0.23	-3.60
Ivory 22	0.54	0.74	-0.03	-0.76	-6.02
Ivory 32	0.18	-0.48	-0.74	3.45	7.76
Ivory 48	-0.09	2.52	1.30	-1.72	-14.63
Joy 12	-1.33	0.63	-0.43	0.75	0.96
Joy 22	-0.84	1.17	-1.07	-0.88	4.32
Joy 32	-0.24	-0.06	-0.19	0.96	3.30
Joy 48	0.74	-1.64	-0.06	1.84	-0.74
Ajax 22	-0.95	3.40	0.57	-3.33	-6.48
Ajax 32	-0.69	-0.30	-1.49	-4.85	9.39
Derm. 32	-0.85	-0.48	-0.04	1.53	10.77
Palm. 22	-0.08	0.63	-0.79	2.23	3.24
Palm. 32	0.78	0.25	0.30	2.13	-18.19
Palm. 48	-3.19	3.86	-1.00	-3.32	19.21
Dove	-1.41	0.64	0.86	-0.14	-0.00
Dawn 22	-0.20	-0.04	-0.35	0.98	1.30
Dawn 32	-0.69	1.34	0.32	0.78	-2.12
Dawn 48	-3.56	0.55	-0.85	1.03	14.68
Sunl. 22	-1.30	-3.33	1.02	-1.57	7.97
Sunl. 32	-1.62	0.11	-3.39	-2.53	14.41
Store	-0.64	0.48	0.16	-1.31	1.21
Generic	-0.55	-0.09	-0.20	3.21	-4.51

Table VI-6 (continued)

Store 12

Brand j

Brand i	Iv 22	Iv 32	Iv 48	Jo 22	Jo 32	Jo 48	De 22	De 32
Ivory 12	-2.58	2.27	-0.73	0.21	-2.16	-0.23	-6.02	-3.06
Ivory 22	-0.08	1.15	-1.17	0.71	-0.17	0.77	-17.34	3.13
Ivory 32	-3.16	-0.28	0.32	0.95	-1.79	-20.44	-9.79	-6.33
Ivory 48	-1.23	0.98	-0.67	2.89	-0.41	-38.85	-17.30	-3.11
Joy 12	-0.71	2.16	-1.02	1.72	-0.26	3.94	-21.14	2.05
Joy 22	-0.74	2.11	0.89	-1.98	-3.55	31.63	5.98	1.63
Joy 32	0.01	0.33	-2.30	1.77	-1.10	-9.79	-26.84	3.38
Joy 48	-0.96	0.88	-1.79	1.35	-1.74	-14.88	-14.59	-4.86
Ajax 22	-5.80	-5.66	-4.53	9.24	-2.50	-44.21	-35.75	-13.15
Derm. 22	-0.19	0.61	0.17	-0.19	1.29	-14.21	-5.84	-4.08
Derm. 32	0.59	0.68	-0.28	-0.18	0.66	3.64	-21.03	3.72
Palm. 12	-0.66	0.87	0.40	-0.58	3.12	-14.74	10.74	-0.60
Palm. 22	-0.53	0.34	1.51	-0.82	-2.03	-3.29	9.28	-1.21
Palm. 32	0.47	-0.38	0.66	2.76	-0.50	-3.43	-26.15	3.50
Palm. 48	4.03	-0.77	3.04	0.30	4.87	-37.50	-13.04	8.97
Dove	-1.23	-0.65	2.43	-2.15	-0.15	-11.49	8.19	-2.20
Lux 22	-1.78	-1.54	0.69	1.37	-1.93	6.14	-12.22	1.22
Lux 32	-0.08	-1.36	0.53	1.25	1.04	-17.57	-10.45	-0.58
Dawn 12	-0.81	1.19	-0.24	0.43	0.07	-30.07	-12.41	-1.85
Dawn 22	-2.92	-0.99	1.09	0.93	-0.89	11.65	-37.11	0.00
Dawn 32	-1.61	0.88	1.16	1.51	1.71	-5.68	-18.81	0.01
Dawn 48	-1.45	-0.10	1.15	0.03	0.70	5.77	-18.61	1.77
Sunl. 12	-2.51	-1.72	0.93	2.29	1.61	-56.48	-0.52	-5.16
Sunl. 22	-0.53	-0.49	-0.21	1.40	-2.36	5.38	-24.85	1.11
Sunl. 32	0.64	1.60	0.63	-0.13	1.83	-7.43	-5.10	0.34
Store	-0.54	0.73	1.49	-1.27	-0.64	13.94	-8.09	2.84
Generic	-0.20	1.72	-1.69	2.44	0.46	-41.24	-22.34	-1.77

Brand j

Brand i	Pa 12	Pa 22	Pa 32	Pa 48	Lu 22	Da 22	Da 32
Ivory 12	4.57	-4.10	-3.28	-3.10	-1.19	0.29	1.97
Ivory 22	1.48	-6.30	-4.62	-2.98	-3.48	-0.86	0.53
Ivory 32	2.46	-2.87	-0.13	-4.90	0.74	-0.49	0.82
Ivory 48	3.91	-1.63	-1.67	-3.92	-5.78	-1.17	-0.82
Joy 12	1.50	-6.01	-2.76	-3.43	0.31	0.12	0.73
Joy 22	-2.03	-1.78	0.82	1.30	4.08	1.44	2.94
Joy 32	3.30	-6.94	-5.95	-2.92	-4.41	-1.65	-0.94
Joy 48	3.23	-3.70	-3.84	-2.61	3.09	-0.13	-1.24
Ajax 22	4.94	4.51	-2.17	-2.04	-2.19	2.64	-5.44
Derm. 22	6.92	0.95	-0.07	-0.37	-3.28	-0.38	-0.88
Derm. 32	6.16	-2.36	-0.62	-1.39	-4.02	-0.96	-1.27
Palm. 12	-3.78	1.95	2.38	1.25	-0.74	-0.03	2.12
Palm. 22	1.08	-1.24	0.97	0.75	0.81	0.41	1.47
Palm. 32	2.41	-1.84	-4.43	-3.23	-2.15	2.76	-0.11
Palm. 48	-2.63	-6.44	-0.83	-2.10	-11.36	1.32	-0.21
Dove	-1.23	0.73	3.40	0.63	0.85	-0.13	1.14

Table VI-6 (continued)

Brand i	Pa 12	Pa 22	Pa 32	Pa 48	Lu 22	Da 22	Da 32
Lux 22	1.55	5.26	1.69	-4.11	-3.17	-0.19	2.26
Lux 32	-2.77	-4.60	0.33	-2.69	-5.11	-0.93	-0.06
Dawn 12	2.87	-3.49	-4.19	-2.96	-5.44	0.01	0.69
Dawn 22	5.40	-3.88	-1.64	-4.59	-1.52	-4.33	-1.20
Dawn 32	2.02	-2.97	-1.65	-4.00	-0.35	-0.13	0.15
Dawn 48	3.59	-2.81	-2.78	-2.47	-5.48	-3.70	-1.20
Sunl. 12	4.16	3.90	2.54	0.55	-3.56	1.46	-0.92
Sunl. 22	3.80	-0.14	-2.76	-3.12	-1.06	0.33	-1.42
Sunl. 32	0.96	-1.84	-0.39	0.93	3.13	2.83	-0.61
Store	3.12	1.86	-0.93	-0.59	0.51	-1.30	-0.41
Generic	10.30	-7.05	-5.50	-2.72	-10.30	0.67	-0.07

Brand j

Brand i	Da 48	Su 12	Su 22	Su 32	Store	Gnrc.	β_{0is}
Ivory 12	1.05	0.58	1.16	-2.31	-0.81	-1.73	29.91
Ivory 22	0.84	-0.16	1.50	-1.60	-4.91	-2.28	54.78
Ivory 32	-1.25	-1.71	2.12	-3.29	2.87	-0.81	82.38
Ivory 48	-2.15	-0.60	2.42	-1.75	-3.99	2.18	122.20
Joy 12	0.94	-2.18	1.40	-1.42	-2.56	-0.67	43.60
Joy 22	5.11	2.33	-1.81	-0.11	-3.11	-2.22	-73.86
Joy 32	0.30	-3.06	3.72	-2.52	-6.73	-1.53	99.99
Joy 48	-2.89	-1.44	2.08	-2.25	-2.98	0.19	80.23
Ajax 22	-5.32	-5.77	10.23	-3.09	9.20	10.35	159.09
Derm. 22	0.14	3.75	-1.17	-1.82	4.18	1.09	25.86
Derm. 32	-0.46	0.13	0.97	-1.43	1.51	0.32	25.56
Palm. 12	-0.15	-1.76	-1.86	1.77	6.11	1.07	1.04
Palm. 22	-1.48	-0.53	-0.35	1.62	3.07	-1.21	-8.47
Palm. 32	-1.93	-1.87	3.48	-1.38	-7.52	-0.54	58.24
Palm. 48	-3.42	-3.61	0.48	2.32	-9.78	-3.86	110.79
Dove	-2.03	-1.06	-0.47	0.54	7.23	-0.33	7.62
Lux 22	-4.15	-4.33	2.31	-1.51	8.70	1.10	12.73
Lux 32	-2.40	0.63	4.99	-0.91	0.43	-0.28	68.93
Dawn 12	-0.96	-0.44	0.80	-1.60	-4.40	-0.48	105.09
Dawn 22	1.32	-0.56	2.59	-3.33	-1.73	2.11	63.67
Dawn 32	-1.41	-1.88	1.58	-1.12	-5.24	-0.01	56.56
Dawn 48	-1.25	-0.84	3.32	-1.24	-7.20	0.26	47.42
Sunl. 12	-6.60	-7.37	-0.77	-0.64	8.69	1.89	113.07
Sunl. 22	-0.70	-1.86	2.95	-1.72	-3.90	0.11	45.93
Sunl. 32	-1.55	0.37	-0.52	1.32	2.98	0.21	5.35
Store	0.79	-0.96	-1.15	0.15	-2.18	-0.12	-12.23
Generic	-1.39	0.83	3.65	-0.93	-3.89	-0.17	128.44

Table VI-6 (continued)

Store 13

Brand j

Brand i	Iv 22	Iv 32	Iv 48	Jo 22	Jo 32	De 22
Ivory 12	-0.80	-0.17	0.24	-0.92	1.86	-1.08
Ivory 22	-0.20	0.63	-0.19	0.07	1.21	-0.92
Ivory 32	-0.23	-0.91	-0.39	-0.84	0.07	-3.69
Ivory 48	-0.34	-0.21	-1.05	0.28	2.30	-0.24
Joy 12	-1.15	-0.23	0.82	0.62	-0.68	-0.68
Joy 22	1.45	0.94	-1.14	-1.99	0.49	5.56
Joy 32	-1.40	-0.47	1.59	0.95	1.09	3.16
Joy 48	-0.47	-1.61	0.53	0.29	4.42	4.17
Ajax 22	2.00	-0.03	2.74	0.92	-1.12	-3.07
Ajax 32	2.99	0.83	-0.67	-6.59	-1.02	-4.18
Derm. 22	-1.86	0.93	1.58	0.03	-1.89	-9.57
Derm. 32	-1.01	1.29	0.86	4.39	0.17	1.41
Palm. 12	-1.29	-0.48	-1.70	-0.52	1.92	4.53
Palm. 22	1.92	3.36	-3.33	-0.14	-2.91	1.29
Palm. 32	2.41	0.56	2.80	2.39	2.95	0.48
Palm. 48	1.08	-0.73	-1.49	-1.60	-1.49	6.75
Dove	-0.15	-0.14	1.76	2.28	2.13	4.05
Lux 22	2.84	-0.27	-0.96	-1.57	4.62	1.16
Lux 32	-1.01	0.72	2.69	2.34	-1.40	1.41
Dawn 12	0.28	0.19	0.00	0.11	-1.76	0.09
Dawn 22	-0.10	1.12	-1.67	-1.80	1.66	-2.53
Dawn 32	-1.08	-0.82	1.34	0.40	2.05	-0.41
Dawn 48	0.25	-0.99	-0.44	0.32	1.54	3.32
Sunl. 12	0.04	0.76	-0.13	0.70	2.14	2.68
Sunl. 22	0.39	-0.93	-0.35	0.67	0.62	-3.54
Sunl. 32	0.70	-0.57	-3.08	-0.06	2.00	4.22
Store	-0.40	0.65	0.22	0.03	-1.94	0.01
Generic	0.04	-1.29	0.84	0.93	1.45	5.08

Brand j

Brand i	Pa 12	Pa 22	Pa 48	Lu 32	Da 22	Da 32
Ivory 12	-0.19	-2.52	0.42	-0.12	0.58	0.28
Ivory 22	0.27	1.03	-0.04	0.78	0.49	-2.57
Ivory 32	-1.77	1.14	0.61	-0.45	-0.32	-2.33
Ivory 48	0.73	-1.11	0.30	0.43	-0.34	-0.11
Joy 12	1.62	1.88	0.29	-0.54	1.08	0.86
Joy 22	-2.07	0.30	-0.26	-1.79	-0.25	0.62
Joy 32	-1.19	-0.74	0.20	2.29	-1.01	0.86
Joy 48	2.08	-0.31	0.14	2.84	0.50	1.05
Ajax 22	-6.88	-4.35	0.72	-3.10	-3.01	2.84
Ajax 32	4.21	-4.71	2.20	-2.69	-0.63	-0.79
Derm. 22	-7.24	7.42	-0.53	-2.05	0.89	-0.14
Derm. 32	-4.68	0.44	-3.75	0.51	-1.53	2.92
Palm. 12	-1.93	-1.20	1.76	-1.14	-0.38	1.39
Palm. 22	-1.31	0.89	-2.93	-0.79	0.26	-0.38
Palm. 32	-1.27	-6.55	0.95	0.81	1.90	2.51

Table VI-6 (continued)

	Brand j					
Brand i	Pa 12	Pa 22	Pa 48	Lu 32	Da 22	Da 32
Palm. 48	0.15	-2.91	-0.25	0.57	-0.65	0.57
Dove	1.41	-0.17	-1.61	4.59	-0.78	2.11
Lux 22	-4.73	-4.44	0.93	-1.21	-2.53	-0.39
Lux 32	-3.14	-3.40	-1.04	-1.02	-1.31	3.18
Dawn 12	-0.15	1.17	0.42	-0.13	0.78	-0.21
Dawn 22	1.84	-1.77	0.70	-1.90	-0.66	2.12
Dawn 32	-0.47	-1.18	0.44	1.32	-0.83	-0.65
Dawn 48	1.47	-0.66	0.43	1.18	1.10	1.05
Sunl. 12	1.62	-1.63	0.22	0.39	-1.29	1.31
Sunl. 22	0.77	2.64	0.77	0.34	1.02	-1.65
Sunl. 32	-0.00	-0.14	2.40	-0.29	0.98	0.22
Store	2.33	1.28	1.65	-0.39	2.55	-1.73
Generic	-0.15	0.46	-0.01	0.56	-0.11	1.74

	Brand j					
Brand i	Da 48	Su 12	Su 22	Store	Gnrc.	β_{0is}
Ivory 12	-0.43	-0.33	0.04	0.16	-1.13	8.26
Ivory 22	-1.54	-0.06	-1.64	3.49	-0.44	5.20
Ivory 32	0.13	0.27	-0.54	2.97	0.57	15.78
Ivory 48	-0.23	-0.73	-1.10	0.48	-1.12	5.16
Joy 12	-1.29	-1.44	-0.19	2.92	-1.71	-0.27
Joy 22	0.61	1.85	-0.13	1.68	-4.23	-1.84
Joy 32	-0.25	0.13	-0.33	-1.86	0.62	-2.40
Joy 48	-0.98	0.80	-1.58	-0.93	-2.97	-14.38
Ajax 22	-1.66	1.22	2.07	-3.34	1.38	22.07
Ajax 32	-0.06	-0.49	0.62	-1.60	-4.88	22.52
Derm. 22	0.26	2.96	3.59	1.85	6.42	6.12
Derm. 32	-1.95	-2.15	0.04	-0.43	2.07	7.41
Palm. 12	1.47	0.82	-0.16	-1.75	-2.98	1.49
Palm. 22	-0.49	-0.18	-0.19	-0.15	-0.43	12.16
Palm. 32	-0.52	-1.10	1.28	-0.56	-1.10	-12.19
Palm. 48	-0.94	-0.40	0.36	1.65	-4.07	5.90
Dove	1.43	-2.25	0.23	2.03	-3.23	-20.20
Lux 22	0.56	2.17	1.48	-0.36	-2.36	7.83
Lux 32	-1.55	-3.07	0.64	2.34	-0.97	11.30
Dawn 12	-0.28	-0.62	-0.16	1.15	-0.31	4.07
Dawn 22	0.59	1.86	-0.26	-1.84	-0.49	5.18
Dawn 32	-0.50	-0.54	0.42	1.62	-0.76	3.94
Dawn 48	-1.98	1.20	-1.11	-2.52	-1.72	-4.76
Sunl. 12	1.05	0.39	0.02	-2.36	-2.29	-7.15
Sunl. 22	-0.15	0.32	-1.63	3.05	-0.16	1.23
Sunl. 32	-2.54	2.39	-0.02	-3.43	-3.99	-1.26
Store	-1.93	-0.21	-1.24	-1.47	-0.23	3.49
Generic	-1.63	0.83	-0.56	-0.46	-2.71	-7.96

Table VI-6 (continued)

Store 14

Brand j

Brand i	Iv 22	Iv 32	Iv 48	Jo 22	Jo 32	Jo 48	Aj 32
Ivory 12	-0.36	-0.95	0.87	0.69	0.56	-0.97	0.72
Ivory 22	-1.62	-0.14	0.75	1.33	2.32	-0.77	0.14
Ivory 32	-1.59	-2.83	2.67	-1.32	0.43	-0.15	0.13
Ivory 48	-0.24	-1.55	-1.02	0.45	2.08	-0.18	-0.08
Joy 12	1.33	-2.69	3.02	-0.46	-0.12	1.47	-0.33
Joy 22	-2.23	0.66	1.35	-2.10	-0.39	0.86	-0.65
Joy 32	-1.23	2.83	2.41	-0.50	-1.11	0.47	0.77
Joy 48	-0.77	-2.65	4.93	0.76	5.41	3.01	0.63
Ajax 22	3.04	-2.36	2.39	1.59	-0.11	-8.20	0.01
Ajax 32	2.05	-2.75	7.40	-1.43	-10.21	0.43	2.06
Derm. 22	-0.47	3.27	0.29	-4.29	5.40	2.27	-2.41
Derm. 32	0.16	-0.55	1.06	3.68	7.08	-2.40	-0.88
Palm. 12	-2.61	0.58	0.09	-0.37	0.63	0.63	0.24
Palm. 22	0.65	-0.80	2.01	1.43	3.80	0.18	-0.58
Palm. 32	-2.33	-2.14	0.09	1.01	-2.58	-0.52	0.30
Palm. 48	-2.26	-1.08	1.64	0.26	0.60	-0.08	0.87
Dove	-1.48	-2.10	0.79	1.19	-1.59	-0.25	0.51
Lux 22	0.85	-1.38	3.45	-3.99	3.18	2.23	-0.20
Lux 32	0.46	-1.77	1.35	3.59	1.89	-1.32	-1.17
Dawn 12	-0.18	-1.51	1.39	-0.40	2.29	-0.07	-0.80
Dawn 22	0.13	-2.22	1.10	0.54	0.91	-0.16	-0.53
Dawn 32	-0.98	-1.72	1.75	-0.38	1.87	-0.55	0.56
Dawn 48	-0.92	-0.98	2.17	1.09	0.92	-1.06	-0.41
Sunl. 12	-1.37	-0.92	-1.66	-1.62	1.69	-0.47	-0.60
Sunl. 22	-1.62	0.07	2.37	-0.48	0.23	0.24	0.68
Sunl. 32	1.69	-4.82	2.20	3.65	6.75	-0.61	0.09
Sunl. 48	8.11	-9.05	3.42	3.16	-0.32	-3.70	3.03
Store	-0.14	-2.25	2.33	0.73	0.91	-0.49	0.13
Generic	-0.50	-0.56	2.26	1.07	0.96	-1.19	-0.21

Brand j

Brand i	De 22	Pa 12	Pa 22	Pa 32	Pa 48	Da 22	Da 32
Ivory 12	-2.66	-0.34	-1.41	-1.15	-0.54	-0.93	1.07
Ivory 22	-1.14	-1.76	-0.71	-2.34	-0.76	-0.63	1.47
Ivory 32	-2.57	-7.71	-1.79	-3.80	-0.15	0.82	-0.85
Ivory 48	-2.52	-5.12	-1.53	-3.50	0.89	-0.25	0.88
Joy 12	0.52	1.22	-1.13	-1.41	-0.76	3.20	1.94
Joy 22	-0.87	1.50	-2.60	-0.61	-1.95	-1.82	1.94
Joy 32	2.26	-7.57	-2.65	-3.52	0.77	-1.57	2.81
Joy 48	-1.36	-4.87	-2.74	-4.15	0.75	-1.75	3.23
Ajax 22	6.11	6.13	0.28	-1.08	-0.31	2.59	-3.42
Ajax 32	2.79	-5.71	3.20	-2.37	1.63	2.53	-1.05
Derm. 22	-4.27	0.63	-4.54	0.54	1.07	0.10	0.32
Derm. 32	-3.82	1.64	1.66	-0.70	3.14	3.84	-3.83
Palm. 12	-2.18	-2.05	-1.52	-0.26	0.96	-1.05	1.03
Palm. 22	-1.98	-1.28	-1.89	-2.22	-0.47	-0.86	0.88

Table VI-6 (continued)

Brand i	De 22	Pa 12	Pa 22	Pa 32	Pa 48	Da 22	Da 32
				Brand j			
Palm. 32	1.02	-3.01	-0.18	-4.91	0.24	-1.31	2.16
Palm. 48	-2.81	-3.44	-1.36	-0.94	-2.64	-2.30	2.13
Dove	-1.98	-7.30	-2.32	-3.46	-0.06	0.26	3.28
Lux 22	-0.66	0.52	-5.46	-2.29	2.60	1.28	2.60
Lux 32	0.85	-2.43	-0.52	-3.55	0.31	-1.75	-0.52
Dawn 12	-1.07	-1.41	-2.99	-2.39	-0.26	-0.36	1.79
Dawn 22	-2.81	-0.44	-2.64	-3.93	-2.13	-0.71	0.72
Dawn 32	-1.94	-4.63	-1.92	-3.83	0.38	0.87	-2.32
Dawn 48	0.01	-1.46	-3.05	-4.01	-0.78	-0.49	-0.00
Sunl. 12	-1.40	-8.04	0.84	-1.15	0.64	0.42	1.82
Sunl. 22	-3.32	-1.85	-0.88	-1.19	-0.48	0.23	1.20
Sunl. 32	-5.61	-1.19	-1.96	-2.23	-0.00	-2.29	-0.41
Sunl. 48	-6.98	-1.92	-7.74	-7.52	-0.62	5.82	0.74
Store	-1.99	-2.84	-2.27	-3.07	0.75	0.22	-0.01
Generic	-0.36	-1.98	-1.86	-3.04	-1.26	-0.51	1.35

Brand i	Da 48	Su 12	Su 22	Su 32	Store	Gnrc.	β^{0is}
					Brand j		
Ivory 12	0.29	-1.34	-0.36	-0.26	-0.12	-0.28	15.88
Ivory 22	0.16	-1.19	-0.96	-1.45	1.70	0.30	14.62
Ivory 32	1.50	-2.30	-0.96	-1.94	-2.73	0.30	46.72
Ivory 48	1.53	-1.46	-0.03	-2.26	0.29	-1.00	27.81
Joy 12	0.28	-2.01	2.40	-1.47	-4.40	0.83	-1.49
Joy 22	1.31	-1.04	-1.36	-1.32	-2.98	-0.78	23.43
Joy 32	2.44	-3.82	0.09	-1.19	4.18	-1.61	16.92
Joy 48	2.16	-3.96	-5.35	-3.12	-7.42	-0.62	28.68
Ajax 22	2.95	-5.59	4.16	1.22	-0.49	-2.87	-9.44
Ajax 32	-0.41	-1.93	0.60	1.38	2.15	2.50	3.19
Derm. 22	0.01	-0.30	-0.50	-1.54	-0.25	-1.04	10.13
Derm. 32	0.29	-4.55	-3.23	3.69	-8.86	-3.99	4.55
Palm. 12	1.30	-1.38	-1.91	-0.80	-0.24	1.19	16.70
Palm. 22	0.55	-2.95	0.10	-0.70	1.92	-0.87	9.70
Palm. 32	0.40	0.42	-1.75	-0.49	-0.67	-1.22	28.31
Palm. 48	1.05	-3.75	0.60	-2.27	1.60	-1.50	29.65
Dove	2.99	-2.65	0.07	-0.81	-3.34	-0.51	34.75
Lux 22	1.32	-5.80	3.92	-4.34	0.41	3.15	3.87
Lux 32	2.41	-4.66	0.11	-0.03	-6.62	-2.55	24.75
Dawn 12	1.37	-2.09	-0.57	-1.54	-0.12	-0.40	19.92
Dawn 22	0.71	-2.36	-0.77	-1.76	-2.48	-0.30	34.96
Dawn 32	1.15	-1.94	-2.47	-1.72	1.08	0.28	33.78
Dawn 48	-0.56	-3.47	0.08	-1.27	0.58	-0.95	28.32
Sunl. 12	-0.19	-8.58	-0.20	-1.21	-3.32	-0.85	47.30
Sunl. 22	1.32	-3.81	-3.46	-0.48	0.28	-0.87	24.23
Sunl. 32	-1.71	-2.87	-0.15	-3.87	4.84	-2.02	21.96
Sunl. 48	1.56	-3.17	11.72	2.78	-1.58	4.70	2.20
Store	0.68	-3.77	0.43	-1.90	-1.10	0.05	27.54
Generic	0.22	-3.32	-0.23	-0.89	0.30	0.36	20.80

cross effects and implies optimal values of the independent variable. Second, the functional form of the LMS model provides a ready mechanism for dealing with multicollinearity. Note that in the CPF, a separate parameter was estimated for the price of each item. If there is some systematic pattern to the pricing of different items, the significance and reliability of the parameter estimates are compromised. The LMS model uses the same set of independent variables, but fits only one parameter (β_{ps}) to all independent variables simultaneously.

To estimate the LMS model, we take advantage of the fact that, for store data, there is no "zero cell" problems: for each week (in general) there is at least one sale of each stocked item in each store. This being the case, a linearized form of the LMS can be approximated by picking one of the items (say k) as a baseline and model the market shares of the remaining items relative to that baseline item:

$$\log(MS_{ist}/MS_{kst}) = (\beta_{0is}-\beta_{0ks}) + \beta_{ps}(PPO_{ist}-PPO_{jst}) \tag{6.9}$$

Weighted least squares estimates of this linear form are generally referred to as minimum logit chi-square (Berkson 1955) and are equivalent to the estimators proposed by Nakanishi and Cooper (1974). It should be noted that some controversy exists in the marketing literature about the quality of these estimators compared to maximum likelihood (Bunch and Batsell 1989; Jones and Zufryden 1980). However, the estimates obtained from weighted least squares will be relatively good approximates of maximum likelihood estimates for large sample sizes (Flath and Leonard 1979).

A separate weighted least squares LMS model was estimated for each store, specifying as alternatives only those items that were in stock during the estimation weeks. In total, the LMS required estimates of 380 logit parameters, one for each item within each store. Parameter estimates are presented in Table VI-7.

Recovery of Item-Level Sales Prediction

The estimated parameters from each of the six models (the SLM, ULM, RSM, SPF, CPF, and the LMS) were used to generate predictions of weekly store-level item sales in the 1986-1987 store data. Sales predictions from the SPF and CPF were obtained by a straightforward application of the each model's parameters to the observed weekly store price data for 1986-1987. If an item was out of stock (indicated by missing shelf price), then the predicted sales for that item were set to zero. In certain instances, sales predictions from the SPF and CPF were enormous, sometimes predicting millions (or even billions) of unit sales for a single item within a given week. This problem was more apparent with the CPF than the SPF, and is indicative of residual multicollinearity problems in the calibration data despite the preventative measures taken in estimation. To reduce the problem of unreasonable predictions, the sales predictions of the SPF and CPF were constrained to be less than 1000 units for any item in any store during any given week. This is approximately 20% higher than the highest observed weekly store sales for any item in the calibration period. This constraining of predictions has the effect of inflating the actual fit of the SPF and the CPF. In total, 713 parameters were used to generate

Table VI-7

Logistic Market Share Model Parameter Estimates

		Store						
		1	2	3	4	5	6	7
Price		-0.71	-0.75	-0.78	-0.53	-0.56	-0.84	-0.59
Brand Constants								
Ivory	12	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Ivory	22	-0.08	-0.28	-0.28	0.04	0.16	-0.23	-0.00
Ivory	32	-0.88	-0.92	-0.97	-0.53	-0.61	-1.04	-0.44
Ivory	48	-1.91	-1.97	-1.92	-1.36	-1.43	-1.81	-1.42
Joy	12	0.02	0.05	0.17	0.06	0.21	-0.01	0.22
Joy	22	-0.47	-0.59	-0.57	-0.21	-0.05	-0.59	-0.25
Joy	32	-1.17	-1.20	-1.21	-0.87	-0.73	-1.25	-0.80
Joy	48	-2.34	-2.27	-2.22	-1.97	-1.82	-2.30	-1.99
Ajax	22	-2.12	-2.02	-1.77	-2.02	-1.77	-2.26	-1.80
Ajax	32	-2.38	-2.53	-2.55	-2.15	-2.21	-2.47	-1.80
Derm.	22	*	*	*	*	*	*	*
Derm.	32	-2.77	-2.77	-2.79	-2.03	-2.13	-2.96	-2.76
Palm.	12	-0.58	-0.54	-0.45	-0.25	-0.28	-0.75	-0.57
Palm.	22	-0.77	-0.94	-0.90	-0.39	-0.54	-1.11	-0.57
Palm.	32	-2.00	-2.15	-2.00	-1.71	-1.47	-2.33	-1.67
Palm.	48	-2.72	-2.69	-2.62	-2.33	-2.15	-2.80	-2.20
Dove		-2.49	-2.68	-2.83	-2.03	-2.60	-2.98	-2.26
Lux	22	-2.59	-3.55	-2.80	-2.62	-2.31	-3.05	-1.96
Lux	32	-2.56	-2.59	-2.66	-2.14	-2.08	-2.90	-2.14
Dawn	12	0.48	0.43	0.90	0.78	0.76	0.36	0.43
Dawn	22	-0.23	-0.27	-0.02	0.20	0.13	-0.46	-0.12
Dawn	32	-1.09	-1.13	-0.73	-0.69	-0.58	-1.42	-0.99
Dawn	48	-1.94	-1.97	-1.62	-1.63	-1.41	-2.65	-1.88
Sunl.	12	-0.86	-0.88	-0.56	-0.42	-0.51	-0.76	-0.48
Sunl.	22	-0.95	-1.06	-1.19	-0.42	-0.58	-1.17	-0.65
Sunl.	32	-1.46	-1.39	-1.61	-1.03	-0.92	-1.74	-0.92
Sunl.	48	-2.71	-2.76	-2.92	-2.18	-2.04	-2.70	-2.38
Store		-2.67	-2.75	-2.82	-1.74	-1.55	-3.18	-1.97
Generic		-4.91	-4.72	-4.60	-3.91	-3.50	-5.36	-4.53

Table VI-7 (continued)

	Store 8	9	10	11	12	13	14
Price	-0.68	-0.42	-0.46	-0.49	-0.27	-0.17	-0.61
Ivory 12	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Ivory 22	-0.06	-0.03	0.17	0.12	0.23	0.01	-0.24
Ivory 32	-0.84	-0.53	-0.50	-0.54	-0.21	0.15	-0.16
Ivory 48	-1.81	-1.56	-1.58	-1.64	-0.97	-0.53	-1.15
Joy 12	0.13	0.18	0.34	0.31	0.43	0.08	-0.89
Joy 22	-0.43	0.01	0.11	-0.01	0.30	-0.13	-0.15
Joy 32	-1.12	-0.81	-0.65	-0.60	-0.24	-0.00	-0.15
Joy 48	-2.44	-1.84	-1.57	-1.69	-1.08	-0.88	-1.89
Ajax 22	-2.03	-1.49	-0.94	-1.05	-1.43	-1.40	-1.50
Ajax 32	-2.05	-2.29	-2.08	-2.19	-1.65	-1.49	-2.99
Derm. 22	*	*	*	*	-1.41	-1.29	-1.79
Derm. 32	-2.53	-2.29	*	-2.09	-1.64	-2.10	-3.10
Palm. 12	-0.43	*	*	*	-0.22	-0.17	-1.52
Palm. 22	-0.66	-0.53	-0.29	-0.31	-0.31	-0.65	-0.66
Palm. 32	-1.96	-1.50	-1.16	-1.32	-1.23	-0.60	-1.14
Palm. 48	-2.77	-2.10	-1.99	-2.22	-2.07	-1.21	-1.72
Dove	-2.40	-2.23	-2.02	-2.14	-0.60	-0.61	-1.25
Lux 22	-3.22	*	*	*	-1.48	-1.44	-2.45
Lux 32	-2.32	*	*	*	-2.35	-1.61	-2.53
Dawn 12	0.69	*	*	*	0.84	1.10	1.00
Dawn 22	0.01	0.10	0.23	0.52	0.71	0.40	0.20
Dawn 32	-0.78	-0.93	-0.63	-0.47	-0.17	0.44	0.04
Dawn 48	-1.67	-1.81	-1.38	-1.40	-0.80	-0.26	-0.74
Sunl. 12	-0.48	*	*	*	-0.47	0.07	-1.37
Sunl. 22	-0.78	-0.49	-0.56	-0.56	-0.23	-0.12	-0.26
Sunl. 32	-1.49	-1.07	-1.15	-1.26	-0.71	-0.10	-0.80
Sunl. 48	-3.17	-2.37	-2.69	-2.88	-2.11	-1.81	-3.57
Store	-2.40	-1.11	-1.03	-1.31	-0.45	0.11	-1.72
Generic	-4.66	-2.37	-2.28	-2.66	-1.73	-1.12	-3.18

predictions from the SPF, and 6877 parameters were used to generate predictions from the CPF.

Sales predictions from the LMS were obtained by first predicting within-store item market share. If items were out of stock in the store in the prediction week, then predicted shares were computed only for the available items. This procedure is consistent with the assumption of independence of irrelevant alternatives. These market shares were then multiplied by the average observed weekly category sales for the store from the calibration period to obtain sales predictions. In total, 394 parameters (380 logit parameters and mean category sales for each of 14 stores) were used to generate predictions from the LMS.

Since the remaining models (ULM, SLM, and RSM) were originally calibrated on panel data, obtaining store-level predictions from these models was somewhat more complicated. Store-level item share predictions were recovered from the ULM by assuming that β parameters do not vary by store, since the model assumes a homogeneous (unsegmented) population. Within each store-week, P_{it}'s were evaluated at observed item price levels. In the case that an item was out of stock, the share predictions were constrained such that the item's choice probability was distributed proportionately among all the remaining items. This is consistent with an assumption of independence of irrelevant alternatives (IIA). As with the LMS, sales predictions were finally obtained by multiplying the P_{it}'s by the average observed category sales for the store in the calibration period. This assumption of constant category sales is consistent with the empirical finding that primary demand is unaffected by price level. In total, 43 parameters (29 logit parameters

and average category sales for each of 14 stores) were used to generate predictions from the ULM.

To recover predictions of observed store data from the SLM and the RSM, it was assumed that (1) the β parameters from the models do not vary by store, and (2) stores are homogeneous in the proportion of sales made to each segment. Store-level item market predictions were obtained with the following equation:

$$(\text{Predicted Market Share})_{ist} = \Sigma\, N_x P_{ixt} \tag{6.10}$$

where P_{ix} is evaluated at the observed levels of price for the items in store s at time t

N_x = the proportion of purchases assigned to segment x.

If a particular item was out of stock, the predicted within-segment purchase probability for the item was distributed proportionally among the remaining items in the segment's consideration set. This method for dealing with censored choices is consistent with an assumption of independence of irrelevant alternatives (IIA) at the segment level. To obtain sales predictions, these predicted market shares were multiplied by the average observed weekly category sales for the store in the calibration period. As with the ULM, this is assumption of constant category demand is consistent with the results of the primary demand model. In total, 218 parameters (156 logit parameters, 58 segment size estimates, and average category sales for each of 14 stores) were used to generate sales predictions from the SLM. Only 216 parameters were used to generate predictions from the RSM because the RSM structure contained 2 fewer segments.

It should be noted that the assumption of homogeneity of stores is rejectable for the SLM. Chi-square for the empirical segment-by-store purchase frequency matrix was 5894 (significant at .001), indicating a relationship between stores and segments. The assumption of homogeneity was still adopted for two reasons. First, preliminary analysis indicated that the sales predictions of the heterogeneous model (i.e., where store-level estimates of N_x were used) were nearly identical to the predictions of the homogeneous model. In fact, the homogeneous model slightly outperformed the heterogeneous model in fitting the observed sales data (r^2 of .655 versus .649). Second, by allowing interstore heterogeneity, the number of parameters needed to generate sales predictions goes up substantially from 218 to 1027.

Each of the six models generated predictions of unit sales for each item in each store in each week of the prediction period, resulting in 19,865 predictions for each model. Since each model was constrained to predict zero sales for out-of-stock items, the final predictive data set was reduced to the 16,425 non-zero predictions. Three different measures of predictive fit were computed between observed item sales and the predictions from each of the six models: predictive r, chi-square based on sales, and chi-square based on weekly store-level item market share. These fit statistics were computed on the overall data set, as well as within store, within item, within week, and within store-item. All fit statistics are presented in Tables VI-8, VI-9, and VI-10.

Several striking results can be found in the fit statistics. First, the proposed dissertation model (SLM) outperformed every tested model except the LMS. In general, the LMS predictions had the highest fit to the observed sales data, slightly outperforming the SLM (r of

Table VI-8

Predictive Correlations of Six Models

		SLM	RSM	ULM	SPF	CPF	LMS
	All	0.655	0.623	0.539	0.323	0.169	0.679
Store							
	1	0.750	0.693	0.573	0.589	0.141	0.742
	2	0.752	0.719	0.567	0.543	0.421	0.739
	3	0.717	0.637	0.524	0.624	0.308	0.723
	4	0.807	0.774	0.662	0.752	0.521	0.827
	5	0.759	0.726	0.580	0.688	0.469	0.722
	6	0.821	0.786	0.570	0.731	0.568	0.812
	7	0.697	0.629	0.583	0.631	0.184	0.686
	8	0.719	0.663	0.577	0.588	0.414	0.723
	9	0.574	0.569	0.511	0.563	0.216	0.668
	10	0.600	0.632	0.483	0.655	0.430	0.642
	11	0.609	0.622	0.563	0.749	0.602	0.778
	12	0.326	0.293	0.449	0.438	-0.071	0.502
	13	0.362	0.273	0.344	0.019	0.309	0.512
	14	0.652	0.562	0.511	0.167	0.336	0.609
Brand							
	1	0.332	0.498	0.226	0.103	-0.116	0.429
	2	0.450	0.490	0.162	0.581	0.400	0.522
	3	0.667	0.665	0.456	0.655	0.102	0.659
	4	0.461	0.406	0.327	0.305	-0.128	0.488
	5	0.309	0.283	0.198	0.417	-0.123	0.497
	6	0.508	0.492	0.356	0.443	0.244	0.490
	7	0.615	0.562	0.283	0.529	0.022	0.584
	8	0.560	0.475	0.266	0.026	0.033	0.542
	9	0.595	0.582	0.421	0.558	-0.112	0.642
	10	0.369	0.354	0.238	0.349	0.077	0.261
	11	0.701	0.731	0.688	0.691	-0.208	0.703
	12	0.551	0.545	0.409	0.795	0.825	0.770
	13	0.210	0.248	0.035	0.196	-0.015	0.265
	14	0.595	0.611	0.288	0.632	0.359	0.648
	15	0.581	0.574	0.337	0.471	0.202	0.549
	16	0.670	0.666	0.534	0.691	0.017	0.699
	17	0.268	0.276	0.187	0.112	0.054	0.220
	18	-0.025	-0.010	-0.069	0.113	0.037	0.367
	19	0.318	0.267	0.167	0.397	-0.032	0.379
	20	0.301	0.299	0.276	0.230	-0.079	0.426
	21	0.440	0.451	0.305	0.415	0.238	0.492
	22	0.621	0.624	0.453	0.560	0.270	0.708
	23	0.608	0.516	0.464	0.636	0.265	0.675
	24	0.211	0.322	0.057	0.170	-0.082	0.266
	25	0.646	0.684	0.393	0.681	0.243	0.687
	26	0.520	0.525	0.299	0.147	0.244	0.543
	27	0.486	0.504	0.150	0.462	0.021	0.471
	28	0.342	0.262	0.417	0.379	0.265	0.418
	29	0.609	0.592	0.604	0.635	-0.016	0.683

Table VI-9

Predictive Chi-Squares Based on Sales

	N	SLM	RSM	ULM	SPF	CPF	LMS
Overall	16425	170407	218158	248268	618176	4816746	163085

Store	N	SLM	RSM	ULM	SPF	CPF	LMS
1	1285	9719	11657	17492	11987	1146186	9793
2	1288	16540	17145	19943	19676	28411	17078
3	1288	29460	36802	47710	37372	236051	28984
4	1297	8608	11107	14555	10632	797547	8458
5	1286	13647	15479	19591	14937	33909	14146
6	1296	5496	5977	10809	7981	22937	5598
7	1292	7002	10450	10346	7681	127740	6615
8	1278	8355	10736	13336	10653	28999	8716
9	1056	5187	5610	6172	4796	19256	4727
10	1021	6818	6932	9394	5960	17920	6415
11	1065	9358	10516	11874	13643	18394	6619
12	1162	22319	28863	21592	18850	2140220	15521
13	725	8999	19048	16521	371289	27930	9375
14	1086	18908	27837	28934	82719	171247	21039

Brand	N	SLM	RSM	ULM	SPF	CPF	LMS
1	685	3696	2269	3419	26226	16448	3880
2	684	10604	12506	15977	16409	149749	12326
3	664	5682	5216	7110	5610	89847	6701
4	685	2827	3487	2398	2239	47942	1976
5	640	2786	4127	3683	3161	15149	3136
6	682	6750	11686	11218	11051	1849065	11263
7	683	4597	6088	8309	6160	50542	5531
8	642	4649	11776	15147	358906	78913	5665
9	236	810	1539	1075	1602	32260	807
10	470	3243	3791	1701	2168	22584	2249
11	118	1371	9352	4661	831	12296	460
12	113	332	338	329	191	258	203
13	486	6912	7393	8718	6306	12889	5596
14	685	18765	18937	35761	22995	73707	21528
15	683	7333	9018	15072	14439	712430	12905
16	670	1706	1814	2499	2140	48651	1638
17	679	6086	6609	7920	12534	90913	8959
18	336	11286	12839	14893	8147	24348	5520
19	442	1313	1875	1324	1216	28897	1186
20	532	7722	9963	6711	37802	61603	5350
21	682	14195	13927	14005	13498	153411	10744
22	681	5966	9411	9741	11553	135684	5757
23	682	3133	3994	3453	3188	32782	2876
24	486	2391	2161	2621	2578	88252	2259
25	685	12513	16476	18272	7881	579779	7756
26	683	12087	16646	19944	29740	130842	7278
27	434	3236	2391	3544	2390	25918	2438
28	685	6606	8517	5772	5212	212480	5279
29	592	1809	4009	2991	2003	39106	1820

Table VI-9 (continued)

Week	N	SLM	RSM	ULM	SPF	CPF	LMS
2	359	4115	4802	5790	8669	6122	3866
3	354	4052	4143	6695	5440	11886	2771
4	355	6655	8325	11197	47304	23829	7558
5	353	3303	4978	6128	30291	59265	3316
6	352	3017	4408	6154	11851	55460	3198
7	352	2973	4212	5415	22714	43806	3370
8	347	4240	6368	7781	31182	74408	6003
9	342	5521	7526	11784	56073	114326	5591
10	344	4127	5674	8265	32143	63531	4377
11	342	3343	4415	4986	38337	166927	2774
12	344	2871	4868	4808	56411	16509	2752
13	344	3004	5578	4399	59045	15616	3235
14	344	5729	7725	6689	16931	26138	4052
15	339	3815	4929	5683	15102	87352	2650
16	343	3063	3908	4817	7616	38519	2945
17	340	3193	4320	5052	5068	70443	2837
18	334	3478	4688	5724	7513	59318	3351
19	339	2949	4315	4961	5246	181964	3192
20	339	2493	3279	3737	3392	68926	2431
21	341	1863	2806	2905	3868	4874	1698
22	340	2169	3086	3277	3522	11175	2038
23	339	2226	2909	3090	3256	9251	2170
24	337	4170	4352	9471	5214	6863	4028
25	336	3839	4512	5406	4302	45722	3807
26	335	3451	4077	4881	4114	36167	3398
27	341	4370	4974	6190	6053	25114	3540
28	340	2666	3288	3352	5478	12752	2426
29	339	13250	16558	14873	14127	20498	9716
30	314	2423	3026	3465	3852	9142	2179
31	307	2353	3406	2957	4427	14276	2761
32	310	2059	2612	2068	3828	15217	2322
33	309	1889	2256	2839	3337	14389	1641
34	306	2079	2595	2882	4017	15598	2074
35	306	1947	2680	2497	3842	21715	1800
36	306	2054	2730	2458	3899	33839	1915
37	302	2161	2826	2310	4087	328830	1911
38	302	3408	3603	3046	4280	993143	2675
39	302	2803	3312	3345	4795	332923	2448
40	300	2999	3527	2867	4797	39319	2942
41	293	3260	3464	4548	4860	102129	3054
42	296	2427	2622	2851	4161	45258	2445
43	296	2629	2990	3115	4726	22820	3200
44	296	4445	5088	3688	5672	37337	3389
45	294	2746	3219	4108	6458	55431	2865
46	299	4297	5141	6018	8297	89772	4987
47	278	2504	3224	3521	5760	153619	2915
48	278	2343	2591	4033	4542	196996	2375
49	277	1971	2436	2765	3882	118230	1846
50	279	2359	2544	2378	4236	102275	2179
51	279	2063	2700	2556	4032	66329	2221
52	282	3243	4546	4445	6125	651399	3852

Table VI-9 (continued)

S	B	N	SLM	RSM	ULM	SPF	CPF	LMS
1	1	51	216	130	183	117	931	171
1	2	51	635	691	1210	1052	105568	818
1	3	51	296	212	362	206	31520	354
1	4	51	153	133	129	116	12532	72
1	5	51	113	188	207	175	445	116
1	6	50	191	792	528	1114	4265	727
1	7	51	203	159	862	159	2827	140
1	8	51	196	224	174	92	1824	111
1	9	1	12	17	13	4	5	3
1	10	51	265	338	102	236	11847	178
1	13	51	136	157	248	168	1324	126
1	14	51	2413	2581	5532	1786	1425	2112
1	15	51	608	967	1412	799	1804	1184
1	16	50	76	70	158	67	11066	67
1	17	51	150	104	166	108	2653	109
1	18	25	349	420	422	230	15628	184
1	19	51	144	295	162	164	22070	135
1	20	51	159	432	113	267	16726	160
1	21	51	177	192	333	856	91983	246
1	22	50	170	224	394	565	8534	183
1	23	51	152	149	136	98	11549	112
1	24	51	153	111	176	214	52555	246
1	25	51	637	970	1441	547	519343	488
1	26	51	1410	1581	2455	2322	12943	1342
1	27	51	432	319	405	280	12217	218
1	28	51	122	105	92	165	188804	110
1	29	38	135	85	66	72	3799	69
2	1	51	115	303	178	359	1046	898
2	2	51	1092	1842	1300	2123	2259	1937
2	3	50	1100	536	1012	831	524	1294
2	4	51	530	167	322	200	251	296
2	5	51	657	1252	871	800	363	735
2	6	51	860	2221	1669	1610	1521	1879
2	7	51	601	538	870	471	1052	650
2	8	51	478	165	336	211	642	217
2	9	1	19	26	19	10	2	8
2	10	51	617	676	146	321	1315	247
2	12	1	20	14	14	13	4	15
2	13	51	236	247	182	274	375	352
2	14	51	778	656	1414	670	1969	716
2	15	51	919	876	1172	878	970	945
2	16	51	170	164	324	139	826	143
2	17	50	551	339	402	298	1138	309
2	18	30	255	274	252	478	478	259
2	19	44	312	126	185	287	1042	347
2	20	51	620	412	714	4320	1498	670
2	21	51	926	1062	1276	1352	1350	1350
2	22	51	794	380	450	588	638	649
2	23	51	458	292	309	284	425	221
2	24	51	338	553	560	303	799	189
2	25	51	1304	1630	2143	497	3659	738

Table VI-9 (continued)

S	B	N	SLM	RSM	ULM	SPF	CPF	LMS
2	26	51	1390	1419	2659	1435	1879	933
2	27	50	639	400	625	292	930	314
2	28	51	517	510	475	237	1140	530
2	29	42	241	64	60	391	314	232
3	1	51	451	221	380	95	1272	80
3	2	51	1573	850	3497	523	10531	946
3	3	51	118	231	397	111	4971	175
3	4	51	73	567	157	279	1792	251
3	5	51	138	101	125	124	1876	137
3	6	51	493	598	365	702	2285	573
3	7	51	343	314	943	449	19193	371
3	8	51	71	503	456	177	7745	232
3	10	51	197	289	466	524	4742	746
3	13	51	233	302	501	209	539	185
3	14	51	5271	5601	9868	6971	12577	5877
3	15	51	2319	3318	4866	5342	39875	4495
3	16	51	63	94	64	96	1852	111
3	17	51	2274	3059	4062	3557	69683	4697
3	18	23	4901	4977	6167	4430	4430	1888
3	19	51	70	329	160	123	989	88
3	20	51	3523	5404	3066	2875	4108	1501
3	21	51	3128	2657	3226	2534	5202	1603
3	22	51	1115	3044	2831	3985	6863	784
3	23	51	271	1031	587	777	3415	442
3	24	51	367	201	267	593	2590	384
3	25	51	1072	825	2380	1296	21354	1798
3	26	51	466	621	1122	345	2385	362
3	27	51	402	387	499	564	1075	700
3	28	51	301	329	346	418	1256	400
3	29	41	225	947	910	272	3451	157
4	1	51	433	246	448	227	2100	94
4	2	50	386	250	1188	223	1727	302
4	3	51	217	352	197	186	397	189
4	4	51	86	950	144	132	746	111
4	5	51	94	223	154	114	622	93
4	6	51	404	427	446	356	1632	324
4	7	51	207	196	703	267	2766	241
4	8	51	97	355	263	153	6813	102
4	10	50	207	222	143	197	1202	172
4	13	51	90	83	216	114	353	107
4	14	51	1086	1114	2946	1506	32765	1213
4	15	51	643	917	1652	2600	639078	1855
4	16	50	44	52	105	69	9338	62
4	17	51	464	801	784	611	494	611
4	18	37	562	642	805	437	437	532
4	19	51	112	148	101	88	1243	92
4	20	51	579	433	422	310	787	173
4	21	51	736	714	727	855	1622	316
4	22	50	348	850	374	535	1365	402
4	23	51	90	187	90	172	3289	160
4	24	51	383	120	205	288	1354	256

Table VI-9 (continued)

S	B	N	SLM	RSM	ULM	SPF	CPF	LMS
4	25	51	382	481	1006	499	1370	423
4	26	50	358	525	877	344	83429	295
4	27	51	281	205	274	181	1580	169
4	28	51	211	521	239	123	529	122
4	29	41	99	92	47	44	511	43
5	1	51	119	180	176	317	395	501
5	2	51	868	1434	1347	2171	1494	1470
5	3	51	900	695	921	419	684	571
5	4	51	372	113	211	129	340	204
5	5	51	486	765	700	638	771	577
5	6	51	516	1650	1190	1567	1000	1514
5	7	51	387	517	723	467	1093	483
5	8	51	334	44	134	149	139	88
5	10	51	515	541	141	185	596	97
5	13	51	222	159	210	385	858	390
5	14	51	783	818	1765	1241	5280	947
5	15	51	359	317	678	330	1054	345
5	16	51	104	97	235	93	203	97
5	17	50	498	352	477	720	720	578
5	18	26	1936	2483	2984	1341	1341	1105
5	19	51	197	119	127	164	865	166
5	20	51	230	152	281	901	1334	523
5	21	51	538	601	600	698	477	751
5	22	51	541	258	382	363	961	473
5	23	51	310	146	199	113	545	153
5	24	51	162	275	369	176	642	148
5	25	51	966	1295	2254	755	4144	680
5	26	51	1457	1830	2687	1002	5664	1166
5	27	51	534	343	578	269	2030	286
5	28	51	149	119	120	210	993	642
5	29	37	149	167	94	123	278	181
6	1	51	78	82	89	181	311	284
6	2	51	311	283	1047	1767	4097	523
6	3	50	233	139	399	212	197	296
6	4	51	90	118	120	88	211	103
6	5	51	186	193	270	220	369	155
6	6	51	220	334	486	578	572	579
6	7	51	149	146	520	135	525	114
6	8	50	98	74	219	148	757	51
6	9	5	38	53	38	9	14	7
6	10	51	189	207	129	75	581	100
6	13	51	95	88	148	95	309	83
6	14	51	310	241	1167	424	1557	361
6	15	50	338	277	1077	915	574	750
6	16	49	69	73	133	68	478	65
6	17	49	261	215	287	228	677	251
6	18	36	567	624	741	348	348	386
6	19	51	72	113	88	82	465	60
6	20	51	142	138	178	160	564	119
6	21	51	265	260	561	468	1979	278
6	22	51	171	120	404	658	728	113

Table VI-9 (continued)

S	B	N	SLM	RSM	ULM	SPF	CPF	LMS
6	23	51	219	98	162	69	720	87
6	24	50	74	82	123	91	742	81
6	25	51	312	604	907	285	1287	203
6	26	50	464	956	986	260	893	152
6	27	48	203	214	293	174	3094	148
6	28	51	247	138	181	187	269	186
6	29	43	81	91	42	44	610	50
7	1	51	256	179	200	98	367	96
7	2	51	286	338	569	627	1363	361
7	3	50	223	195	174	305	329	343
7	4	51	71	479	134	119	586	73
7	5	51	66	81	91	122	379	104
7	6	50	260	354	341	441	1557	464
7	7	51	145	114	400	148	633	112
7	8	51	104	111	115	54	238	35
7	9	7	40	60	40	10	22	9
7	10	51	158	206	164	147	405	145
7	12	3	17	7	13	6	6	6
7	13	51	99	102	161	107	693	95
7	14	51	858	992	1254	1199	2541	1016
7	15	51	569	561	1196	916	5158	1032
7	16	49	53	54	60	55	202	49
7	17	51	178	307	307	272	2268	283
7	18	27	1651	2553	2313	314	314	349
7	19	48	122	520	252	130	493	111
7	20	51	159	336	110	244	1188	200
7	21	51	250	266	397	564	2027	247
7	22	50	161	272	161	344	86581	192
7	23	50	100	190	82	115	763	118
7	24	51	217	134	136	210	2551	169
7	25	51	254	667	517	313	9173	253
7	26	51	253	685	669	246	3423	202
7	27	51	204	239	266	292	1374	303
7	28	51	135	263	132	129	475	136
7	29	40	97	172	80	140	2620	100
8	1	51	258	99	216	95	316	86
8	2	51	256	577	452	2647	3212	602
8	3	51	289	213	301	391	392	269
8	4	51	137	62	77	381	189	56
8	5	51	117	295	206	199	247	142
8	6	50	218	589	445	351	1005	438
8	7	51	111	96	417	195	1124	93
8	8	51	200	74	97	111	96	57
8	10	50	288	418	208	233	395	253
8	13	51	90	85	197	115	469	92
8	14	51	1828	1718	3858	1527	5334	1727
8	15	51	367	532	1231	781	2656	1067
8	16	50	60	62	126	99	171	79
8	17	50	415	478	696	498	1273	579
8	18	28	412	391	582	264	264	467
8	19	51	99	116	88	128	335	119

Table VI-9 (continued)

S	B	N	SLM	RSM	ULM	SPF	CPF	LMS
8	20	51	606	1348	438	198	1303	332
8	21	48	434	526	281	499	493	623
8	22	50	307	973	499	494	1102	357
8	23	51	152	270	66	116	491	161
8	24	51	224	171	168	195	861	193
8	25	51	416	620	1016	413	1092	294
8	26	51	509	614	1220	177	2146	154
8	27	50	300	172	290	288	3560	257
8	28	51	145	185	110	185	331	164
8	29	35	104	38	39	61	128	43
9	1	51	98	85	66	77	148	263
9	2	51	339	377	478	422	1200	438
9	3	41	251	184	449	363	3432	365
9	4	51	163	102	124	86	282	90
9	5	51	100	192	105	200	757	311
9	6	51	312	406	402	377	2191	492
9	7	50	312	477	293	242	1189	290
9	8	48	251	202	311	255	548	165
9	9	47	121	158	225	77	514	127
9	10	13	91	131	12	20	20	15
9	12	51	103	67	60	66	66	60
9	14	51	517	436	624	292	755	326
9	15	50	109	90	143	95	746	85
9	16	45	84	91	95	78	240	81
9	17	50	125	68	59	47	104	51
9	21	51	187	191	340	269	479	277
9	22	51	238	227	440	428	1533	286
9	23	49	264	140	168	138	783	123
9	25	51	773	952	707	511	784	307
9	26	51	490	672	748	347	2380	177
9	28	51	154	216	109	282	789	217
9	29	51	94	136	205	117	305	169
10	1	51	210	115	216	80	350	152
10	2	51	249	361	351	275	545	424
10	3	49	405	503	563	387	1106	417
10	4	51	192	133	108	47	171	62
10	5	51	125	97	116	117	464	171
10	6	51	474	552	588	495	712	674
10	7	51	318	302	390	324	983	325
10	8	51	234	81	94	62	369	87
10	9	51	77	172	156	197	1085	86
10	10	3	33	40	10	32	41	9
10	14	51	682	615	1719	797	964	1227
10	15	51	141	156	214	182	3589	154
10	16	51	118	137	110	113	774	116
10	17	51	153	72	52	57	90	61
10	21	51	470	447	549	500	1023	403
10	22	51	532	538	952	611	2420	584
10	23	51	289	328	533	338	1118	365
10	25	51	1112	1181	1002	398	813	253
10	26	51	637	534	1117	546	883	485

Table VI-9 (continued)

S	B	N	SLM	RSM	ULM	SPF	CPF	LMS
10	28	51	293	251	170	264	182	189
10	29	51	65	305	375	128	231	160
11	1	51	427	148	425	138	455	76
11	2	51	184	663	249	184	925	361
11	3	45	335	332	490	331	759	331
11	4	51	232	141	202	133	2447	139
11	5	51	206	185	151	146	227	183
11	6	51	350	535	847	985	881	574
11	7	51	649	642	1071	667	511	685
11	8	51	387	612	759	6942	490	361
11	9	51	130	182	312	105	1901	135
11	12	51	95	189	149	99	164	108
11	14	51	525	555	1061	728	965	971
11	15	51	195	286	312	270	3153	288
11	16	51	123	138	112	89	721	102
11	17	51	173	69	56	59	91	60
11	21	51	1331	1110	867	419	737	395
11	22	51	457	614	1023	556	1118	448
11	23	51	255	275	385	298	528	263
11	25	51	1479	1545	1310	618	517	518
11	26	51	1281	1284	1599	518	1222	338
11	28	51	462	738	256	207	162	132
11	29	51	67	263	229	144	421	143
12	1	45	159	104	140	126	6293	130
12	2	45	1263	993	869	1092	11042	1046
12	3	45	363	314	399	307	26070	326
12	4	45	93	317	116	111	25873	99
12	5	45	170	197	290	233	8453	324
12	6	45	1467	1222	1313	1310	1816535	1232
12	7	45	370	454	184	985	14646	272
12	8	45	305	675	1349	1089	20148	444
12	9	45	238	504	204	672	28454	237
12	10	11	56	48	29	16	16	29
12	11	40	689	2860	1497	428	10443	241
12	12	2	4	1	4	2	2	2
12	13	45	5463	5992	6402	4652	6337	4028
12	14	45	868	894	691	1236	2698	1200
12	15	45	130	95	114	140	8710	128
12	16	43	137	156	94	67	19925	65
12	17	45	176	318	263	538	4916	295
12	18	45	182	193	232	171	469	203
12	19	21	85	18	63	18	1295	16
12	20	45	542	834	419	360	24822	313
12	21	45	3433	3585	1665	1594	16963	1515
12	22	45	439	924	296	292	9951	298
12	23	45	156	304	174	146	2738	129
12	24	45	272	278	248	307	26034	276
12	25	45	1193	1630	887	301	8135	318
12	26	45	1129	1462	674	704	2435	690
12	27	12	67	29	101	6	6	5
12	28	45	2672	3555	2567	1835	12879	1537

Table VI-9 (continued)

S	B	N	SLM	RSM	ULM	SPF	CPF	LMS
12	29	43	181	895	292	112	23926	110
13	1	28	58	56	44	47	48	60
13	2	28	921	978	618	1972	1672	1591
13	3	28	195	198	164	148	667	177
13	4	28	37	76	50	58	45	79
13	5	28	90	182	131	52	148	67
13	6	28	339	584	572	72	227	132
13	7	27	195	186	157	277	668	254
13	8	28	1614	7347	8073	341718	11085	3052
13	9	28	132	362	65	517	261	189
13	10	9	41	45	21	19	29	20
13	11	28	348	3234	888	140	879	93
13	12	1	7	4	8	2	4	2
13	13	28	151	134	357	182	1627	132
13	14	28	590	521	659	348	511	283
13	15	28	45	49	42	41.4	302	56
13	16	28	42	87	58	76.3	121	46
13	17	28	42	40	77	2958.7	312	101
13	18	9	16	16	29	32.7	45	27
13	19	19	28	47	32	28.8	93	33
13	20	28	311	312	230	170.7	256	205
13	21	28	906	878	1043	1480.7	7476	1299
13	22	28	322	557	544	149.4	193	162
13	23	28	137	233	245	141.4	130	89
13	24	28	58	112	162	176.8	99	294
13	25	28	983	1020	1019	323.7	320	378
13	26	28	683	924	483	19959	313	242
13	27	16	126	62	132	22.9	22	17
13	28	28	497	482	415	125.3	307	213
13	29	28	70	307	188	49.8	56	68
14	1	51	813	315	653	24265	2411	984
14	2	51	2234	2864	2797	1328	4109	1502
14	3	51	753	1108	1278	1407	18794	1590
14	4	51	591	123	499	358	2473	337
14	5	6	231	171	260	16	26	16
14	6	51	639	1419	2020	1090	14681	1655
14	7	51	601	1940	771	1370	3330	1496
14	8	12	272	1305	2761	7741	28015	658
14	10	28	583	624	125	160	1391	233
14	11	50	332	3256	2274	263	974	124
14	12	4	84	55	78	2	11	8
14	13	5	91	42	93	4	4	3
14	14	51	2252	2189	3198	4264	4362	3545
14	15	51	584	572	957	1144	4757	516
14	16	51	555	531	819	1026	2729	553
14	17	51	619	383	227	2579	6490	967
14	18	50	451	262	363	98	591	116
14	19	4	69	41	63	0	5	14
14	20	51	847	158	737	27992	9014	1150
14	21	51	1405	1432	2136	1406	21598	1437
14	22	51	365	425	985	1981	13695	821

Table VI-9 (continued)

S	B	N	SLM	RSM	ULM	SPF	CPF	LMS
14	23	51	273	344	311	379	6285	446
14	24	6	138	120	205	22	22	18
14	25	51	1625	3051	1677	1120	7785	1099
14	26	51	1555	3533	2643	1533	10844	734
14	27	3	44	18	79	16	26	18
14	28	51	695	1099	554	842	4360	696
14	29	51	193	441	359	300	2451	290

Table VI-10

Predictive Chi-Squares Based on Market Share

	N	SLM	RSM	ULM	SPF	CPF	LMS
Overall	16425	220.972	305.080	350.604	1973.47	124548	207.220

Brand	N	SLM	RSM	ULM	SPF	CPF	LMS
1	685	8.448	4.903	8.046	11.10	22	2.436
2	684	12.108	11.946	19.471	29.34	698	12.933
3	664	5.762	6.328	7.730	7.14	179	6.114
4	685	2.475	7.101	2.788	4.37	42	2.104
5	640	3.572	3.900	4.164	3.81	29	2.513
6	682	11.835	13.007	13.039	14.78	116838	11.861
7	683	4.958	5.530	11.395	6.84	49	4.893
8	642	7.383	23.430	30.918	1634.49	120	11.138
9	236	1.843	2.409	3.261	5.14	15	2.668
10	470	3.311	4.161	2.430	3.97	54	3.266
11	118	3.082	18.482	9.693	2.97	5	0.866
12	113	0.577	1.165	0.921	0.95	2	0.794
13	486	2.664	3.002	4.749	3.22	52	2.136
14	685	24.269	24.792	49.630	55.71	222	35.263
15	683	9.762	12.140	23.037	23.51	1307	19.110
16	670	3.304	3.312	4.575	3.64	45	2.914
17	679	6.507	8.142	9.910	12.04	88	10.362
18	336	15.810	19.138	21.872	11.09	76	8.185
19	442	1.512	3.645	2.071	1.74	50	1.348
20	532	7.719	14.688	5.538	17.00	101	4.099
21	682	22.976	22.794	17.735	29.60	820	15.017
22	681	8.389	16.280	20.016	19.38	755	9.438
23	682	4.186	7.627	6.525	5.90	139	4.996
24	486	3.121	2.082	2.694	3.60	175	3.520
25	685	14.280	18.394	22.786	11.71	857	8.899
26	683	14.150	19.295	24.697	34.04	229	10.396
27	434	3.947	3.258	5.030	4.59	45	3.760
28	685	9.819	13.837	6.984	8.48	1504	4.170
29	592	3.192	10.281	8.886	3.30	33	2.011

Store	N	SLM	RSM	ULM	SPF	CPF	LMS
1	1285	11.89	14.95	22.22	15.34	3344	12.08
2	1288	13.55	15.86	25.06	21.48	56	15.43
3	1288	17.94	24.15	30.92	25.35	287	17.42
4	1297	9.78	12.81	16.82	12.14	1425	9.43
5	1286	13.75	17.45	24.87	17.13	45	14.65
6	1296	12.63	14.40	27.89	18.04	62	13.34
7	1292	15.26	23.28	22.64	15.94	802	14.13
8	1278	13.78	18.47	21.32	17.73	56	14.29
9	1056	21.42	24.48	28.96	20.81	146	18.17
10	1021	17.29	17.83	24.85	13.99	53	15.41
11	1065	15.94	17.93	19.94	21.72	28	10.47
12	1162	25.57	36.51	21.62	20.65	118061	15.95
13	725	16.82	36.48	31.75	1668.59	55	17.69
14	1086	15.37	30.38	31.68	84.57	128	18.70

Table VI-10 (continued)

Week	N	SLM	RSM	ULM	SPF	CPF	LMS
2	359	5.801	7.272	8.701	24.40	9	5.134
3	354	6.305	7.026	10.342	9.10	26	3.945
4	355	7.564	9.811	12.481	140.17	43	8.075
5	353	5.591	8.721	10.887	142.75	589	5.280
6	352	4.742	7.382	10.873	27.82	801	5.108
7	352	4.104	6.384	7.851	98.43	410	3.961
8	347	6.551	10.357	12.937	132.22	278	9.281
9	342	5.359	8.372	12.297	177.93	639	5.455
10	344	4.912	8.005	10.148	137.47	585	4.781
11	342	5.821	8.319	8.598	218.92	892	4.251
12	344	4.314	8.213	7.747	266.67	42	3.889
13	344	3.937	8.134	5.997	199.47	30	4.182
14	344	8.107	11.968	9.595	70.67	61	5.517
15	339	4.991	7.116	8.080	67.92	863	2.950
16	343	4.390	5.975	7.364	19.51	396	4.164
17	340	4.352	6.321	7.226	6.86	448	3.637
18	334	5.617	7.569	9.515	17.88	118	5.238
19	339	2.852	4.693	5.960	6.00	276	3.249
20	339	3.342	4.856	5.589	3.75	113	2.982
21	341	2.610	4.225	4.330	5.18	6	2.328
22	340	2.828	4.458	4.955	4.57	18	2.671
23	339	3.023	4.655	4.998	4.39	17	2.823
24	337	5.289	5.907	12.932	6.08	12	5.210
25	336	5.155	6.415	8.044	5.50	56	5.081
26	335	5.016	5.976	7.817	5.58	63	4.771
27	341	6.484	7.796	9.886	11.69	61	5.396
28	340	4.166	5.838	6.014	14.87	26	3.367
29	339	5.079	6.559	6.881	10.14	26	3.726
30	314	3.371	4.368	5.801	4.15	12	3.055
31	307	2.976	4.847	4.458	4.76	42	3.594
32	310	2.785	3.936	2.902	4.44	37	3.047
33	309	2.517	3.215	4.484	3.72	35	1.937
34	306	2.750	3.900	4.817	5.15	27	2.829
35	306	2.546	3.828	3.786	4.51	42	2.360
36	306	2.816	4.722	4.441	4.56	845	2.781
37	302	2.964	4.343	3.442	4.43	20438	2.227
38	302	4.811	5.703	4.358	4.49	33712	3.219
39	302	3.442	4.058	4.176	5.55	41645	2.871
40	300	4.518	5.447	4.118	6.98	981	3.874
41	293	4.110	4.533	6.211	7.17	9579	4.474
42	296	3.680	4.047	4.709	6.57	2540	3.513
43	296	3.736	4.566	4.710	6.99	72	4.727
44	296	7.520	8.641	5.921	8.75	883	5.010
45	294	4.321	5.193	6.694	10.29	160	4.348
46	299	6.097	7.640	9.841	11.76	3109	7.627
47	278	3.388	4.854	5.672	8.30	477	4.429
48	278	3.066	3.599	6.489	4.96	637	3.224
49	277	2.304	3.359	4.367	3.99	396	2.212
50	279	3.265	3.601	3.278	4.67	905	2.655
51	279	2.743	4.050	3.806	4.50	186	3.258
52	282	2.948	4.293	4.066	6.80	882	3.485

Table VI-10 (continued)

S	B	N	SLM	RSM	ULM	SPF	CPF	LMS
1	1	51	0.459	0.265	0.428	0.140	6	0.129
1	2	51	0.622	0.535	1.434	0.980	565	0.652
1	3	51	0.301	0.277	0.416	0.270	124	0.391
1	4	51	0.133	0.229	0.202	0.250	28	0.074
1	5	51	0.110	0.168	0.189	0.180	5	0.116
1	6	50	0.240	0.807	0.544	1.020	15	0.746
1	7	51	0.298	0.225	1.184	0.200	3	0.216
1	8	51	0.211	0.380	0.272	0.210	8	0.178
1	9	1	0.016	0.023	0.016	0.000	0	0.004
1	10	51	0.302	0.401	0.156	0.378	21.8	0.293
1	13	51	0.205	0.265	0.383	0.237	3.8	0.178
1	14	51	3.077	3.426	6.845	2.823	9.8	2.765
1	15	51	0.797	1.314	2.061	1.945	7.3	1.665
1	16	50	0.091	0.097	0.175	0.097	15.8	0.099
1	17	51	0.181	0.186	0.282	0.201	2.6	0.205
1	18	25	0.524	0.623	0.665	0.271	39.0	0.226
1	19	51	0.136	0.406	0.186	0.125	41.0	0.131
1	20	51	0.314	0.855	0.221	0.303	81.3	0.281
1	21	51	0.442	0.364	0.525	0.799	399.6	0.456
1	22	50	0.213	0.461	0.812	1.406	12.4	0.317
1	23	51	0.150	0.242	0.162	0.165	75.3	0.170
1	24	51	0.304	0.168	0.261	0.555	145.6	0.495
1	25	51	0.580	0.951	1.573	0.551	734.0	0.478
1	26	51	1.441	1.715	2.530	1.670	43.8	1.350
1	27	51	0.490	0.359	0.500	0.315	19.3	0.287
1	28	51	0.095	0.091	0.097	0.149	928.7	0.090
1	29	38	0.146	0.111	0.091	0.096	9.8	0.080
2	1	51	0.541	0.200	0.542	0.242	1.015	0.149
2	2	51	1.143	0.904	2.071	1.238	4.876	0.902
2	3	50	0.270	0.311	0.529	0.298	0.726	0.381
2	4	51	0.151	0.218	0.091	1.102	0.509	0.143
2	5	51	0.145	0.410	0.241	0.196	0.349	0.160
2	6	51	0.541	0.918	0.887	0.602	1.670	0.793
2	7	51	0.317	0.299	0.987	0.386	1.992	0.268
2	8	51	0.321	0.545	1.275	0.373	2.019	0.348
2	9	1	0.017	0.024	0.017	0.008	0.002	0.007
2	10	51	0.320	0.417	0.237	0.598	3.313	0.431
2	12	1	0.017	0.011	0.011	0.012	0.001	0.012
2	13	51	0.180	0.167	0.406	0.215	0.330	0.124
2	14	51	1.346	1.548	3.659	2.038	7.868	1.469
2	15	51	1.576	2.107	3.027	2.996	1.965	2.781
2	16	51	0.168	0.176	0.238	0.212	1.092	0.196
2	17	50	0.692	0.825	1.104	1.207	4.094	0.948
2	18	30	0.866	0.757	1.218	0.470	0.483	0.946
2	19	44	0.153	0.279	0.183	0.220	1.072	0.167
2	20	51	0.571	0.884	0.464	2.284	1.304	0.598
2	21	51	0.865	0.813	1.189	0.746	1.191	0.753
2	22	51	0.327	0.713	0.590	0.597	1.766	0.502
2	23	51	0.221	0.299	0.230	0.206	0.343	0.385
2	24	51	0.210	0.195	0.221	0.310	1.623	0.391
2	25	51	0.601	0.608	1.956	1.505	7.696	0.513

Table VI-10 (continued)

S	B	N	SLM	RSM	ULM	SPF	CPF	LMS
2	26	51	1.250	1.185	2.598	1.542	3.841	1.105
2	27	50	0.503	0.554	0.661	1.117	1.986	0.710
2	28	51	0.136	0.152	0.127	0.618	2.533	0.151
2	29	42	0.085	0.336	0.289	0.128	0.311	0.088
3	1	51	0.145	0.079	0.166	0.163	1.464	0.186
3	2	51	0.690	0.805	1.999	1.143	5.849	0.626
3	3	51	0.441	0.192	0.540	0.430	5.003	0.402
3	4	51	0.134	0.265	0.116	0.127	1.618	0.116
3	5	51	0.154	0.335	0.273	0.258	3.802	0.161
3	6	51	0.530	1.432	0.839	1.345	1.343	1.088
3	7	51	0.193	0.176	0.701	0.238	20.830	0.174
3	8	51	0.175	0.274	0.257	0.098	11.320	0.131
3	10	51	0.327	0.451	0.222	0.474	7.660	0.496
3	13	51	0.108	0.109	0.255	0.172	0.870	0.108
3	14	51	2.393	2.699	5.228	3.403	8.530	2.948
3	15	51	0.966	1.519	2.560	3.104	46.080	2.244
3	16	51	0.068	0.071	0.165	0.078	5.010	0.066
3	17	51	1.716	2.169	2.909	2.355	58.650	3.305
3	18	23	3.075	3.132	3.951	3.364	8.080	1.063
3	19	51	0.122	0.171	0.107	0.175	1.220	0.103
3	20	51	1.610	2.915	1.310	1.631	10.250	0.507
3	21	51	1.563	1.353	1.542	1.458	6.920	1.015
3	22	51	0.448	1.644	1.736	2.620	10.610	0.411
3	23	51	0.135	0.539	0.308	0.397	3.520	0.220
3	24	51	0.183	0.164	0.254	0.290	7.950	0.179
3	25	51	0.917	1.180	2.159	0.796	40.270	0.839
3	26	51	0.958	1.322	2.067	0.503	5.520	0.315
3	27	51	0.628	0.481	0.599	0.415	1.240	0.483
3	28	51	0.154	0.156	0.151	0.168	2.430	0.146
3	29	41	0.092	0.514	0.499	0.128	11.170	0.085
4	1	51	0.419	0.219	0.468	0.209	2.09	0.117
4	2	50	0.417	0.349	1.492	0.346	2.05	0.393
4	3	51	0.358	0.415	0.265	0.234	0.46	0.292
4	4	51	0.088	1.038	0.142	0.128	0.77	0.098
4	5	51	0.147	0.409	0.267	0.189	0.59	0.140
4	6	51	0.442	0.716	0.573	0.565	1.45	0.499
4	7	51	0.205	0.186	0.770	0.255	4.26	0.229
4	8	51	0.176	0.431	0.320	0.207	8.15	0.126
4	10	50	0.340	0.348	0.162	0.249	2.01	0.187
4	13	51	0.086	0.125	0.224	0.181	0.39	0.180
4	14	51	1.008	1.050	3.128	1.424	46.42	1.138
4	15	51	0.613	0.912	1.801	2.888	1189.8	1.987
4	16	50	0.060	0.063	0.163	0.075	11.66	0.070
4	17	51	0.565	0.896	0.911	0.714	0.73	0.701
4	18	37	0.612	0.691	0.838	0.604	0.93	0.582
4	19	51	0.180	0.144	0.135	0.129	1.90	0.137
4	20	51	0.511	0.361	0.347	0.443	0.58	0.166
4	21	51	0.778	0.775	0.719	0.895	1.13	0.379
4	22	50	0.373	0.861	0.379	0.540	0.99	0.390
4	23	51	0.132	0.179	0.108	0.190	3.73	0.143
4	24	51	0.412	0.180	0.244	0.340	2.11	0.270

Table VI-10 (continued)

S	B	N	SLM	RSM	ULM	SPF	CPF	LMS
4	25	51	0.541	0.829	1.321	0.568	1.16	0.538
4	26	50	0.563	0.856	1.417	0.359	138.18	0.325
4	27	51	0.439	0.277	0.420	0.196	2.57	0.190
4	28	51	0.123	0.409	0.150	0.125	0.54	0.084
4	29	41	0.173	0.086	0.050	0.072	0.58	0.061
5	1	51	0.185	0.135	0.194	0.126	0.440	0.147
5	2	51	0.509	0.564	1.434	0.937	0.830	0.599
5	3	51	0.304	0.216	0.500	0.168	0.550	0.167
5	4	51	0.142	0.254	0.113	0.125	0.430	0.099
5	5	51	0.169	0.304	0.276	0.317	0.280	0.211
5	6	51	0.321	0.753	0.594	0.770	0.660	0.644
5	7	51	0.199	0.219	0.674	0.235	0.750	0.229
5	8	51	0.128	0.157	0.279	0.054	0.261	0.040
5	10	51	0.272	0.312	0.225	0.260	0.649	0.235
5	13	51	0.109	0.121	0.208	0.226	0.951	0.151
5	14	51	1.851	1.851	3.897	3.254	8.657	2.879
5	15	51	0.382	0.407	1.255	0.699	1.158	0.737
5	16	51	0.095	0.122	0.139	0.106	0.382	0.098
5	17	50	0.530	0.619	0.895	1.827	1.788	1.480
5	18	26	4.080	5.139	6.091	3.066	4.301	2.404
5	19	51	0.122	0.490	0.206	0.145	1.715	0.126
5	20	51	0.319	0.770	0.219	0.872	0.265	0.148
5	21	51	0.860	0.928	0.682	0.630	2.259	0.688
5	22	51	0.238	0.418	0.586	0.530	0.827	0.229
5	23	51	0.125	0.201	0.130	0.118	1.147	0.111
5	24	51	0.193	0.128	0.195	0.196	0.985	0.205
5	25	51	0.862	0.760	2.641	0.972	6.691	1.244
5	26	51	1.046	1.237	2.138	0.771	3.881	1.064
5	27	51	0.379	0.367	0.745	0.432	4.213	0.466
5	28	51	0.235	0.543	0.291	0.181	0.453	0.151
5	29	37	0.080	0.430	0.260	0.101	0.187	0.086
6	1	51	0.290	0.235	0.317	0.225	0.799	0.331
6	2	51	0.979	0.484	3.104	1.486	1.979	0.543
6	3	50	0.408	0.502	1.019	0.417	1.412	0.482
6	4	51	0.174	0.500	0.440	0.269	1.452	0.190
6	5	51	0.238	0.244	0.363	0.353	1.539	0.208
6	6	51	0.641	0.512	0.997	0.759	0.947	0.805
6	7	51	0.328	0.238	1.214	0.237	4.314	0.187
6	8	50	0.134	0.306	0.726	0.208	3.753	0.150
6	9	5	0.071	0.101	0.071	0.009	0.079	0.012
6	10	51	0.282	0.322	0.408	0.220	4.085	0.209
6	13	51	0.208	0.275	0.428	0.376	0.939	0.225
6	14	51	1.498	1.156	4.029	2.027	7.129	1.801
6	15	50	0.945	0.776	2.993	3.154	2.588	2.358
6	16	49	0.144	0.137	0.242	0.185	0.661	0.163
6	17	49	0.626	0.683	0.870	0.791	2.034	0.896
6	18	36	1.505	1.653	2.059	1.026	1.660	0.818
6	19	51	0.179	0.555	0.347	0.186	0.701	0.189
6	20	51	0.196	0.672	0.226	0.315	0.341	0.277
6	21	51	0.845	0.854	1.447	0.865	1.402	0.827
6	22	51	0.267	0.579	1.260	2.473	3.402	0.451

Table VI-10 (continued)

S	B	N	SLM	RSM	ULM	SPF	CPF	LMS
6	23	51	0.330	0.168	0.259	0.349	4.758	0.321
6	24	50	0.268	0.136	0.229	0.317	4.175	0.352
6	25	51	0.535	0.765	2.033	0.556	4.773	0.522
6	26	50	0.724	1.606	1.717	0.473	1.991	0.315
6	27	48	0.346	0.338	0.683	0.333	4.025	0.323
6	28	51	0.332	0.228	0.227	0.238	0.256	0.272
6	29	43	0.121	0.366	0.175	0.178	0.923	0.106
7	1	51	0.782	0.560	0.607	0.320	1.690	0.259
7	2	51	0.592	0.653	1.243	1.149	1.601	0.697
7	3	50	0.449	0.415	0.349	0.637	0.599	0.674
7	4	51	0.174	1.233	0.363	0.303	0.902	0.187
7	5	51	0.141	0.164	0.172	0.300	0.653	0.195
7	6	50	0.580	0.768	0.698	0.865	3.681	0.929
7	7	51	0.333	0.259	0.772	0.310	1.399	0.244
7	8	51	0.208	0.303	0.278	0.147	0.380	0.086
7	9	7	0.081	0.125	0.082	0.017	0.034	0.019
7	10	51	0.330	0.422	0.457	0.336	1.460	0.324
7	12	3	0.037	0.015	0.027	0.012	0.012	0.013
7	13	51	0.229	0.235	0.378	0.270	2.330	0.220
7	14	51	1.892	2.200	2.917	2.242	8.875	2.272
7	15	51	0.924	0.910	2.256	1.728	20.308	1.851
7	16	49	0.126	0.131	0.122	0.119	0.325	0.114
7	17	51	0.402	0.733	0.738	0.701	8.929	0.684
7	18	27	3.882	5.998	5.388	0.826	1.625	0.814
7	19	48	0.254	1.173	0.554	0.217	1.476	0.226
7	20	51	0.390	0.855	0.263	0.596	0.990	0.503
7	21	51	0.521	0.583	0.719	0.887	9.621	0.526
7	22	50	0.322	0.643	0.373	0.763	680.88	0.450
7	23	50	0.209	0.493	0.180	0.340	2.529	0.314
7	24	51	0.514	0.256	0.301	0.485	8.952	0.401
7	25	51	0.541	1.357	1.074	0.641	30.040	0.599
7	26	51	0.478	1.396	1.343	0.425	4.244	0.376
7	27	51	0.445	0.512	0.597	0.744	5.088	0.716
7	28	51	0.233	0.529	0.223	0.257	0.680	0.228
7	29	40	0.180	0.346	0.160	0.286	2.592	0.199
8	1	51	0.639	0.247	0.582	0.275	1.293	0.106
8	2	51	0.342	0.659	0.783	2.393	2.074	0.686
8	3	51	0.364	0.427	0.401	0.603	0.996	0.373
8	4	51	0.153	0.132	0.110	0.507	0.290	0.076
8	5	51	0.141	0.354	0.241	0.191	0.472	0.168
8	6	50	0.376	0.761	0.596	0.453	0.811	0.575
8	7	51	0.191	0.159	0.699	0.413	3.479	0.185
8	8	51	0.264	0.159	0.189	0.171	0.245	0.118
8	10	50	0.417	0.609	0.378	0.424	0.695	0.411
8	13	51	0.173	0.154	0.415	0.231	1.441	0.133
8	14	51	2.606	2.607	4.978	2.598	6.985	2.575
8	15	51	0.671	1.006	2.232	1.895	8.320	2.003
8	16	50	0.093	0.099	0.180	0.233	0.175	0.160
8	17	50	0.560	0.669	0.966	0.821	1.649	0.819
8	18	28	0.856	0.808	1.206	0.754	0.992	0.967
8	19	51	0.138	0.275	0.168	0.123	0.422	0.158

Table VI-10 (continued)

S	B	N	SLM	RSM	ULM	SPF	CPF	LMS
8	20	51	1.600	3.185	1.240	0.837	4.455	0.910
8	21	48	0.920	1.026	0.498	0.765	1.067	1.034
8	22	50	0.688	2.198	1.231	1.537	4.276	0.762
8	23	51	0.214	0.534	0.105	0.250	1.615	0.287
8	24	51	0.434	0.282	0.284	0.386	1.281	0.351
8	25	51	0.562	0.814	1.512	0.609	2.758	0.461
8	26	51	0.662	0.815	1.718	0.283	3.526	0.218
8	27	50	0.399	0.224	0.413	0.667	5.577	0.501
8	28	51	0.164	0.208	0.131	0.224	0.492	0.182
8	29	35	0.139	0.056	0.058	0.071	0.302	0.056
9	1	51	1.859	1.491	1.771	1.080	1.863	0.261
9	2	51	1.167	1.018	1.613	1.957	4.507	1.121
9	3	41	0.542	0.473	1.026	0.906	36.076	0.781
9	4	51	0.372	0.952	0.418	0.409	1.939	0.369
9	5	51	0.603	0.307	0.431	0.402	0.785	0.343
9	6	51	2.504	1.283	1.500	1.960	34.395	1.137
9	7	50	0.711	1.005	0.809	0.941	1.658	0.683
9	8	48	0.893	1.788	2.889	1.042	5.144	1.315
9	9	47	0.738	0.469	1.564	0.372	3.886	1.238
9	10	13	0.228	0.349	0.018	0.037	0.491	0.028
9	12	51	0.282	0.641	0.424	0.701	1.589	0.495
9	14	51	1.078	1.031	1.812	1.017	1.935	1.543
9	15	50	0.364	0.452	0.650	0.451	1.902	0.586
9	16	45	0.342	0.423	0.422	0.373	0.606	0.358
9	17	50	0.236	0.226	0.182	0.203	1.376	0.216
9	21	51	1.690	1.984	1.574	1.581	3.132	1.051
9	22	51	1.237	1.890	3.800	3.560	24.635	2.675
9	23	49	0.659	0.571	0.605	0.561	3.125	0.587
9	25	51	1.614	2.040	1.570	1.246	9.831	0.989
9	26	51	1.108	1.445	2.052	1.064	3.377	1.496
9	28	51	2.185	2.870	1.189	0.472	1.263	0.367
9	29	51	0.992	1.763	2.632	0.461	2.824	0.527
10	1	51	1.216	0.634	1.317	0.349	0.467	0.213
10	2	51	0.404	0.477	0.427	0.391	1.840	0.475
10	3	49	0.817	1.520	0.979	0.798	3.611	0.787
10	4	51	0.290	0.583	0.156	0.191	0.539	0.190
10	5	51	0.743	0.250	0.695	0.274	0.499	0.191
10	6	51	1.257	0.997	0.982	0.993	1.496	1.071
10	7	51	0.551	0.558	0.810	0.581	1.566	0.575
10	8	51	0.369	0.338	0.416	0.156	1.023	0.160
10	9	51	0.237	0.240	0.550	0.329	4.132	0.228
10	10	3	0.061	0.076	0.015	0.063	0.431	0.014
10	14	51	1.347	1.224	4.105	1.864	2.101	3.256
10	15	51	0.494	0.690	0.765	0.502	13.542	0.612
10	16	51	0.373	0.413	0.321	0.350	2.012	0.378
10	17	51	0.218	0.121	0.111	0.103	0.210	0.111
10	21	51	1.468	1.064	1.324	1.418	2.631	1.003
10	22	51	1.313	1.368	2.941	1.647	7.121	1.742
10	23	51	0.946	1.311	2.166	1.075	2.087	1.481
10	25	51	1.876	2.011	1.738	0.689	3.456	0.621
10	26	51	1.178	0.989	2.216	1.601	3.231	1.797

Table VI-10 (continued)

S	B	N	SLM	RSM	ULM	SPF	CPF	LMS
10	28	51	1.786	1.586	1.143	0.400	0.441	0.294
10	29	51	0.340	1.375	1.663	0.205	1.447	0.206
11	1	51	1.172	0.455	1.225	0.368	1.293	0.081
11	2	51	0.244	0.711	0.260	0.266	2.074	0.355
11	3	45	0.589	0.623	0.693	0.556	1.996	0.536
11	4	51	0.300	0.244	0.302	0.242	4.290	0.273
11	5	51	0.572	0.480	0.452	0.277	0.472	0.204
11	6	51	0.609	0.696	1.055	1.278	1.811	0.696
11	7	51	0.918	0.894	1.823	0.932	1.479	0.961
11	8	51	0.487	0.913	1.139	11.587	1.245	0.482
11	9	51	0.229	0.220	0.597	0.170	3.695	0.266
11	12	51	0.164	0.452	0.383	0.221	0.441	0.260
11	14	51	0.730	0.861	1.738	1.112	1.985	1.828
11	15	51	0.330	0.549	0.535	0.461	5.320	0.551
11	16	51	0.196	0.256	0.167	0.186	1.175	0.215
11	17	51	0.205	0.102	0.080	0.092	0.649	0.088
11	21	51	2.833	2.372	1.561	0.442	1.992	0.490
11	22	51	0.712	1.072	1.674	0.812	1.422	0.684
11	23	51	0.454	0.649	0.855	0.519	1.455	0.596
11	25	51	2.015	2.112	1.787	0.922	1.067	0.825
11	26	51	1.796	1.798	2.293	0.808	2.276	0.675
11	28	51	1.256	1.855	0.743	0.285	0.615	0.237
11	29	51	0.112	0.616	0.569	0.216	1.281	0.160
12	1	45	0.367	0.163	0.158	0.10	3	0.092
12	2	45	2.122	1.502	1.226	1.81	102	1.653
12	3	45	0.268	0.215	0.303	0.17	1	0.185
12	4	45	0.126	0.764	0.120	0.12	2	0.107
12	5	45	0.189	0.193	0.281	0.28	15	0.339
12	6	45	3.015	2.133	2.447	2.17	116773	2.204
12	7	45	0.285	0.412	0.143	1.26	2	0.161
12	8	45	0.361	1.129	2.467	2.00	1	0.656
12	9	45	0.283	0.741	0.216	1.15	3	0.354
12	10	11	0.095	0.082	0.048	0.03	0	0.047
12	11	40	1.078	4.989	2.647	0.47	1	0.258
12	12	2	0.007	0.002	0.008	0.00	0	0.004
12	13	45	1.024	1.335	1.243	0.69	38	0.588
12	14	45	1.393	1.532	1.102	2.34	74	2.454
12	15	45	0.228	0.295	0.316	0.52	5	0.462
12	16	43	0.192	0.225	0.124	0.13	5	0.119
12	17	45	0.271	0.645	0.456	0.72	4	0.391
12	18	45	0.159	0.174	0.198	0.15	16	0.172
12	19	21	0.138	0.030	0.100	0.04	0	0.037
12	20	45	1.323	2.088	0.568	0.37	0	0.340
12	21	45	6.830	7.171	2.659	2.69	374	2.410
12	22	45	0.469	1.504	0.475	0.43	5	0.270
12	23	45	0.288	0.796	0.247	0.21	38	0.156
12	24	45	0.446	0.373	0.383	0.51	1	0.480
12	25	45	1.744	2.504	1.218	0.28	14	0.296
12	26	45	1.456	2.015	0.825	1.01	12	0.773
12	27	12	0.115	0.050	0.175	0.01	0	0.009
12	28	45	0.895	1.414	0.821	0.80	563	0.772

Table VI-10 (continued)

S	B	N	SLM	RSM	ULM	SPF	CPF	LMS
12	29	43	0.387	2.026	0.636	0.18	2	0.151
13	1	28	0.164	0.133	0.098	0.70	0.136	0.078
13	2	28	1.898	1.987	1.240	13.95	2.419	3.465
13	3	28	0.270	0.358	0.150	0.68	0.889	0.168
13	4	28	0.061	0.205	0.066	0.24	0.090	0.084
13	5	28	0.072	0.175	0.116	0.57	0.270	0.059
13	6	28	0.400	0.739	0.713	0.79	0.495	0.149
13	7	27	0.213	0.205	0.163	0.23	1.002	0.292
13	8	28	3.264	14.430	15.892	1600.35	23.542	6.111
13	9	28	0.166	0.462	0.144	3.07	0.622	0.537
13	10	9	0.055	0.061	0.027	0.07	0.059	0.026
13	11	28	0.783	6.627	1.905	1.53	1.242	0.225
13	12	1	0.010	0.006	0.012	0.00	0.008	0.003
13	13	28	0.283	0.190	0.749	0.62	3.568	0.224
13	14	28	0.831	0.745	0.922	2.35	0.903	0.683
13	15	28	0.100	0.124	0.094	0.76	0.808	0.067
13	16	28	0.149	0.122	0.226	0.67	0.397	0.153
13	17	28	0.088	0.140	0.265	1.86	0.805	0.135
13	18	9	0.057	0.054	0.112	0.14	0.156	0.103
13	19	19	0.040	0.092	0.039	0.36	0.124	0.062
13	20	28	0.748	0.751	0.550	1.87	0.318	0.192
13	21	28	2.148	2.108	2.177	9.67	14.335	2.998
13	22	28	0.852	1.418	1.587	1.35	0.353	0.297
13	23	28	0.166	0.427	0.555	0.80	0.326	0.087
13	24	28	0.066	0.120	0.185	0.15	0.256	0.369
13	25	28	1.413	1.470	1.469	0.20	0.445	0.501
13	26	28	0.928	1.294	0.587	22.65	0.677	0.267
13	27	16	0.171	0.080	0.181	0.31	0.071	0.037
13	28	28	1.214	1.185	1.013	2.15	0.282	0.218
13	29	28	0.196	0.765	0.504	0.45	0.121	0.086
14	1	51	0.202	0.081	0.167	6.805	0.265	0.279
14	2	51	0.972	1.291	1.138	1.290	1.309	0.760
14	3	51	0.375	0.377	0.551	0.979	1.878	0.488
14	4	51	0.170	0.476	0.143	0.349	0.204	0.089
14	5	6	0.141	0.100	0.161	0.010	0.059	0.011
14	6	51	0.373	0.485	0.606	1.204	1.536	0.518
14	7	51	0.210	0.690	0.638	0.618	2.348	0.483
14	8	12	0.385	2.271	4.510	17.888	53.857	1.231
14	10	28	0.277	0.305	0.073	0.829	11.167	0.559
14	11	50	1.221	6.866	5.141	0.975	2.552	0.381
14	12	4	0.057	0.036	0.053	0.000	0.007	0.004
14	13	5	0.053	0.020	0.055	0.002	0.006	0.001
14	14	51	3.212	2.855	5.263	27.210	37.361	7.645
14	15	51	1.365	1.073	2.486	2.390	2.298	1.201
14	16	51	1.200	0.970	1.885	0.816	1.077	0.718
14	17	51	0.210	0.119	0.133	0.432	0.863	0.376
14	18	50	0.189	0.104	0.141	0.404	2.337	0.085
14	19	4	0.044	0.025	0.040	0.003	0.001	0.007
14	20	51	0.131	1.348	0.126	7.478	0.478	0.172
14	21	51	1.207	1.394	1.112	6.762	1.515	1.380
14	22	51	0.921	1.505	2.566	1.113	0.806	0.253

Table VI-10 (continued)

S	B	N	SLM	RSM	ULM	SPF	CPF	LMS
14	23	51	0.151	1.212	0.608	0.711	0.474	0.133
14	24	6	0.088	0.075	0.134	0.056	0.049	0.021
14	25	51	0.472	0.985	0.727	2.164	0.564	0.465
14	26	51	0.557	1.614	1.189	0.883	2.697	0.312
14	27	3	0.027	0.010	0.052	0.045	0.015	0.032
14	28	51	1.005	2.603	0.674	2.412	1.519	0.972
14	29	51	0.143	1.485	1.295	0.729	0.244	0.113

.679 versus .655, frequency chi-square 163,085 versus 170,407, market share chi-square 207.22 versus 220.97). However, the SLM provided better chi-square fit than the LMS for 12 of the 29 brands, 7 of the 14 stores, and 17 of the 51 weeks. Thus, the SLM compares quite favorably with the best store model in predicting store sales, even though the SLM parameters were based on panel data, and as such are prone to sampling error. Further, the SLM used fewer parameters to generate predictions than the LMS (218 vs. 380). One might still argue that the LMS model is preferable to the SLM on the basis that the LMS is much easier to estimate and requires only store data to for calibration. However, unlike the SLM, the LMS cannot convey segment information beyond the level of the store. As such, the LMS model is less useful for examining behavior at a more disaggregate level.

The third and fourth best fitting models were the RSM and ULM, respectively. While the RSM was relatively close to the SLM in terms of r (.623 versus .655), the RSM chi-squares were substantially higher than the SLM (218,158 versus 170,407 for sales; 305 versus 221 for market share). The RSM chi-square fit was actually closer to the ULM than to the SLM: going from the ULM to the RSM reduced the overall market share chi-square from 350.6 to 305.1, while the SLM reduced it further to 221. This indicates that the inclusion of additional parameters to a simple panel choice model, while helpful, cannot account alone for the increased fit of the SLM. Thus, the increased fit of the SLM compared to the RSM appears to be much more than an artifact of the number of choice parameters.

The two remaining models had generally poor fit to the observed data, especially the CPF (r = .169, frequency chi-square = 4,816,746,

market share chi-square = 124,548). Upon closer inspection, a large proportion of the chi-square for the CPF is due to its inability to fit particular groups of cells (for example, store 12). However, even if these cells are eliminated, the CPF remains the poorest fitting model. It is tempting to attribute the lack of fit on the part of the power function models to the fact that the other models constrain primary demand to be constant. However, if the data are normalized to eliminate the effect of primary demand (as in the case of market share chi-square) the SPF and the CPF are still the poorest fitting models.

The poor fit of the power function models obtains despite the fact that they fit the calibration data rather well: the average calibration multiple r^2 for the item level models was .902 for the CPF and .830 for the SPF. The results here indicate that while power-function models perform very well in fitting calibration data, the presence of multicollinearity drastically hampers their predictive validity. In most applications of these models, predictive validity is seldom examined (e.g., Reibstein and Gatignon 1984). It would seem prudent, based on these results, to examine multicollinearity and predictive fit before investing much faith in the results of models of this type.

Validity of IIA Assumptions

A final set of analyses were performed to test the validity of assumptions regarding independence of irrelevant alternatives (IIA). Recall that the SLM is assumed to obey IIA within segment, while the ULM is assumed to obey IIA over all choices. To test the validity of both

these assumptions, the McFadden, Train, and Tye (1977) test was employed on the empirical data.

The McFadden, Train, and Tye test for IIA proceeds by first computing the probability of choice for each alternative for each observed choice occasion. One of the alternatives (say i) is selected and the choice records are sorted in order of predicted choice probability for i. The records are then divided into M cells, with an equal number of observations in each cell. A chi-square statistic is computed over all M cells:

$$X^2 = \sum^{M} (O_m - E_m)^2 / E_m$$

where O_m = the observed number of choices of j in m

E_m = the expected number of choices of j in m, computed by the average predicted probability

This process is then repeated for each alternative. X^2 has an assymptomatic distribution bounded by chi-squares with M-1 and M-K-1 degrees of freedom, where K = the number of alternatives.

Separate tests of IIA assumptions were conducted for the ULM and for each multi-item segment identified in the SLM. In each test, the number of cells (M) was chosen as to be twice the number of alternatives. For example, M was chosen as 58 for the ULM and the all-item segment (30) from the SLM. Segment 31 from the SLM contained four items, so M was set at 8 for the IIA test for segment 31. This approach was adopted so that each test would have an adequate number of degrees of freedom. Results of the IIA tests are presented in Table VI-11.

Results of the IIA test for the ULM indicate that IIA is rejectable at the .05 level for each item at the lower d.f. level (27). At the higher d.f. level (57), IIA is still rejectable for all but seven items. Such severe violations of call into question the use of a simple logit model calibrated on the panel data.

Turning to the segment level SLM test, IIA appears to be rejectable for the all-item segment (30). At the lower d.f. level, IIA is rejectable for all but one item, and is rejectable for all but 13 items at the higher d.f. level. This is not particularly surprising, as the segment is theoretically composed of households with the full choice set as well as those consumers with censored choice sets that were not well-represented by the identified SLM partitions. The Bayesian segment assignment method tends to lump all of these households into the all-item segment. However, the results indicate that the chi-squares for the all-item segment are substantially lower than the corresponding chi-squares for the ULM for nearly every brand, even though the tests are based on the same number of cells and the same set of alternatives. This result gives some indication that IIA violations are reduced by the segmentation of individual households.

IIA test results for the remaining segments are much more encouraging. In 16 of the 28 multi-item segments, IIA could not be rejected at either d.f. level for any of the constituent items. In the remaining 12 segments, IIA could only be rejected at the lower d.f. level, and then only for a subset of the items. The only multi-item segments in which there appeared to be consistently strong violations of IIA were segment 40 (all 22-ounce items), segment 54 (22 and 32-ounce medium and strong power) and segment 57 (22 and 32-ounce medium and

Table VI-11

McFadden, Train, and Tye Test for IIA Assumptions

* = Significant at .05, lower d.f.
* * = Significant at .05, both lower and higher d.f.

Unstructured Logit Model

Item	X^2	Item	X^2	Item	X^2
1	50.66*	11	779.77**	21	86.37**
2	162.47**	12	97.00**	22	88.03**
3	115.76**	13	89.07**	23	97.55**
4	86.24**	14	175.40**	24	72.63*
5	65.61*	15	100.61**	25	128.81**
6	152.50**	16	96.19**	26	355.59**
7	72.51*	17	99.59**	27	157.57**
8	69.08*	18	535.57**	28	51.01*
9	691.50**	19	103.48**	29	63.39*
10	96.77**	20	76.36**		

Test for IIA Within Segment: Structured Logit Model

Segment 30

Item	X^2	Item	X^2	Item	X^2
1	61.13*	11	170.71**	21	74.50*
2	77.40**	12	48.74*	22	59.49*
3	55.37*	13	100.67**	23	76.37**
4	53.22*	14	115.67**	24	71.76*
5	59.45*	15	91.80**	25	82.51**
6	118.26**	16	75.91*	26	107.01**
7	63.04*	17	74.37*	27	152.20**
8	36.96	18	327.53**	28	54.44*
9	221.51**	19	56.41**	29	48.40*
10	69.74*	20	76.78**		

Table VI-11 (continued)

S	I	X^2
31	1	2.42
	2	4.99
	3	4.48
	4	3.62
32	5	2.11
	6	3.46
	7	5.17
	8	2.27
33	9	1.13
	10	.82
34	11	5.48*
	12	4.85*
35	13	11.03*
	14	2.83
	15	10.27*
	16	3.21
36	18	0.54
	19	2.18
37	20	4.01
	21	8.62*
	22	10.27*
	23	1.82
38	24	7.13
	25	5.68
	26	18.40**
	27	7.72
39	1	5.14
	5	6.96
	13	5.15
	20	8.16
	24	6.71
40	2	17.00*
	6	14.69*
	9	28.11**
	11	21.61*
	14	25.37**
	18	62.19**
	21	22.40*

S	I	X^2
41	3	11.45
	7	23.28*
	10	22.01*
	12	31.61*
	15	18.08
	17	17.90
	19	11.30
	22	15.15
	26	13.05
	28	11.40
	29	21.05*
42	4	6.30
	8	3.14
	16	5.47
	23	5.26
	27	8.73
43	1	7.43
	2	3.38
	3	10.88
44	2	1.66
	3	3.88
	4	0.75
45	6	1.64
	7	1.41
46	13	0.85
	14	1.61
47	14	4.09*
	15	5.38*
48	15	1.27
	16	0.54
49	20	2.39
	21	1.01
50	21	1.69
	22	0.70
51	22	0.37
	23	0.52

S	I	X^2
52	26	1.03
	27	2.55
53	5	20.31*
	6	22.80*
	9	19.93*
	13	15.25
	14	12.70
	20	15.46
	21	9.18
	24	19.85*
	25	13.94
54	2	2.99
	3	32.89*
	6	67.26**
	7	78.52**
	9	59.74**
	10	30.37*
	14	58.62**
	15	16.78
	17	34.15**
	21	21.01
	22	27.43*
	25	16.54
	26	26.54*
55	3	9.33
	4	18.74*
	7	9.98
	8	22.01*
	15	19.90*
	16	13.84
	25	14.01
	26	20.89*
56	5	10.43*
	6	6.45
	24	6.90
	25	7.50

S	I	X^2
57	2	35.51*
	3	27.93*
	6	28.70*
	7	29.83*
	11	56.60**
	12	19.37
	14	38.53**
	15	27.24*
	17	25.19*
	18	126.30**
	19	18.82
	25	20.25
	26	41.28**
58	28	2.59
	29	1.10

gentle power). Over all multi-item segments (excluding 30), IIA was not rejectable for 77 items, rejectable at the lower d.f. level only for 35 items, and rejectable at the higher d.f. level for only 15 items. If segments 40, 54, and 57 are excluded, the violations drop to 21 and 3 for the lower and higher d.f. levels respectively. It might be added that IIA violation were somewhat more prevalent in the a priori segments (30-42) than in the segments that were identified via overlapping clustering.

Thus, it appears that the assumption of IIA is much more robust in the SLM than in the simple ULM. Violations of IIA were found to be widespread in the ULM, while violations of IIA only occurred in a few isolated segments in the SLM. These results suggest that the SLM approach is better suited to situations where there exist a large number of alternatives or a strong structure. For example, Guadagni and Little (1983) performed the same IIA test on a logit model for the coffee data set. Their model assumed IIA and that all households had fully enumerated choice sets. Results of Guadagni and Little's IIA test were not sufficient to reject IIA. However, it should be noted that Guadagni and Little's model was calibrated on only ground, caffeinated brands of coffee. Other researchers (Hutchinson and Zenor 1987; Grover and Srinivasan 1987) have identified this subset of brands as a significant partition in the coffee market. As such, there is good reason to believe that IIA violations would have been much more severe had their model been calibrated on all items of coffee (decaffeinated, freeze-dried, etc.). The SLM approach would be much more amenable to an analysis of the larger coffee market.

CHAPTER VII

SEGMENT CHARACTERISTICS AND ELASTICITIES

In this chapter, two additional analyses of the identified segments are performed. First, the households assigned to each of the segments are profiled in terms of demographic characteristics. In the second analysis, the self-price elasticity of sales each item are compared both on the margin and within segment. The analysis of segment characteristics lends additional face validity to the assignment method, and the elasticity analysis confirms hypotheses regarding the effect of choice set size on price sensitivity.

Segment Characteristics

The parameter estimates from the segmented logit model revealed that the segments varied considerably in terms of price sensitivity. While this is encouraging evidence that the model was successful in identifying segments that differed in terms of behavior, it is still unclear whether the identified segments are managerially actionable.

Fortunately, data are available for examining this issue. The original UPC tape included a file containing demographic descriptor variables for each of the panel households. Included in this file were such variables as household income, number of children, weekly expenditures on groceries, media usage, and ownership of appliances.

To examine whether the segments varied on the basis of demographic as well as behavioral variables, the mean levels of three demographic variables were computed for each segment. The demographic variables utilized in this analysis were household income, weekly grocery expenditures, and ownership of an automatic dishwasher. These profiles are presented in Table VII-1.

The segment profiles give a preliminary indication that the segments differ on demographic as well as behavioral variables, and that these differences lend additional face validity to the segments. Note first that single-item loyal households have a significantly higher mean income (\$28,397 versus \$26,962), lower weekly grocery expenditures (\$63.31 versus \$68.33), and are much more likely to own automatic dishwashers (63.7% versus 54.0%) than the households assigned to the switching segments. These results are very consistent with a priori expectations. As a higher proportion of a household's income is spent on groceries, one would expect that the household would seek to economize by engaging in comparison shopping and brand switching. Further, those households without automatic dishwashers would tend to purchase LDD items more frequently and spend a higher proportion of household income within the LDD category. Households with high purchase frequency might be seen as more familiar with the available alternatives in the category, as well as having greater opportunity and motivation to engage in brand switching behavior and product trial.

Further segment differences can be found within the households assigned to the single-item loyal segments (1 through 29). Those households assigned to 12-ounce single-item loyal segments were more likely to own automatic dishwashers than households assigned to 22, 32,

Table VII-1

Demographic Profile of Households Assigned to Each Segment

Segment	Price β	Average Income	Weekly Grocery $	Proportion owning dishwasher	N assigned
1	0	27167	58.9	.75	16
2	0	27940	64.3	.67	45
3	0	30663	71.7	.71	52
4	0	25448	65.2	.53	32
5	0	30947	60.8	.76	21
6	0	29484	59.3	.64	36
7	0	27395	58.9	.57	23
8	0	33500	76.1	.50	16
9	0	30250	68.5	.58	12
10	0	27562	73.4	.56	9
11	0	25000	53.7	.33	3
12	0	30818	66.0	.68	25
13	0	31875	55.8	.50	4
14	0	25345	58.4	.38	32
15	0	31538	56.5	.77	13
16	0	33594	72.9	.69	16
17	0	28500	75.5	.67	15
18	0	21500	52.6	.40	5
19	0	31500	66.7	.75	16
20	0	28031	57.3	.73	41
21	0	28565	60.8	.60	57
22	0	28784	60.9	.68	41
23	0	24817	59.2	.47	34
24	0	25000	54.9	.71	14
25	0	28635	64.5	.81	52
26	0	30181	65.6	.66	35
27	0	24056	57.1	.63	19
28	0	27052	65.1	.56	43
29	0	25455	64.9	.53	11
30	-.740	26525	64.9	.53	312
31	-.663	36500	87.0	.50	6
32	-.118	29423	72.4	.63	27
33	-.046	26500	62.0	.71	7
34	-6.621	20833	49.5	.25	4
35	-.890	26326	76.7	.42	23
36	-.763	25500	98.1	.22	9
37	-.612	25315	75.9	.54	52
38	-.814	28244	66.5	.62	89
39	-.118	28415	67.4	.59	46

Table VII-1 (continued)

Segment	Price β	Average Income	Weekly Grocery \$	Proportion owning dishwasher	N assigned
40	-.473	27006	63.6	.56	103
41	-.711	24278	69.1	.62	39
42	-.810	26437	73.8	.35	17
43	-.314	27972	63.2	.60	94
44	-.661	30576	72.2	.56	27
45	-.570	27938	74.9	.53	47
46	-.744	30156	77.6	.56	16
47	-.524	29750	76.4	.52	44
48	-.867	26944	73.4	.45	11
49	-.212	27920	71.7	.58	50
50	-.398	25528	68.8	.55	56
51	-.788	22545	67.6	.43	23
52	-1.016	27981	67.4	.33	33
53	-.599	25963	65.8	.61	89
54	-.593	27407	71.1	.50	114
55	-.798	29333	76.5	.55	31
56	-.919	23227	61.2	.64	28
57	-.675	26172	68.7	.48	153
58	-3.067	23791	55.8	.67	12

or 48-ounce single-item loyal segments (72.7% versus 62.7%, 65.4%, and 55.4% respectively). On average, the 12-ounce single-item loyal households also had lower weekly grocery expenditures ($57.92 versus $61.83, $65.84, and $62.88). Among the single-item loyal households, there were no significant size-based differences in income. Again, the demographic profiles are consistent with expectations. Households with automatic dishwashers, for example, may only use liquid detergents for cleaning of a few special items, such as non-dishwasher safe containers or good silverware. These households would have little need to maintain a large inventory of LDD items, and thus would be more likely to purchase smaller sizes when buying in the category.

Additional relationships between demographic variables and segment behavior were found within the multi-item loyal switching segments by correlating the price sensitivity β_p parameter for each segment with the mean levels of income, grocery expenditures, incidence of automatic dishwasher ownership, and consideration set size. Significant positive correlations were found between the price sensitivity parameter and income ($r = .485$, $prob(r) = .008$), grocery expenditures ($r = .483$, $prob(r) = .008$), and dishwasher ownership ($r = .413$, $prob(r) = .026$). The results indicate that households with higher incomes, grocery expenditures, and incidence of dishwasher ownership were assigned to less price-sensitive switching segments (smaller negative β_p's). The relationships between price sensitivity and income, and price sensitivity and dishwasher ownership were in the expected direction. However, the relationship between grocery expenditures and price sensitivity was counter to expectations, as households with higher grocery expenditures were found to be less sensitive to price. This is

partially explained by the significant positive relationship between household income and grocery expenditures. The relationship between the price sensitivity parameter and consideration set size was very small (.05), but consideration set size was marginally associated with income ($r = -.173$, $prob(r) = .195$), dishwasher ownership ($r = -.184$, $prob(r) = .167$), and grocery expenditures ($r = .202$, $prob(r) = .128$). These results are in the expected direction: households with higher incomes and higher incidences of automatic dishwasher ownership were determined to have smaller consideration sets. Households with higher weekly grocery expenditures (and presumably greater motivation for comparison shopping) were determined to have larger consideration sets.

An Analysis of Variance (ANOVA) test found that 59.2% of the variance in switching segment price sensitivities could be accounted for by the average segment income, grocery expenditures, and dishwasher ownership alone ($F = 8.938$, $prob(F) = .0001$). Thus, while the original segments were formed on the basis of behavioral (consideration set) similarity, segment membership is also associated with demographic variables. Generally, the relationships between segment membership and demographic were very intuitive and confirmed a priori expectations about the segments. These relationships are important for two reasons: first, the segment profiles lend some additional face validity to the consideration set interpretation of the segments. Second, in order to be useful, segments should differ in behavior and be accessible through information links with media (Frank, Massey, and Wind 1972). Since most media vehicle audience data use only simple demographic descriptors, it would be difficult to differentially access each segment through a targeted media program had the they not been related to demographic

variables. Since the revealed segments did relate to demographic variables, a manager could potentially select particular media vehicles in order to disproportionately access a desired segment.

Elasticity Analysis

A particularly appealing feature of the segmented logit model is that it can be employed to obtain self- and cross-price elasticities, both marginally and within segment. Models based on simple store data (such as the SPF, CPF, and LMS models examined in Chapter VI) at best can specify elasticities at the store or aggregate market level. In Chapter II, it was shown that aggregate elasticity could be expressed as a weighted sum of the elasticities within segment. Thus, the segmented logit model can provide a detailed breakdown of the contribution each segment makes toward an item's overall self-price elasticity or cross elasticity with other items.

Table VII-2 presents each item's marginal self-price sales elasticity estimated by the SLM as well as the three store models estimated in Chapter VI. The SLM elasticity estimates were generated by applying equation (2.11) to the SLM parameter estimates. The weighted averages of all negative within-store self-price elasticities for each item were computed to obtain marginal elasticity estimates from the SPF and CPF models. Marginal elasticity estimates from the LMS model were obtained by applying the unsegmented case of equation (2.11) to its parameters.

The marginal elasticities demonstrate several interesting points. First, the self-price elasticity estimates from each model were in approximately the same range, lying roughly between -1 and -6. The

Table VII-2

Estimated Self-Price Sales Elasticities

	Model			
Item	SPF	CPF	LMS	SLM
Ivory 12	-1.84	.	-4.25	-2.25
Ivory 22	-3.62	-3.97	-3.46	-1.94
Ivory 32	-2.34	-2.69	-3.17	-1.82
Ivory 48	-1.42	-1.12	-3.11	-1.68
Joy 12	-7.66	.	-4.17	-2.88
Joy 22	-3.10	-2.74	-3.29	-2.2
Joy 32	-2.75	-1.98	-3.26	-2.38
Joy 48	-4.02	-3.13	3.13	-1.79
Ajax 22	-2.62	-3.21	-3.44	-2.90
Ajax 32	-3.87	-3.81	-3.2	-2.17
Derm. 22	-5.34	-5.21	-2.03	-3.52
Derm. 32	-1.27	-5.00	-2.56	-2.37
Palmolive 12	-1.46	-2.02	-3.65	-3.17
Palmolive 22	-2.69	-2.53	-3.2	-2.28
Palmolive 32	-3.27	-3.74	-3.05	-2.79
Palmolive 48	-3.30	-2.96	-3.01	-2.97
Dove 32	-4.07	-5.96	-2.42	-2.21
Lux 22	-6.40	-3.17	-1.94	-2.16
Lux 32	-5.61	-4.86	-2.33	-1.92
Dawn 12	-5.12	.	-3.71	-2.03
Dawn 22	-2.89	-2.59	-3.37	-1.86
Dawn 32	-2.33	-2.22	-3.21	-1.95
Dawn 48	-2.54	-1.77	-3.07	-1.89
Sunlight 12	-2.56	-3.97	-3.42	-3.13
Sunlight 22	-3.33	-3.60	-3.08	-2.48
Sunlight 32	-4.51	-3.83	-2.84	-2.67
Sunlight 48	-3.13	-2.32	-2.67	-2.20
Store 32	-4.71	-3.30	-1.44	-1.22
Generic 32	-2.16	-1.79	-0.81	-1.24

absolute size of these elasticities indicate that the LDD market is rather sensitive to pricing: on this point, all the models are in general agreement. Further, the least elastic item (on average) was 32-ounce generic. This item was also the least expensive in each store. The relative insensitivity of generic sales to generic price may be partially due to ordinality of prices. At its normal price, generic is the least expensive of all items (on a per ounce basis), and minor fluctuations in its price may not significantly affect choice probabilities.

A second interesting feature of the elasticity estimates is that the dissertation model (SLM) implies lower price sensitivities that the store models. This difference can be partially attributed to the inclusion of single-item loyal segments in the SLM. Because pricing was found to have no effect on primary demand, the single-item loyal segments have, by definition, elasticities of zero. Thus, the single-item loyal segments would act to dampen the magnitude of marginal elasticities.

A third interesting feature of the marginal elasticities is the correlation patterns among the various models. The SLM elasticities correlated significantly with the elasticities from the LMS ($r = .345$, $prob(r) = .067$) and the CPF ($r = .334$, $prob(r) = .095$), but were unrelated to the SPF elasticities. The correlation between the SLM and LMS elasticities was expected, as both models had similar forms and predictive fit. The correlation between the SLM and CPF was somewhat surprising, as the two models had different forms and the SLM greatly outperformed the CPF in predictive validity. While the The CPF elasticities were also correlated with the SPF elasticities ($r = .506$,

prob(r) = .008), but were unrelated to the LMS elasticities. The LMS elasticities were only correlated with the SLM elasticities. These results demonstrate a number of points. First, all the models generally imply that the LDD market is rather elastic, as nearly all self-price elasticities were less than -1. Secondly, despite the pattern of correlations among the recovered self-price elasticities, the various models varied significantly in their ability to predict. This may be due in part to the assumptions the models make regarding the form of the elasticities. In the SPF and CPF, for example, elasticities are assumed constant. The SLM and LMS, on the other hand, allow elasticities to be a function of the prices of the alternatives. The less restrictive elasticities assumptions made by the SLM and LMS may help explain their greater predictive power.

Table VII-3 presents the within-segment self-price elasticity of sales for each item computed from the SLM. These elasticities exhibited a great deal of variation between segments and indicated a relationship between consideration-set size and elasticity. Note, for example, that self-price elasticities in the all-items segment were rather large, with all but three items less than -3. In segments with smaller consideration sets, the estimated self-price elasticities were generally smaller: for example, in segment 32 (all Joy), elasticities were all greater than -1. This effect was further borne out by the fact that for 19 of the 29 items, the largest absolute self-price elasticity was in the all-items segment (30). To further investigate this effect, two correlation analyses were conducted. First, the weighted average of the self-price elasticities within each switching segment was correlated with the number of items within the segment. The resultant correlation

Table VII-3

Self-Price Elasticity for Each Item Within Each Switching Segment
Evaluated at Each Item's Average Price

	Segment					
Item	30	31	32	33	34	35
Ivory 12	-4.9578	-3.9166	0	0	0	0
Ivory 22	-3.9684	-2.7372	0	0	0	0
Ivory 32	-4.0272	-2.4728	0	0	0	0
Ivory 48	-4.0751	-2.8666	0	0	0	0
Joy 12	-4.8527	0	-0.55494	0	0	0
Joy 22	-3.9761	0	-0.39881	0	0	0
Joy 32	-4.2259	0	-0.58583	0	0	0
Joy 48	-4.2754	0	-0.61279	0	0	0
Ajax 22	-4.3521	0	0	-0.18005	0	0
Ajax 32	-4.0589	0	0	-0.09274	0	0
Derm. 22	-3.4827	0	0	0	-14.650	0
Derm. 32	-3.3130	0	0	0	-16.038	0
Palmolive 12	-4.9712	0	0	0	0	-5.1794
Palmolive 22	-3.8668	0	0	0	0	-3.1980
Palmolive 32	-4.0777	0	0	0	0	-3.8734
Palmolive 48	-4.0421	0	0	0	0	-3.5979
Dove 32	-3.5014	0	0	0	0	0
Lux 22	-2.9900	0	0	0	0	0
Lux 32	-3.3354	0	0	0	0	0
Dawn 12	-4.8822	0	0	0	0	0
Dawn 22	-4.0666	0	0	0	0	0
Dawn 32	-4.1916	0	0	0	0	0
Dawn 48	-4.1292	0	0	0	0	0
Sunlight 12	-4.7509	0	0	0	0	0
Sunlight 22	-3.8082	0	0	0	0	0
Sunlight 32	-3.6136	0	0	0	0	0
Sunlight 48	-3.4047	0	0	0	0	0
Store 32	-2.2267	0	0	0	0	0
Generic 32	-1.3751	0	0	0	0	0

Table VII-3 (continued)

Item	36	37	38	39	40	41
Ivory 12	0	0	0	-0.75711	0	0
Ivory 22	0	0	0	0	-2.1984	0
Ivory 32	0	0	0	0	0	-3.1260
Ivory 48	0	0	0	-0.67436	0	0
Joy 12	0	0	0	0	-2.3069	0
Joy 22	0	0	0	0	0	0
Joy 32	0	0	0	0	0	-3.8329
Joy 48	0	0	0	0	-2.6805	0
Ajax 22	0	0	0	0	0	0
Ajax 32	0	0	0	0	-2.1916	-3.8263
Derm. 22	0	0	0	0	0	0
Derm. 32	0	0	0	-0.68521	0	-3.1148
Palmolive 12	0	0	0	0	-2.2154	0
Palmolive 22	0	0	0	0	0	0
Palmolive 32	0	0	0	0	0	-3.8208
Palmolive 48	0	0	0	0	0	0
Dove 32	0	0	0	0	-1.8571	-3.1785
Lux 22	-1.4443	0	0	0	0	- 0
Lux 32	-1.8887	0	0	-0.52918	0	-3.0668
Dawn 12	0	-3.0703	0	0	-2.3337	0
Dawn 22	0	-2.6290	0	0	0	0
Dawn 32	0	-2.5875	0	0	0	-3.6063
Dawn 48	0	-2.8895	-4.6773	-0.61178	0	0
Sunlight 12	0	0	-2.8366	0	-2.1267	0
Sunlight 22	0	0	-2.6503	0	0	0
Sunlight 32	0	0	-3.3771	0	0	-3.0873
Sunlight 48	0	0	0	0	0	0
Store 32	0	0	0	0	0	-1.9929
Generic 32	0	0	0	0	0	-1.2820

Table VII-3 (continued)

Item	42	43	44	45	46	47
Ivory 12	0	-1.7954	0	0	0	0
Ivory 22	0	-0.8942	-2.9259	0	0	0
Ivory 32	0	-1.2081	-2.2475	0	0	0
Ivory 48	-2.9952	0	-2.3233	0	0	0
Joy 12	0	0	0	0	0	0
Joy 22	0	0	0	-1.3557	0	0
Joy 32	0	0	0	-1.9553	0	0
Joy 48	-4.1035	0	0	0	0	0
Ajax 22	0	0	0	0	0	0
Ajax 32	0	0	0	0	0	0
Derm. 22	0	0	0	0	0	0
Derm. 32	0	0	0	0	0	0
Palmolive 12	0	0	0	0	-2.8315	0
Palmolive 22	0	0	0	0	-1.8450	-1.0561
Palmolive 32	0	0	0	0	0	-1.8682
Palmolive 48	-3.8964	0	0	0	0	0
Dove 32	0	0	0	0	0	0
Lux 22	0	0	0	0	0	0
Lux 32	0	0	0	0	0	0
Dawn 12	0	0	0	0	0	0
Dawn 22	0	0	0	0	0	0
Dawn 32	0	0	0	0	0	0
Dawn 48	-3.6137	0	0	0	0	0
Sunlight 12	0	0	0	0	0	0
Sunlight 22	0	0	0	0	0	0
Sunlight 32	0	0	0	0	0	0
Sunlight 48	-3.1195	0	0	0	0	0
Store 32	0	0	0	0	0	0
Generic 32	0	0	0	0	0	0

Table VII-3 (continued)

Item	48	49	50	51	52	53
Ivory 12	0	0	0	0	0	0
Ivory 22	0	0	0	0	0	0
Ivory 32	0	0	0	0	0	0
Ivory 48	0	0	0	0	0	0
Joy 12	0	0	0	0	0	-3.6497
Joy 22	0	0	0	0	0	-3.0969
Joy 32	0	0	0	0	0	0
Joy 48	0	0	0	0	0	0
Ajax 22	0	0	0	0	0	-3.4931
Ajax 32	0	0	0	0	0	0
Derm. 22	0	0	0	0	0	0
Derm. 32	0	0	0	0	0	0
Palmolive 12	0	0	0	0	0	-3.8601
Palmolive 22	0	0	0	0	0	-3.0726
Palmolive 32	-2.9562	0	0	0	0	0
Palmolive 48	-1.8805	0	0	0	0	0
Dove 32	0	0	0	0	0	0
Lux 22	0	0	0	0	0	0
Lux 32	0	0	0	0	0	0
Dawn 12	0	-0.74831	0	0	0	-3.4416
Dawn 22	0	-0.62219	-0.9569	0	0	-3.1690
Dawn 32	0	0	-1.3756	-2.7125	0	0
Dawn 48	0	0	0	-1.8286	0	0
Sunlight 12	0	0	0	0	0	-3.4908
Sunlight 22	0	0	0	0	0	-2.7291
Sunlight 32	0	0	0	0	-3.1422	0
Sunlight 48	0	0	0	0	-1.9309	0
Store 32	0	0	0	0	0	0
Generic 32	0	0	0	0	0	0

Table VII-3 (continued)

Item	54	55	56	57	58
Ivory 12	0	0	0	0	0
Ivory 22	-3.1314	0	0	-3.4186	0
Ivory 32	-3.1737	-3.4199	0	-3.3787	0
Ivory 48	0	-4.1909	0	0	0
Joy 12	0	0	-4.1064	0	0
Joy 22	-3.4058	0	-4.5154	-3.3655	0
Joy 32	-3.0027	-4.0915	0	-3.6514	0
Joy 48	0	-4.3299	0	0	0
Ajax 22	-3.3950	0	0	0	0
Ajax 32	-3.1656	0	0	0	0
Derm. 22	0	0	0	-3.1595	0
Derm. 32	0	0	0	-2.9231	0
Palmolive 12	0	0	0	0	0
Palmolive 22	-2.9187	0	0	-3.3275	0
Palmolive 32	-3.2037	-4.0143	0	-3.6985	0
Palmolive 48	0	-4.2285	0	0	0
Dove 32	-2.7860	0	0	-3.1571	0
Lux 22	0	0	0	-2.7521	0
Lux 32	0	0	0	-3.0092	0
Dawn 12	0	0	0	0	0
Dawn 22	-3.0548	0	0	0	0
Dawn 32	-3.2790	0	0	0	0
Dawn 48	0	0	0	0	0
Sunlight 12	0	0	-4.5484	0	0
Sunlight 22	-2.8605	0	-3.8796	-3.2895	0
Sunlight 32	-2.6373	-3.3660	0	-2.8518	0
Sunlight 48	0	-3.0746	0	0	0
Store 32	0	0	0	0	-2.8147
Generic 32	0	0	0	0	-4.1342

was -.161 (prob = .40). Second, the marginal self-price elasticity of each brand was correlated with the average consideration set size of purchasers of the brand. This correlation was -.085 (prob = .660). While not approaching significance, these correlations reveal that larger consideration sets are associated with more price sensitivity.

The observed relationship between elasticity and consideration set size is very consistent with prior expectations. Those consumers for whom price is an important attribute might increase the number of considered items in order to obtain the best deal. Consumers who have strong preferences based on product attributes such as size and efficacy might limit their consideration set to only a small set of alternatives and pay relatively little attention to price information. In general, this seems to have been the case with the empirical LDD data. While consistent with expectations, these results are somewhat at odds with the results of Finn and Louviere (1989), who found that "low price" was slightly more important in determining choice among consumers with smaller identified choice sets. One notable exception to the positive relationship between consideration set size and price sensitivity is the case of the price brands (store and generic). In the all brands segment, self-price elasticities of sales were -2.23 and -1.38 for store brand and generic, respectively. In the price brands segment (58), these self-price elasticities increased in absolute value to -2.81 and -4.13. Because the price brand segment contains a much smaller number of brands, one would expect self-price elasticity to decrease in absolute value. However, the price brand segment might be seen as a special case: while the other segments are related to product attributes such as brand name, size, and strength, the price brand segment is most strongly

related to item price. Households assigned to this segment theoretically have chosen item price as the basis for their consideration set, and thus might be even more attuned to price information than households that consider a superset of these brands. Another explanation for this result is price ordinality. For households who consider all items, small fluctuation in the price of the least expensive items may not effect the ordinality of prices across the consideration set, thus leaving the choice probability of those items relatively unaffected. Small fluctuations in the prices of a moderately priced item, on the other hand, could potentially affect the ordinality of that item's price vis-a-vis the other considered items.

CHAPTER VIII

OPTIMIZATION

In this chapter, the SLM parameter estimates are employed to obtain optimal prices for all items within the Procter and Gamble (hereafter P&G) product line. Optimality is defined here with respect to the objectives of two conceptually different players: hypothetical independent brand managers and a hypothetical category manager. These optimi are further examined under the case of segment-level price discrimination and the case of no price discrimination, as well as under different levels of information about competitive behavior. Results of the optimality analysis indicate that several tangible benefits are derived from category management as opposed to brand management, and price discrimination versus no price discrimination. The results also demonstrate how the SLM is useful for product-line pricing decision problems.

Perfect Information

Four different optimization analyses were conducted assuming perfect knowledge of competitor prices: i.e., it is assumed here that decision makers can perfectly anticipate the prices of all competitors. Clearly, this assumption is weak, but it acts to define an upper bound for managerial knowledge. This assumption will subsequently be relaxed.

Within the perfect knowledge condition, price discrimination ability (no versus yes) was crossed with decision-making level (brand versus category) to obtain weekly optimal price vectors for all sizes of all three P&G brands (Ivory, Joy, and Dawn) in store 1. All conditions and optimization results are discussed in detail below.

No Price Discrimination

In the no price discrimination condition, decision makers can only charge one fixed price for each item (i.e., each customer faces the same set of prices regardless of segment). Both brand-level and category-level objective functions were examined under the no price discrimination condition.

The first type of objective function examined was the that of a brand manager. The form of the objective function for the brand manager is

$$\text{MAX} \quad R_{bst} = \sum_{i \in B} (PPO_{ist})(size_i)Q_{ist} \tag{8.1}$$

where R_{bst} = revenue from brand b items

B = the set of all items with the brand name

Q_{ist} = SLM model for item i

Brand-level revenue equations were used as the objectives in this analysis since unit cost data are necessary to specify profit. Unfortunately, no reliable unit cost statistics could be obtained for the LDD category.

Jointly optimal prices for the four different sizes of each brand were found with the GRG2 optimizer, employing a gradient search algorithm. Each price was constrained to be less than or equal to 120% of its observed average in the empirical data. Prices were constrained in this was because of the lack of primary demand effects. While the lack of influence of price on primary demand appears reasonable given the estimation data, it renders each optimization problem degenerate. Under a strict interpretation of the parameters, for example, the optimal price of 12-ounce Ivory would be infinity: there would be a small but positive number of item-loyal consumers willing to purchase at any price. By adding constraints, search is limited to price levels close to the range of observed prices.

A vector of optimal prices for each brand was generated for each week in the observed data, assuming perfect knowledge of competitor prices and no competitive reaction. The results of the weekly brand-level optimizations are presented in Tables VIII-1 through VIII-3. Among the three P&G brands, Dawn items on average had the highest optimal prices, followed by Ivory and Joy. The per-ounce price of 12-ounce Dawn was highest among all items. In each week, 12-ounce Dawn was priced at the upper bound of the constraint. The high price of 12-ounce Dawn is due to the relatively high proportion of sales it draws from single-item loyal segments. Note that Dawn is strongly positioned as a high strength, grease cutting formula. The large proportion of sales that 12-ounce Dawn obtains from its single-item loyal segment may be due to consumers who occasionally purchase a small bottle for infrequent heavy cleaning jobs. This hypothesis is bolstered by the fact that 73% of

Table VIII-1

Optimal Per-Ounce Prices of Ivory Items from the Perspective of Ivory Brand Manager: No Price Discrimination, Perfect Information (Store 1)

Week	2	3	4	5	6	7	8	9
Ivory 12	5.99	5.98	6.05	5.72	6.21	6.11	6.10	5.98
Ivory 22	3.93	3.93	3.95	3.88	4.07	4.05	4.04	3.95
Ivory 32	3.50	3.49	3.52	3.52	3.67	3.61	3.60	3.56
Ivory 48	3.08	3.05	3.05	3.15	3.11	3.03	3.04	3.05

Week	10	11	12	13	14	15	16	17
Ivory 12	6.06	5.99	5.84	5.84	5.84	5.95	5.95	5.92
Ivory 22	3.97	3.92	3.84	3.84	3.84	3.91	3.91	3.90
Ivory 32	3.64	3.60	3.57	3.57	3.51	3.54	3.54	3.53
Ivory 48	3.15	3.13	3.14	3.14	3.09	3.08	3.08	3.08

Week	18	19	20	21	22	23	24	25
Ivory 12	5.92	5.91	5.91	5.81	5.81	5.91	6.06	6.04
Ivory 22	3.90	3.89	3.89	3.82	3.82	3.88	3.99	3.97
Ivory 32	3.53	3.53	3.53	3.47	3.47	3.56	3.64	3.63
Ivory 48	3.08	3.08	3.08	3.07	3.07	3.09	3.10	3.10

Week	26	27	28	29	30	31	32	33
Ivory 12	6.02	6.00	5.90	5.89	5.98	6.04	5.92	6.32
Ivory 22	3.95	3.95	3.86	3.86	3.92	3.98	3.98	4.07
Ivory 32	3.58	3.57	3.52	3.51	3.52	3.63	3.58	3.61
Ivory 48	3.09	3.07	3.11	3.08	3.05	3.09	3.04	3.03

Week	34	35	36	37	38	39	40	41
Ivory 12	6.32	6.44	6.50	6.35	6.41	6.22	5.56	5.60
Ivory 22	4.10	4.13	4.15	4.08	4.10	4.02	3.93	3.95
Ivory 32	3.63	3.64	3.67	3.61	3.65	3.62	3.49	3.46
Ivory 48	3.03	3.03	3.08	3.03	3.05	3.06	2.88	2.86

Week	42	43	44	45	46	47	48	49
Ivory 12	5.46	5.49	6.06	6.90	6.90	6.73	6.32	6.57
Ivory 22	3.87	3.88	3.95	4.29	4.29	4.25	4.06	4.13
Ivory 32	3.42	3.43	3.59	3.79	3.79	3.75	3.67	3.71
Ivory 48	2.90	2.89	3.13	3.13	3.13	3.08	3.09	3.13

Week	50	51	52	min	max	max cut
Ivory 12	6.41	6.16	5.58	5.46	6.90	$0.17
Ivory 22	4.07	3.95	3.72	3.72	4.29	$0.13
Ivory 32	3.69	3.51	3.39	3.39	3.79	$0.13
Ivory 48	3.14	3.01	3.07	2.86	3.15	$0.15

Table VIII-2

Optimal Per-Ounce Prices of Joy Items from the Perspective of Joy Brand Manager: No Price Discrimination, Perfect Information (Store 1)

Week	2	3	4	5	6	7	8	9
Joy 12	4.93	4.93	4.97	4.76	5.02	4.86	4.89	4.78
Joy 22	3.65	3.65	3.69	3.62	3.77	3.75	3.75	3.69
Joy 32	3.09	3.08	3.15	3.22	3.24	3.21	3.21	3.17
Joy 48	3.02	2.97	3.00	3.19	3.04	2.95	2.94	2.99

Week	10	11	12	13	14	15	16	17
Joy 12	4.83	4.90	4.59	4.59	4.57	4.65	4.65	4.58
Joy 22	3.70	3.71	3.60	3.60	3.57	3.63	3.63	3.62
Joy 32	3.22	3.21	3.15	3.15	3.09	3.14	3.14	3.14
Joy 48	3.15	3.09	3.16	3.16	3.10	3.06	3.06	3.08

Week	18	19	20	21	22	23	24	25
Joy 12	4.58	4.57	4.57	4.57	4.57	4.60	4.64	4.64
Joy 22	3.62	3.61	3.61	3.56	3.56	3.63	3.65	3.65
Joy 32	3.14	3.13	3.13	3.09	3.09	3.18	3.20	3.19
Joy 48	3.08	3.08	3.08	3.10	3.10	3.13	3.12	3.12

Week	26	27	28	29	30	31	32	33
Joy 12	4.80	4.89	4.84	4.84	4.84	4.78	4.59	4.90
Joy 22	3.64	3.70	3.64	3.64	3.64	3.71	3.73	3.81
Joy 32	3.15	3.20	3.13	3.13	3.12	3.24	3.22	3.24
Joy 48	3.07	3.02	3.08	3.08	3.04	3.10	3.01	2.96

Week	34	35	36	37	38	39	40	41
Joy 12	4.75	4.83	4.87	4.72	4.74	4.62	3.95	4.40
Joy 22	3.79	3.82	3.83	3.77	3.79	3.72	3.60	3.67
Joy 32	3.24	3.25	3.28	3.23	3.26	3.20	3.07	3.10
Joy 48	2.97	2.96	3.04	2.98	3.01	3.04	3.06	2.98

Week	42	43	44	45	46	47	48	49
Joy 12	4.14	4.28	4.89	5.33	5.33	5.22	4.80	4.97
Joy 22	3.54	3.56	3.69	3.97	3.97	3.93	3.76	3.81
Joy 32	3.03	3.04	3.19	3.39	3.39	3.35	3.23	3.29
Joy 48	3.03	3.04	3.11	3.07	3.07	3.00	3.07	3.12

Week	50	51	52	min	max	max cut
Joy 12	4.90	4.79	4.47	3.95	5.03	$0.13
Joy 22	3.79	3.67	3.45	3.45	3.97	$0.10
Joy 32	3.27	3.13	3.03	3.03	3.39	$0.12
Joy 48	3.14	2.97	3.14	2.94	3.19	$0.12

Table VIII-3

Optimal Per-Ounce Prices of Dawn Items from the Perspective of Dawn Brand Manager: No Price Discrimination, Perfect Information (Store 1)

Week	2	3	4	5	6	7	8	9
Dawn 12	8.78	8.78	8.78	8.78	8.78	8.78	8.78	8.78
Dawn 22	7.52	7.52	4.46	7.52	4.49	4.47	4.47	4.44
Dawn 32	6.94	6.97	3.95	7.36	3.96	3.96	3.95	3.96
Dawn 48	3.32	3.31	2.97	4.15	2.97	2.95	2.95	2.95

Week	10	11	12	13	14	15	16	17
Dawn 12	8.78	8.78	8.78	8.78	8.78	8.78	8.78	8.78
Dawn 22	4.45	4.45	7.52	7.52	7.52	7.52	7.52	7.52
Dawn 32	3.96	3.96	6.71	6.71	6.95	6.45	6.45	6.55
Dawn 48	2.98	2.98	3.46	3.46	3.46	3.24	3.24	3.29

Week	18	19	20	21	22	23	24	25
Dawn 12	8.78	8.78	8.78	8.78	8.78	8.78	8.78	8.78
Dawn 22	7.52	7.52	7.52	7.52	7.52	7.52	4.45	7.52
Dawn 32	6.55	6.61	6.61	7.21	7.21	6.57	3.97	6.05
Dawn 48	3.29	3.31	3.31	3.57	3.57	3.31	2.97	3.20

Week	26	27	28	29	30	31	32	33
Dawn 12	8.78	8.78	8.78	8.78	8.78	8.78	8.78	8.78
Dawn 22	7.52	7.52	7.52	7.52	7.52	4.45	7.52	4.47
Dawn 32	6.35	6.25	6.83	6.84	6.61	3.97	6.16	3.94
Dawn 48	3.25	3.25	3.43	3.41	3.25	2.96	3.24	2.95

Week	34	35	36	37	38	39	40	41
Dawn 12	8.78	8.78	8.78	8.78	8.78	8.78	8.78	8.78
Dawn 22	4.47	4.49	4.51	4.48	4.49	4.48	7.52	7.52
Dawn 32	3.94	3.95	3.96	3.95	3.95	3.95	7.38	7.38
Dawn 48	2.95	2.95	2.97	2.95	2.95	2.95	6.46	6.36

Week	42	43	44	45	46	47	48	49
Dawn 12	8.78	8.78	8.78	8.78	8.78	8.78	8.78	8.78
Dawn 22	7.52	7.52	4.45	4.60	4.60	4.59	4.46	4.51
Dawn 32	7.38	7.38	3.95	4.00	4.00	3.99	3.95	3.96
Dawn 48	6.63	6.61	2.98	3.01	3.01	2.99	2.96	2.99

Week	50	51	52	min	max	max cut
Dawn 12	8.78	8.78	8.78	8.78	8.78	$0.00
Dawn 22	4.48	4.44	7.52	4.44	7.52	$0.68
Dawn 32	3.95	3.94	7.38	3.94	7.38	$1.10
Dawn 48	2.98	2.98	4.87	2.95	6.63	$1.77

households in the 12-ounce Dawn-loyal segment owned automatic dishwashers.

While Ivory and Joy had lower optimal prices than Dawn on average, the ranges of optimal prices for Ivory and Joy items were rather small. For example, the highest optimal price for 12-ounce Ivory was 6.90 cents-per-ounce while the lowest optimal price was 5.46 cents-per-ounce, or a total package discount of 10 cents (5.46 cents-per-ounce times 12 ounces). Across all Ivory and Joy items, the package discounts ranged from 10 to 17 cents. While the 12-ounce size of Dawn had a constant optimal price, the other sizes of Dawn ranged quite dramatically. For instance, the difference between the highest and lowest optimal price of 22-ounce Dawn was 3.08 cents-per-ounce, or a total package discount of 68 cents. The discounts for 32 and 48-ounce Dawn were $1.10 and $1.77, respectively.

Suggested promotion periods under optimality were very frequent compared to the actual promotion periods observed in the empirical store data. For example, optimal prices for 48-ounce Dawn suggest that deep price discounts (more than $0.25 less than the average weekly package price) should be offered in 29 weeks (weeks 4, 6 through 10, 24, 31, 33 through 39, and 41 through 51). In the empirical data, 48-ounce Dawn was discounted in only two weeks (weeks 51 and 52). The frequent suggested price discounts under brand level optimization reflect the reaction of each brand to the promotions of all other competitive brands, including other P&G brands.

A additional point of interest regarding the optimal item prices is that quantity discounts are strictly obeyed. For instance, the optimal per-ounce price of 48-ounce Joy is smaller than the optimal per-

ounce price of 32-ounce Joy, which is smaller than the optimal per-ounce price of 22-ounce Joy, and so forth. The monotonicity of per-ounce prices held for each brand in each week, despite the fact that there was no constraint insuring this result. Further, for Ivory and Joy, the total optimal package prices (cents-per-ounce times number of ounces) were also monotonic, such that the prices of smaller items were always less than the prices of larger items with the same brand name. There were a number of weeks in which the optimal price of 12-ounce Dawn was slightly larger than the optimal price of 22-ounce Dawn. For instance, in week 9, the total optimal price of 12-ounce Dawn was $1.05 (8.78 cents-per-ounce times 12 ounces), while the optimal price of 22-ounce Dawn was $0.98. These violations can again be traced to the disproportionately large size of the 12-ounce Dawn loyal segment. No other violations of package price monotonicity were detected. The observed monotonicity of optimal prices is somewhat important in that the order of prices agree with accepted managerial policy: discounts are given to those who buy larger quantities, and the total price for smaller items is less than the total price for larger items with the same brand name. It is worth noting that violations of monotonic per-unit and total prices in the work of Reibstein and Gatignon (1984) were severe enough to warrant additional optimization constraints: here the violations are isolated and rather small.

The second objective function examined was that of a hypothetical P&G category manager. The objective for this manager is to find jointly optimal prices for all 12 P&G items. The objective of the category manager is represented by the equation

$$\text{MAX} \quad R_{st} = \sum_{b \in P} R_{bst} \tag{8.2}$$

where P = set of all P&G brands

The major difference between the more global category manager objective and the individual brand manager objectives is that the former takes into account the level of intra-product line competition. An alternative way of conceptualizing the category-level objective is that it represents the collusive objective of a number of brand managers.

GRG2 was used to find jointly optimal per-ounce prices for all 12 P&G items for each week. Again, upper constraints on prices were employed and perfect information on competitive pricing was assumed. The optimal price vectors for the category objective are presented in Table VIII-4.

Note that the optimal item price levels under category objectives are substantially higher than under brand-level objectives. For example, the optimal price for 12-ounce Ivory in week 2 increased from 6 cents-per-ounce in the brand management case to 8.85 cents-per-ounce in the category management case. This effect did not hold for 12-ounce Dawn, since optimal 12-ounce Dawn prices were already at the upper price constraint in the brand management case. Except for a few discount periods, Ivory and Dawn item prices were at or near their respective upper constraints. Optimal price ranges tended to be larger for the larger package sizes: for example, the range of optimal prices for 22-ounce Ivory was from 7.48 cents-per-ounce to 6.89 cents-per-ounce, while the largest suggested price discount for 48-ounce Ivory was from 7.11 cents-per-ounce to 4.27 cents per ounce.

Table VIII-4

Optimal Per-Ounce Prices of all P&G Items from the Perspective of P&G Category Manager: No Price Discrimination, Perfect Information (Store 1)

Week	2	3	4	5	6	7	8	9	10
Ivory 12	8.85	8.85	8.85	8.85	8.85	8.85	8.85	8.85	8.85
Ivory 22	7.48	7.48	7.48	7.48	7.23	7.48	7.39	7.48	7.48
Ivory 32	7.29	7.29	7.14	7.29	5.54	5.72	5.64	7.12	6.71
Ivory 48	6.54	6.54	6.33	6.65	4.60	4.70	4.59	6.38	6.00
Joy 12	6.36	6.32	6.40	6.06	6.63	6.22	6.38	6.00	6.11
Joy 22	4.46	4.45	4.51	4.38	4.87	4.75	4.80	4.45	4.52
Joy 32	3.68	3.66	3.75	3.90	4.20	4.10	4.11	3.79	3.94
Joy 48	3.68	3.44	3.48	3.90	3.83	4.04	3.46	3.41	3.94
Dawn 12	8.78	8.78	8.78	8.78	8.78	8.78	8.78	8.78	8.78
Dawn 22	7.38	7.52	7.52	7.38	7.52	7.38	7.52	7.52	7.38
Dawn 32	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38
Dawn 48	6.29	6.32	6.23	6.48	6.13	6.24	6.28	6.35	6.25

Week	11	12	13	14	15	16	17	18	19
Ivory 12	8.85	8.85	8.85	8.85	8.85	8.85	8.85	8.85	8.85
Ivory 22	7.48	7.48	7.48	7.48	7.48	7.48	7.48	7.48	7.48
Ivory 32	6.42	7.29	7.29	7.29	7.29	7.29	7.29	7.29	7.29
Ivory 48	5.58	6.64	6.64	6.76	7.01	7.01	6.90	6.90	6.94
Joy 12	6.31	5.57	5.57	5.55	5.68	5.68	5.55	5.55	5.59
Joy 22	4.56	4.25	4.25	4.18	4.32	4.32	4.27	4.27	4.27
Joy 32	3.94	3.73	3.73	3.62	3.64	3.64	3.73	3.73	3.63
Joy 48	3.93	3.73	3.73	3.62	3.62	3.64	3.73	3.73	3.63
Dawn 12	8.78	8.78	8.78	8.78	8.78	8.78	8.78	8.78	8.78
Dawn 22	7.38	7.52	7.52	7.52	7.38	7.38	7.38	7.38	7.38
Dawn 32	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38
Dawn 48	6.23	6.49	6.49	6.53	6.41	6.41	6.46	6.46	6.46

Week	20	21	22	23	24	25	26	27	28
Ivory 12	8.85	8.85	8.85	8.85	8.85	8.85	8.85	8.85	8.85
Ivory 22	7.48	7.48	7.48	7.48	7.48	7.48	7.48	7.48	7.48
Ivory 32	7.29	7.29	7.29	7.29	6.55	6.68	6.92	5.78	7.29
Ivory 48	6.94	6.91	6.91	6.62	5.77	5.91	6.12	4.83	6.52
Joy 12	5.59	5.55	5.55	5.61	5.81	5.79	6.12	6.41	6.18
Joy 22	4.27	4.16	4.16	4.30	4.49	4.46	4.44	4.74	4.40
Joy 32	3.63	3.61	3.61	3.77	3.96	3.93	3.83	4.07	3.71
Joy 48	3.63	3.61	3.61	3.77	3.92	3.90	3.62	3.64	3.71
Dawn 12	8.78	8.78	8.78	8.78	8.78	8.78	8.78	8.78	8.78
Dawn 22	7.38	7.52	7.52	7.38	7.38	7.38	7.52	7.52	7.38
Dawn 32	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38
Dawn 48	6.46	6.52	6.52	6.41	6.31	6.33	6.30	6.22	6.34

Table VIII-4 (continued)

Week	29	30	31	32	33	34	35	36	37
Ivory 12	8.85	8.85	8.85	8.85	8.85	8.85	8.85	8.85	8.85
Ivory 22	7.48	7.48	7.48	7.48	6.99	7.48	6.89	7.05	7.48
Ivory 32	7.29	7.29	6.24	7.03	5.33	5.73	5.28	5.39	5.67
Ivory 48	6.52	6.52	5.40	6.39	4.34	4.73	4.29	4.48	4.64
Joy 12	6.18	6.16	6.03	5.58	6.30	5.99	6.24	6.30	5.99
Joy 22	4.40	4.40	4.63	4.53	4.97	4.86	5.06	5.04	4.82
Joy 32	3.71	3.71	4.05	3.89	4.18	4.07	4.23	4.26	4.06
Joy 48	3.71	3.67	4.03	3.47	3.68	3.63	3.69	3.85	3.61
Dawn 12	8.78	8.78	8.78	8.78	8.78	8.78	8.78	8.78	8.78
Dawn 22	7.38	7.38	7.52	7.52	7.52	7.52	7.52	7.52	7.52
Dawn 32	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38
Dawn 48	6.34	6.33	6.24	6.39	6.02	6.10	6.03	5.87	6.09

Week	38	39	40	41	42	43	44	45	46
Ivory 12	8.85	8.85	8.85	8.85	8.85	8.85	8.85	8.85	8.85
Ivory 22	7.29	7.48	7.48	7.48	7.48	7.48	7.48	7.02	7.02
Ivory 32	5.53	5.95	7.29	7.29	7.29	7.29	6.99	5.41	5.41
Ivory 48	4.51	4.99	7.11	7.11	7.11	7.11	6.23	4.40	4.40
Joy 12	6.08	5.88	4.78	5.35	5.31	5.60	6.22	7.25	7.25
Joy 22	4.90	4.68	4.42	4.54	4.32	4.32	4.48	5.45	5.45
Joy 32	4.17	4.00	3.72	3.72	3.65	3.64	3.84	4.58	4.58
Joy 48	3.71	3.69	2.91	2.86	3.08	2.84	3.84	4.06	4.06
Dawn 12	8.78	8.78	8.78	8.78	8.78	8.78	8.78	8.78	8.78
Dawn 22	7.52	7.52	7.52	7.52	7.52	7.52	7.38	7.52	7.52
Dawn 32	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38
Dawn 48	6.03	6.13	6.91	6.74	6.91	6.93	6.24	5.20	5.20

Week	47	48	49	50	51	52	min	max	max cut
Ivory 12	8.85	8.85	8.85	8.85	8.85	8.85	8.85	8.85	$0.00
Ivory 22	6.95	7.48	7.13	7.48	7.48	7.48	6.89	7.48	$0.13
Ivory 32	5.32	5.88	5.52	5.86	7.11	7.29	5.28	7.29	$0.64
Ivory 48	4.27	4.93	4.63	4.95	6.24	7.09	4.27	7.11	$1.36
Joy 12	7.19	6.09	6.51	6.28	6.02	5.39	4.78	7.25	$0.30
Joy 22	5.46	4.70	4.97	4.84	4.49	4.12	4.12	5.46	$0.29
Joy 32	4.54	4.04	4.26	4.14	3.71	3.56	3.56	4.58	$0.32
Joy 48	3.91	4.01	4.23	4.14	3.71	3.56	2.84	4.23	$0.67
Dawn 12	8.78	8.78	8.78	8.78	8.78	8.78	8.78	8.78	$0.00
Dawn 22	7.52	7.38	7.38	7.38	7.38	7.52	7.38	7.52	$0.03
Dawn 32	7.38	7.38	7.38	7.38	7.38	7.38	7.38	7.38	$0.00
Dawn 48	5.29	6.12	5.72	5.87	6.15	5.20	5.20	6.91	$0.82

Interestingly, while the optimal price levels for Joy items remained well below the price constraint, even under the category management objective. One way of interpreting this result is that Joy has less of a "captive market" than Ivory and Dawn. In general, a higher proportion of Ivory and Dawn sales were to brand-loyal segments compared to Joy. Joy, on the other hand, derived a larger proportion of its sales from segments that also considered competitive (non-P&G) items, especially Sunlight. Thus, even under category management objectives, the price of Joy was kept relatively low in order to compensate for external competition.

Under category management objectives, the range of optimal prices for Ivory and Joy items was much larger than under brand management objectives. The range in optimal price for 48-ounce Ivory, for example, was 2.84 cents under category management objectives and only 0.29 cents-per-ounce under brand management objectives. The opposite case held for Dawn, as the range in prices for Dawn items was somewhat smaller under category management objectives.

Under category management objectives, suggested price discounts were still more frequent than empirical price discounts, but much less frequent than under brand management objectives. For example, deep price discounts for 48-ounce Dawn are suggested in only 6 weeks (weeks 45, 46, 47, 49, 50, and 52) versus 29 weeks under brand management objectives. The relative infrequency of discount price periods under category management objectives reflects one of the advantages of category level coordination. By coordinating pricing, the various P&G brands need only react to outside competitors rather than both outside competitors and sister brands.

Within brand, the monotonicity of both per-ounce and total package prices was strictly obeyed in the category management optimal prices. The per-ounce price of each item was always higher than the per-ounce price of all larger items with the same brand name, while the total package price (cents-per-ounce times number of ounces) of each item was always smaller than the total price sizes of all larger items with the same brand name. However, price monotonicity did not strictly hold across brands. For example, the per-ounce price of 22-ounce Ivory in week 9 (7.39 cents) was greater than the per-ounce price of 12-ounce Joy (6 cents). Further, the total package price of 12-ounce Dawn in week 9 ($1.06) was greater than the total package price of 22-ounce Joy ($0.98).

In the observed store data, prices for like sizes of Ivory, Joy, and Dawn were quite similar except for particular promotion periods. This may reflect a policy on the part of P&G to price all offered brands at identical levels. The results of the first set of optimizations imply that such a policy is suboptimal, especially for Joy. According to the results here, Joy items should be priced substantially less than both Dawn and Ivory items.

Price Discrimination

In an additional set of analyses, optimal prices were obtained assuming that different prices could be charged to the households in each segment. Of course, it would be pragmatically impossible for the firm to discriminate among consumers with shelf price. Cents-off coupons, on the other hand, can be judiciously distributed such that different consumers are effectively charged different prices. Since the

segments identified in this dissertation vary significantly in terms of demographic characteristics, such a strategy might be feasible with a segmented media strategy.

Segment-level optimal prices were first obtained using brand-level objectives for the three P&G brands. The form of the objective functions in this analysis was equivalent to (8.1), except that separate objective functions were specified for each segment/week. Within each segment, item prices were constrained to be less than or equal to 120% of their mean observed level in the store. Tables VIII-5 through VIII-7 present the segment-level optimal prices for each of the three P&G brands. In the interest of parsimony, only the results for week 2 are presented.

For each brand, the optimal price levels varied significantly between segments, and were inversely related to the number of outside competitors within each segment. For example, the optimal level of price for 22-ounce Ivory was 3.18 cents-per-ounce in segment 30 (all brands) and 5.42 cents-per-ounce in segment 54 (22 and 32-ounce medium and high strength items). If a segment contained only those items within the objective function, then the optimal price vector for the segment was set at the (upper) constraint. For example, the optimal price of 22-ounce Ivory was the maximum 7.48 cents-per-ounce for segments 2, 31, 43, and 44. These segments represent households which are strictly Ivory loyal, and thus the optimal strategy from the point of the Ivory brand manager is to charge the households the maximum allowable price for all Ivory items. Under optimality, significant price discounts are offered to segments which also include competitive items in their respective

Table VIII-5

Optimal Per-ounce Prices of Ivory Items from the Perspective of Ivory Brand Manager: Price Discrimination, Perfect Information (Store 1, Week 2)

Segment

Item	1	2	3	4	30	31	39	40
12-ounce	8.85	-	-	-	4.71	8.85	8.85	-
22-ounce	-	7.48	-	-	3.18	7.48	-	3.61
32-ounce	-	-	7.29	-	2.61	7.29	-	-
48-ounce	-	-	-	7.11	2.19	7.11	-	-

Segment

Item	41	42	43	44	54	55	57
12-ounce	-	-	8.85	-	-	-	-
22-ounce	-	-	7.48	7.48	5.24	-	3.75
32-ounce	3.19	-	7.29	7.29	3.18	3.80	3.03
48-ounce	-	4.11	-	7.11	2.72	2.95	-

Table VIII-6

Optimal Per-ounce Prices of Joy Items from the Perspective of Joy Brand Manager: Price Discrimination, Perfect Information (Store 1, Week 2)

Segment

Item	5	6	7	8	30	32	39	40
12-ounce	8.85	-	-	-	4.47	8.85	8.85	-
22-ounce	-	7.34	-	-	3.05	7.34	-	3.28
32-ounce	-	-	7.31	-	2.52	7.31	-	-
48-ounce	-	-	-	7.28	2.13	7.28	-	-

Segment

Item	41	42	45	53	54	55	56	57
12-ounce	-	-	-	4.98	-	-	7.11	-
22-ounce	-	-	7.34	3.47	3.54	-	4.37	3.45
32-ounce	2.35	-	7.31	-	2.96	3.37	-	2.83
48-ounce	-	3.19	-	-	-	2.66	-	-

Table VIII-7

Optimal Per-ounce Prices of Dawn from the Perspective of Dawn Brand Manager: Price Discrimination, Perfect Information (Store 1, Week 2)

Segment

Item	20	21	22	23	30	37	39
12-ounce	8.78	-	-	-	4.89	8.78	8.78
22-ounce	-	7.52	-	-	3.28	7.52	-
32-ounce	-	-	7.38	-	2.68	7.38	-
48-ounce	-	-	-	7.23	2.24	5.99	-

Segment

Item	40	41	42	49	50	51	53	54
12-ounce	-	-	-	8.78	-	-	5.39	-
22-ounce	3.43	-	-	7.52	7.52	-	3.70	3.48
32-ounce	-	2.71	-	-	7.38	7.38	-	2.92
48-ounce	-	-	3.65	-	-	7.32	-	-

consideration sets. In general, consumers with larger consideration sets are offered larger discounts.

As was the case for the no price discrimination/category management objective optimi, both per-ounce and total package price monotonicity were strictly obeyed for each brand within each segment. The frequency of suggested price discounts under price discrimination/brand-level objectives was the same as under no price discrimination/brand-level objectives: however, suggested temporal price discounts were only offered to segments containing outside competitors.

Using segment-level objective functions equivalent to (8.2), weekly optimal prices were obtained for each P&G item from the perspective of a category manager. Table VIII-8 presents the optimal prices for all P&G items under a category manager's objectives. In the interest of parsimony, only the results for week 2 are reported. Once again, the maximum item price is charged to P&G-loyal segments. However, in segments with outside competition, the optimal price of each item is substantially larger compared to the brand management case. For instance, the optimal price of 22-ounce Ivory in the all-brands segment (30) using the brand-management objective is 3.18 cents-per-ounce. Using the category management objective, the optimal price for the same item increases to 4.63 cents-per-ounce. A dramatic demonstration of this effect was spotted in the 48-ounce loyal segment. Under the individual brand management objective functions, the optimal segment prices for Ivory, Joy, and Dawn were 4.11, 3.19, and 3.65 cents-per-ounce, respectively. Under the category management objective, the optimal price in the 48-ounce segment was 5.79 cents-per-ounce for each brand. The

Table VIII-8

Optimal Per-ounce Prices of all P&G items from the Perspective of P&G Brand Manager: Price Discrimination, Perfect Information (Store 1, Week 2)

Segment

Item	1	2	3	4	5	6	7	8	20
Ivory 12	8.85	-	-	-	-	-	-	-	-
Ivory 22	-	7.48	-	-	-	-	-	-	-
Ivory 32	-	-	7.29	-	-	-	-	-	-
Ivory 48	-	-	-	7.11	-	-	-	-	-
Joy 12	-	-	-	-	8.85	-	-	-	-
Joy 22	-	-	-	-	-	7.34	-	-	-
Joy 32	-	-	-	-	-	-	7.31	-	-
Joy 48	-	-	-	-	-	-	-	7.28	-
Dawn 12	-	-	-	-	-	-	-	-	8.78
Dawn 22	-	-	-	-	-	-	-	-	-
Dawn 32	-	-	-	-	-	-	-	-	-
Dawn 48	-	-	-	-	-	-	-	-	-

Segment

Item	21	22	23	30	31	32	37
Ivory 12	-	-	-	7.35	8.85	-	-
Ivory 22	-	-	-	4.63	7.48	-	-
Ivory 32	-	-	-	3.60	7.29	-	-
Ivory 48	-	-	-	2.85	6.23	-	-
Joy 12	-	-	-	7.35	-	8.85	-
Joy 22	-	-	-	4.62	-	7.34	-
Joy 32	-	-	-	3.60	-	7.31	-
Joy 48	-	-	-	2.85	-	7.28	-
Dawn 12	-	-	-	7.35	-	-	8.78
Dawn 22	7.52	-	-	4.62	-	-	7.52
Dawn 32	-	7.38	-	3.60	-	-	7.38
Dawn 48	-	-	7.23	2.85	-	-	5.99

Table VIII-8 (continued)

Segment

Item	39	40	41	42	43	44	45
Ivory 12	8.85	-	-	-	8.85	-	-
Ivory 22	-	5.03	-	-	7.48	7.48	-
Ivory 32	-	-	3.96	-	7.29	7.29	-
Ivory 48	-	-	-	5.79	-	6.23	-
Joy 12	8.85	-	-	-	-	-	-
Joy 22	-	5.03	-	-	-	-	7.34
Joy 32	-	-	3.96	-	-	-	7.31
Joy 48	-	-	-	5.79	-	-	-
Dawn 12	8.85	-	-	-	-	-	-
Dawn 22	-	5.03	-	-	-	-	-
Dawn 32	-	-	3.96	-	-	-	-
Dawn 48	-	-	-	5.79	-	-	-

Segment

Item	49	50	51	53	54	55	56	57
Ivory 12	-	-	-	-	-	-	-	-
Ivory 22	-	-	-	-	5.10	-	-	4.62
Ivory 32	-	-	-	-	4.04	4.64	-	3.63
Ivory 48	-	-	-	-	-	3.51	-	-
Joy 12	-	-	-	7.11	-	-	7.11	-
Joy 22	-	-	-	4.63	5.10	-	4.37	4.62
Joy 32	-	-	-	-	4.04	4.64	-	3.63
Joy 48	-	-	-	-	-	3.51	-	-
Dawn 12	8.85	-	-	7.11	-	-	-	-
Dawn 22	7.52	7.52	-	4.63	5.10	-	-	-
Dawn 32	-	7.38	7.38	-	4.04	-	-	-
Dawn 48	-	-	7.32	-	-	-	-	-

increase in optimal price in this segment is attributable to the joint dominance of P&G brands among 48-ounce size packages of LDD items.

One striking feature of the price discrimination/category-level objective optimal prices is that equivalent sizes of the various P&G brands are given equal prices for each segment, even during suggested discount weeks. For example, in segment 54 in week 2, the suggested price for each 22-ounce P&G item is 5.1 cents-per-ounce, while the suggested price for each 32-ounce P&G item is 4.04 cents-per-ounce. This equivalence in optimal item price held in each segment in each week. Recall that in the no price discrimination/category objective case, it was determined that Joy items should be priced lower than like-sized Ivory or Dawn items. This led to the conclusion that the observed policy of equivalent prices for all P&G brands is suboptimal. The results here suggest that a policy of equivalent pricing is optimal, but <u>only</u> for those segments that consider multiple P&G brands.

As was the case for the other optimization methods, price discrimination/category-level objective optimal prices were strictly monotonic, both in per-ounce terms and for total package price. Further, price monotonicity held both within brand as well as across brands: the optimal per-unit price of 12-ounce Joy, for example, was always greater than the optimal price of larger sizes of Joy, Ivory, or Dawn.

No Information

In the each of the previous optimization analyses it was assumed that the decision maker, whether a brand manager or category manager, had perfect knowledge of competitor prices. Thus, weekly

optimal prices were generated in each instance by plugging observed competitor prices into the objective function.

Such an assumption could rightly be called overly optimistic, as it presumes that the decision maker can perfectly foresee competitor prices. A much less restrictive assumption is that the decision maker has no information about competitor prices, and uses a best guess that each competitive item will be priced at its mean level in each week. This assumption, in fact, might be seen as overly pessimistic because the decision maker might employ a time-series model to obtain relatively good predictions of competitor prices. Nevertheless, the perfect information and no information assumptions define logical upper and lower bounds on the level of knowledge a manager can have about his competitor's behavior. A time-series model represents a level of knowledge falling somewhere between these two extremes.

The four previous optimization analyses were replicated using the no information assumption. In each replication, optimal prices for the P&G brands were obtained assuming competitive items would be priced at their observed mean levels. Since the mean level of competitor prices are constant over time, the optimal prices in each analysis are the same for each week.

Results of the optimizations under the no information assumption are reported in Tables VIII-9 through VIII-13. Table VIII-9 compares the optimal prices P&G items using individual brand manager objectives with optimal prices for the same items using a single category management objective, allowing no price discrimination. The optimal price values echo the results found in the perfect information condition. Once again, optimal prices under brand management objectives

Table VIII-9

Optimal Values of P&G Items, No Price Discrimination and No Competitor Price Information

Item	Brand Level Optimal Price	Category Level Optimal Price
Ivory 12	6.56	8.85
Ivory 22	4.13	7.05
Ivory 32	3.67	5.30
Ivory 48	3.07	4.33
Joy 12	5.07	6.62
Joy 22	3.83	5.10
Joy 32	3.26	4.39
Joy 48	3.02	4.37
Dawn 12	8.78	8.78
Dawn 22	4.51	7.52
Dawn 32	3.96	7.38
Dawn 48	2.98	5.75

Table VIII-10

Optimal Per-ounce Prices of Ivory Items from the Perspective of Ivory Brand Manager: Price Discrimination, No Information

	Segment							
Item	1	2	3	4	30	31	39	40
12-ounce	8.85	-	-	-	5.22	8.85	8.85	-
22-ounce	-	7.48	-	-	3.46	7.48	-	3.69
32-ounce	-	-	7.29	-	2.80	7.29	-	-
48-ounce	-	-	-	7.11	2.32	7.11	-	-

	Segment						
Item	41	42	43	44	54	55	57
12-ounce	-	-	8.85	-	-	-	-
22-ounce	-	-	7.48	7.48	3.28	-	4.05
32-ounce	3.46	-	7.29	7.29	2.78	4.10	3.25
48-ounce	-	3.91	-	7.11	2.72	3.15	-

Table VIII-11

Optimal Per-ounce Prices of Joy Items from the Perspective of Joy Brand Manager: Price Discrimination, No Information

Segment

Item	5	6	7	8	30	32	39	40
12-ounce	8.85	-	-	-	4.96	8.85	8.85	-
22-ounce	-	7.34	-	-	3.32	7.34	-	3.36
32-ounce	-	-	7.31	-	2.71	7.31	-	-
48-ounce	-	-	-	7.28	2.25	7.28	-	-

Segment

Item	41	42	45	53	54	55	56	57
12-ounce	-	-	-	4.69	-	-	6.29	-
22-ounce	-	-	7.34	3.31	3.69	-	3.92	3.81
32-ounce	2.61	-	7.31	-	3.06	3.82	-	3.08
48-ounce	-	3.02	-	-	-	2.96	-	-

Table VIII-12

Optimal Per-ounce Prices of Dawn Items from the Perspective of Dawn Brand Manager: Price Discrimination, No Information

Segment

Item	20	21	22	23	30	37	39
12-ounce	8.78	-	-	-	5.46	8.78	8.78
22-ounce	-	7.52	-	-	3.66	7.52	-
32-ounce	-	-	7.38	-	2.89	7.38	-
48-ounce	-	-	-	7.23	2.38	5.99	-

Segment

Item	40	41	42	49	50	51	53	54
12-ounce	-	-	-	8.78	-	-	5.06	-
22-ounce	3.51	-	-	7.52	7.52	-	3.52	3.62
32-ounce	-	3.02	-	-	7.38	7.38	-	3.02
48-ounce	-	-	3.47	-	-	7.23	-	-

Table VIII-13

Optimal Per-ounce Prices of All P&G Items from the Perspective of P&G Brand Manager: Price Discrimination, No Information

Segment

Item	1	2	3	4	5	6	7	8	20
Ivory 12	8.85	-	-	-	-	-	-	-	-
Ivory 22	-	7.48	-	-	-	-	-	-	-
Ivory 32	-	-	7.29	-	-	-	-	-	-
Ivory 48	-	-	-	7.11	-	-	-	-	-
Joy 12	-	-	-	-	8.85	-	-	-	-
Joy 22	-	-	-	-	-	7.34	-	-	-
Joy 32	-	-	-	-	-	-	7.31	-	-
Joy 48	-	-	-	-	-	-	-	7.28	-
Dawn 12	-	-	-	-	-	-	-	-	8.78
Dawn 22	-	-	-	-	-	-	-	-	-
Dawn 32	-	-	-	-	-	-	-	-	-
Dawn 48	-	-	-	-	-	-	-	-	-

Segment

Item	21	22	23	30	31	32	37
Ivory 12	-	-	-	8.45	8.85	-	-
Ivory 22	-	-	-	5.23	7.48	-	-
Ivory 32	-	-	-	4.01	7.29	-	-
Ivory 48	-	-	-	3.13	6.23	-	-
Joy 12	-	-	-	8.45	-	8.85	-
Joy 22	-	-	-	5.23	-	7.34	-
Joy 32	-	-	-	4.01	-	7.31	-
Joy 48	-	-	-	3.13	-	7.28	-
Dawn 12	-	-	-	8.45	-	-	8.78
Dawn 22	7.52	-	-	5.23	-	-	7.52
Dawn 32	-	7.38	-	4.01	-	-	7.38
Dawn 48	-	-	7.23	3.13	-	-	5.99

Table VIII-13 (continued)

Segment

Item	39	40	41	42	43	44	45
Ivory 12	8.85	-	-	-	8.85	-	-
Ivory 22	-	5.28	-	-	7.48	7.48	-
Ivory 32	-	-	4.36	-	7.29	7.29	-
Ivory 48	-	-	-	5.25	-	6.23	-
Joy 12	8.85	-	-	-	-	-	-
Joy 22	-	5.28	-	-	-	-	7.34
Joy 32	-	-	4.36	-	-	-	7.31
Joy 48	-	-	-	5.25	-	-	-
Dawn 12	8.85	-	-	-	-	-	-
Dawn 22	-	5.28	-	-	-	-	-
Dawn 32	-	-	4.36	-	-	-	-
Dawn 48	-	-	-	5.25	-	-	-

Segment

Item	49	50	51	53	54	55	56	57
Ivory 12	-	-	-	-	-	-	-	-
Ivory 22	-	-	-	-	5.35	-	-	5.10
Ivory 32	-	-	-	-	4.21	5.08	-	3.97
Ivory 48	-	-	-	-	-	3.81	-	-
Joy 12	-	-	-	6.45	-	-	7.11	-
Joy 22	-	-	-	4.33	5.35	-	4.37	5.10
Joy 32	-	-	-	-	4.21	5.08	-	3.97
Joy 48	-	-	-	-	-	3.81	-	-
Dawn 12	8.85	-	-	6.45	-	-	-	-
Dawn 22	7.52	7.52	-	4.33	5.35	-	-	-
Dawn 32	-	7.38	7.38	-	4.21	-	-	-
Dawn 48	-	-	7.32	-	-	-	-	-

are substantially lower than under the category management objective. Optimal prices of Joy items are less than both Dawn and Ivory under either objective. Monotonicity of per-ounce optimal prices is strictly obeyed for both brand and category objectives. Monotonicity of total package prices is strictly obeyed for optimal prices using the category-level objective, and is violated only in the case of 12-ounce and 22-ounce Dawn under brand-level objectives.

Tables VIII-10 through VIII-12 present the within-segment optimal prices of each item under brand-level objectives, assuming no information about competitor prices. Results again are equivalent to the perfect information condition: significant variability in optimal prices was evident among the various segments. Monotonicity of both per-ounce and total package price was strictly obeyed within each segment.

Finally, Table VIII-13 presents the within-segment optimal prices of each item under the category-level objective, assuming no information about competitor prices. As in the case of the perfect information condition, monotonicity of both per-ounce and total prices was strictly obeyed within each segment as well as across brands. Equality of optimal prices was evident for all pairs of P&G items of like size.

Predicted Outcomes Under Different Optimal Prices

In total, optimal prices for each P&G item were obtained under eight different conditions, completely crossing level of competitive pricing information (none versus perfect), level of decision making (brand versus category), and ability to price discriminate (no versus yes). In order to understand the effect each condition exerts on total

firm outcomes, weekly dollar sales for all P&G items in store 1 were projected using the optimal prices suggested under each of the eight conditions.

The weekly dollar sales projections from each set of optimal prices were then analyzed using the ANOVA procedure of SAS. The independent measures in this analysis were the three optimization condition variables: level of competitor price information, decision-making level, and ability to price discriminate. ANOVA results are presented in Table VIII-14. Mean values of total P&G sales at each level of the independent variables are presented in Table VIII-15.

The ANOVA table and the means present several interesting results. First, the effect of competitor price knowledge is positive, but surprisingly small. Ability to foresee competitor pricing resulted in a mean weekly sales increase of only $7.28 for all P&G items, or a sales increase of only 1%. This rather modest increase can be attributed in part to the flat maximum principle (Tull, et al. 1986). The flat maximum principle states that the value of the objective function is relatively constant over wide ranges of the independent variables. This indeed seems to have been the case for the objective function here. Figure VIII-1 shows a three dimensional view of the joint sales function of 22 and 32-ounce Ivory. Note that the joint sales of the two items (vertical axis) is very flat over a wide range of individual item prices. Due to this flat maximum, the ability to immediately respond to competitor pricing (as in the perfect information condition) does not significantly improve total firm outcomes.

Decision-making level, on the other hand, had a very large and positive effect on total P&G sales. Under brand management objectives,

Table VIII-14

ANOVA Results

Source	DF	SS	MS	F Value
Model	7	2662781.78	380397.35	1186.51
Error	401	128240.39	320.60	
Total	407	2791021.87		

Prob(F) = 0.0001
R-square = 0.954

Individual Effects

Source	DF	ANOVA SS	F Value	PR > F
Information (I)	1	5406.85	16.86	0.0001
Price Discrimination (PD)	1	2017157.34	6291.80	0.0001
Decision-Making Level(DM)	1	552596.16	1718.93	0.0001
I*PD	1	27.39	0.09	0.7705
I*DM	1	3789.56	11.79	0.0007
PD*DM	1	83132.46	258.60	0.0001
I*PD*PM	1	671.70	2.10	0.1486

Table VIII-15

Outcome Means at Different Levels of the Independent Variables

Main Effects

Information	
None	709.36
Perfect	716.64

Decision Making	
Brand	676.20
Category	749.80

Ability to Price Discriminate	
No	642.69
Yes	783.32

Two-Way Interactions

Decision Making	Information: None	Information: Perfect
Brand	669.51	682.89
Category	749.21	750.40

Price Discrimination	Information: None	Information: Perfect
No	638.79	646.59
Yes	779.94	786.70

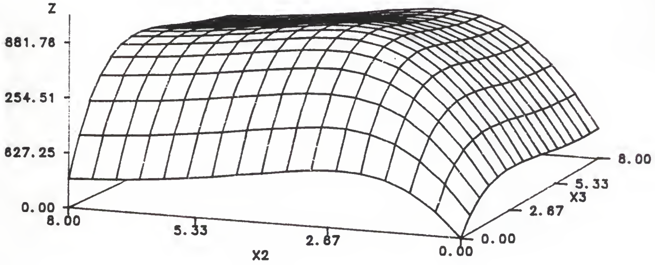
Decision Making	Ability to Price Discriminate: No	Ability to Price Discriminate: Yes
Brand	591.61	760.79
Category	693.77	805.84

Three-Way Interaction

Information	Decision Making	Ability to Price Discriminate: No	Ability to Price Discriminate: Yes
None	Brand	583.38	755.64
	Category	694.20	804.23
Perfect	Brand	599.84	765.93
	Category	693.38	807.46

Figure VIII-1

Joint Sales of 22-Ounce and 32-Ounce Ivory Liquid



Z = Total Sales of Both 22-Ounce and 32-Ounce Ivory

X2 = Per-Ounce Price of 22-Ounce Ivory

X3 = Per-Ounce Price of 32-Ounce Ivory

the average weekly sales for P&G items was $676.20. Under category management objectives, the average weekly sales increased to $749.80, or an increase of 11%. Thus, the result of the analysis confirm that coordinated or category management pricing has a appreciable advantage over brand management pricing as viewed from the perspective of the firm.

While a independent brand management system may offer some benefits to the firm in terms of managerial motivation and knowledge, the results here suggest that the bottom-line costs of such a system are potentially high. These results are consisted with the results of Zenor (1988) who uses a simulation approach to demonstrate a strong negative impact of brand management on total firm profits. To be certain, these results do not in and of themselves constitute a conclusive indictment of brand management decision making, but they demonstrate how the SLM model can provide a metric by which the costs of brand management can be compared to its benefits.

The ability to price discriminate exhibited the largest effect of any variable on total firm sales. When no price discrimination was allowed, the average weekly sales for all P&G items was $638.74; when price discrimination was allowed, average weekly sales increased to $779.94, or an increase of 22%. The large observed effect of price discrimination ability should be interpreted with some caution, since the optimal prices were derived assuming that the firm could perfectly isolate and charge different prices to each segment. Clearly, this is beyond the ability of even the most astute marketer. However, the optimal within-segment prices and demographic profiles can be employed

in targeted media selection, such that the offered discounts result in the least amount of wastage.

The dramatic observed effect of the ability to price discriminate underscores perhaps the biggest managerial advantages of the SLM. Typical models calibrated on simple store data (such as the SPF, CPF, and LMS) or simple unsegmented consumer panel models (such as the ULM) cannot address the issue of optimal pricing at the consumer segment level: at best, these models can only address the issue of differential optimal pricing across stores. The SLM, on the other hand, provides some guidance as to which groups of consumers should receive price discounts, as well as the optimal sizes of these offered discounts. Because of this ability, the SLM model can be of particular service in national media selection and promotional decisions, a feature not present in the other models investigated in this dissertation.

Turning to the interaction effects in the ANOVA table, the two-way interaction between information and price discrimination and the three-way interaction between all independent variables proved to be insignificant. Among the remaining interaction effects, a small but significant negative interaction was found between information and decision-making level. While the effect of information is positive at both levels of decision making, it has a smaller impact when decision making takes place at the category, rather than brand level. This may arise because under the brand management condition, each P&G brand manager considers each of the other P&G brands as competition. Information about the weekly pricing of all other brands, including competing P&G brands, is valuable to the to the decision maker. When category decision making is adopted, weekly pricing of all P&G brands is

known to the decision maker: the only residual uncertainty is about the weekly prices of competing non-P&G brands.

A much stronger negative interaction was observed between the ability to price discriminate and decision-making level. The effect of the ability to price discriminate was positive at both levels of decision making, but was less positive when pricing took place at the category level. One interpretation of this result is that directed price discounts are more effective under a brand management system than under a category management system. It is tempting to conclude based on the means for these interaction cells that price discrimination is an attractive alternative to category management. For example, the mean outcome in the no price discrimination/brand management cell is $591. In the no price discrimination/category management cell the mean outcome increases to $694, and in the price discrimination/brand management cell mean outcome increase further to $761. Based on these means, it appears that a firm using brand management pricing decisions would be better off to adopt price discrimination than to implement category management price decisions. However, the effect of price discrimination was derived here assuming that the firm could perfectly price discriminate. A practical implementation of a perfect price discrimination policy can be seen as nearly impossible for most firms. Category management decisions, on the other hand, can be realistically achieved by a number of mechanisms. The firm could adopt strategies such as assigning decisions to a category manager, assigning decisions to field salespeople (who presumably sell all the firm's brands), or by tying each brand manager's compensation and evaluation to their joint rather than individual

outcomes. Which one of these strategies is the most optimal is an intriguing question, but beyond the scope of this dissertation.

As a final note to the optimization analyses, the power of corporate managers to directly make retail pricing decisions is very much limited, as retailers are the final arbiters as to the offered price of any item. The corporate manager only indirectly affects the final retail price of the item, either by trade promotions (i.e., retailer discounts) or consumer promotions (i.e., coupons). In this sense, the optimizations reported in this chapter might be seen as incomplete because they neglect the intervening influence of the retailer in pricing decisions. Presumably, however, the firm has some knowledge of the size of trade promotions and discounts necessary to obtain a particular reduction in retail price. This knowledge could be integrated into the firm objective function to obtain optimal sizes of trade promotions and discounts. Unfortunately, such trade promotion data were not available.

CHAPTER IX

CONCLUSIONS

The segmented logit model presented in this dissertation has hopefully proved to be a step forward in normative and descriptive marketing models, both with respect to conceptual development and managerial utility. In terms of conceptual development the model has drawn on, and hopefully helped unite, three disparate areas in the marketing literature: market structuring models, demand estimation models, and resource allocation models.

With regard to market structuring, the approach used here provides a less ambiguous interpretation of "structure" than do most available methods. For example, Kalwani and Morrison (1977), Rao and Sabavala (1981), Grover and Srinivasan (1988), and Hutchinson and Zenor (1987) all derived aggregate market structure representation using observed switching data and different clustering methods. These representations, while providing some insights into competitive relationships, are very difficult to interpret meaningfully. For example, Rao and Sabavala characterize a derived hierarchical brand structure as representing the choice process of (presumably homogeneous) consumers. Another interpretation of this structure is that it represents the choice _sets_ of a heterogeneous population of consumers. A third interpretation is that the structure is a mixture of both choice sets and choice processes. Unfortunately, since none of these

interpretations are tested, it is difficult to determine which is most valid. The problem of structure interpretation is present in all of the above models because all utilize only the aggregated switching matrix.

The model in this dissertation, on the other hand, goes one step further by directly assigning individual households to identified partitions based on the households' observed purchase sets. Because IIA assumptions were non-rejectable at the segment level, there is greater reason to believe that the identified structure represents choice sets rather than choice processes.

It should be noted that certain methods, such as k-means clustering, jointly identify structure and assign households to segments. However, direct clustering on households into segments, and the subsequent interpretation of these segments as representing consideration sets, is only advisable if a large number of purchases is made by each household. The vagarities of the data used in this dissertation prevented such an approach; in particular, only a small number of purchases were made by each household. If the purchase rate for the category had been very high, then the intervening step of aggregate structuring may not have been necessary.

With regard to demand estimation, the SLM model stakes out a middle ground between models assuming a completely homogeneous population and individual choice models, with very good results. The predictive validity test results of Chapter VI indicated that the SLM outperformed a homogeneous panel choice model and that this improvement could not be attributed solely to the difference in number of parameters. The SLM also compared quite favorably with the best store

model, even though the SLM was subject to sampling error not present in the store model estimation.

One might be tempted to point out that the SLM was not compared to a true individual-level choice model. However, the small number of choice observations per household (average < 7) renders model estimation nearly impossible for most households. Another type of choice model not examined here is purchase-event feedback logit models such as the models of Guadagni and Little (1983) and Jones and Landwehr (1988). In these models, heterogeneity of choice parameters is relaxed by modelling choice probabilities conditioned on last purchase. While these models have some commendable features, they do not direct identify market structure. Further, purchase event feedback models are of limited use in prediction, as they require knowledge (at the household level) of previous purchase to make choice predictions.

In Chapter VII, within-segment elasticity results indicated a positive relationship between consideration set size and elasticity. This result is consistent with a priori expectations about the behavior of segments with varying consideration set sizes.

Using the SLM parameters, the optimal price values obtained in Chapter VIII provided some interesting conclusions regarding decision-making structure within the firm. First, it was found that category management decision making results in significantly greater firm outcomes compared to brand-management decision making. Second, it was found that knowledge about competitor prices has a large impact on optimal price levels, but a surprisingly small impact on total firm outcomes. Third, the ability to price discrimination was found to have a very large impact on total firm outcomes. These conclusions, while not

all surprising, will hopefully contribute to the knowledge base of managerial marketing.

Finally, with respect to managerial usefulness, the segmented logit model provides a wealth of information for the decision maker. First, the identification of structure and the assignment of households to consideration set segments provides the decision maker a relatively unambiguous snapshot of the distribution of consideration sets across the population. Such information reveals the competitive relationships between brands, both between and within product lines, as well as the customers for whom brands are competing. The identification of structure and segment membership also provides valuable guidance as to product line extension and repositioning decisions. For instance, the decision maker may want to develop a new brand or reposition an existing brand such that the degree of cannibalization among the product line is minimized. The distribution of consideration sets offers clues as to whom repositioning efforts should be addressed. With additional panel data, the decision maker could potentially track structural changes over time to gauge whether repositioning efforts had the desired effect.

Beyond the simple identification of structure, the SLM uses segment membership information to estimate disaggregate (segment-level) demand models. The demand estimation phase of the SLM allows the manager not only to identify who belongs to each segment, but how sensitive these segments are to marketing mix variables. In the optimization analyses of Chapter VIII, it was shown how these disaggregate models can be used in finding optimal values of decision variables across a product line with interrelated demand, both marginally and within segment. In short, the SLM provides a useful framework for evaluating product line

decisions, as it allows the decision maker to identify the specific loci of cannibalization and a mechanism for optimally coping with it through resource allocation.

As a concluding note, a great deal of work is left to be done in the areas of product line decision models, disaggregate demand models, and market structuring methods. it is hoped that the model presented in this paper will provoke additional efforts in this direction.

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BIOGRAPHICAL SKETCH

I was born February 24, 1959, in Sioux City, Iowa, and grew up on my family's farm near Lawton, Iowa. I attended Lawton-Bronson Community Schools, graduating in May of 1977. I matriculated at the University of Iowa, and received a baccalaureate degree in Business Administration in December 1980. Soon after, I returned to the university to pursue a Master of Business Administration degree. As a graduate student, I had the good fortune of becoming a research assistant to Dr. Jordan Louviere, whose work stimulated my interest in pursuing an academic career in marketing. I received my degree in August 1984, shortly after my marriage to Kim Henning.

I moved to Gainesville, Florida, later that month to pursue a doctoral degree in marketing at the University of Florida. In my four year stay at the university, I was able to benefit from the extraordinary knowledge and capability of the marketing faculty.

I left the University of Florida in September 1988 to join the Marketing Department at the University of Texas as an Assistant Professor, where I hope to establish a long and productive career.

I certify that I have read this study and that in my opinion it conforms to the acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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This dissertation was submitted to the Graduate Faculty of the Department of Marketing of the College of Business Administration and to the Graduate School and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

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